Quantum logic & probability: Formal solutions to quantum paradoxes?

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Axiomatic thinking

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- 3 Logic, set theory, probability
- 4 Foundations



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Abstract

For almost a century, impressive efforts have been devoted the foundational problems of quantum mechanics. The efforts have been made in different directions, giving rise to the formation of various foundational sectors of research. Each sector has developed its own jargon, idiosyncrasies, and especially certainties: devout votaries have even been known to insist that their approach, unlike the others, 'solves all the problems of quantum mechanics.' What is true is that the various approaches are not pointless, each offering its own perspective, which can shed light from a particular angle. Quantum logic and quantum probability in their effort to provide formal solutions of quantum paradoxes, have been represented as axiomatisations in the spirit of Hilbert's programme. We can consider them as such, to assess their contributions to the logical clarification of crucial questions in the foundational debate.





2 Introduction

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- a handful are often singled out as being 'logically primitive' and called *axioms*
- but there is much freedom in the choice of axioms (the very notion of 'logically upstream' being somewhat arbitrary)
- Quine aptly wondered "which points of Ohio are starting points?"

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Quantum peculiarities

complex-valued wavefunctions (in configuration space)

- minimum quantities related to Planck's h
- incompatible observables
- failure of distributivity
- trouble with joint distributions

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- quantum logicians emphasise the failure of distributivity, Rovelli the Planck-sized cell, quantum probablists the trouble with joint distributions and so on

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From classical to quantum

- here we'll start with Boolean algebras and Kolmogorovian probability (with its joint distributions), and generalise with departures from commutativity, distributivity *etc*.
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Boolean algebras

- set theory and propositional logic are structurally similar, being both Boolean algebras
- we can begin with a (denumerable) set Ω on which we take unions, intersections and complements
- if we think of Ω as an orthonormal basis $\{\Omega_k\}$, operators

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(uncountable measure spaces)

separable Hilbert spaces are typically used in quantum mechanics

- a space L₂(Ω, C) of square-integrable functions can be separable even if the configuration space Ω is uncountable
- appropriate real-valued functions on Ω give self-adjoint operators with 'eigenset' Ω (in the sense that they're multiplication operators on Ω, 'diagonal' on Ω)
- even if such an operator has no eigenvectors $|\Omega_k\rangle \in \mathscr{L}_2(\Omega, \mathbb{C})$, there are the projectors χ_{Δ} , where Δ is a (measurable) subset of Ω

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(uncountable measure spaces)

- separable Hilbert spaces are typically used in quantum mechanics
- a space L₂(Ω, C) of square-integrable functions can be separable even if the configuration space Ω is uncountable
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Four Boolean algebras

■ in fact there are four equivalent Boolean algebras

- 1 the sets $\Delta \subset \Omega$
- 2 the corresponding propositions
- 3 the characteristic functions χ_{Δ} equal to one on Δ and zero elsewhere
- 4 the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

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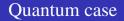
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- if we now allow the basis {Ω_k} to rotate, we obtain a similar classical, Kolmogorovian scheme for every {Ω'_k}, {Ω''_k},...
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2 Introduction

3 Logic, set theory, probability

4 Foundations

5 Hilbert

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• one can wonder about the exact nature of quantum logic & probability

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- or are they downright *ontic*?
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but if the formalism were complete, with hidden variables ruled out, quantum logic & probability would assume a more *ontic* character—there would be no underlying, empirically inaccessible, Boolean logic and Kolmogorovian probability

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Joint distributions

• the trouble is that the spectra of operators diagonal on Ω label subsets of a common set

• one operator just refines the partition of the other

- a joint distribution corresponds to a finer partition
- but with incompatible operators one has to choose—each spectrum now labels subspaces of a common space V, which changes everything

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6. Mathematische Behandlung der Axiome der Physik

Durch die Untersuchungen über die Grundlagen der Geometrie wird uns die Aufgabe nahegelegt, nach diesem Vorbilde diejenigen physikalischen Disciplinen axiomatisch zu behandeln, in denen schon heute die Mathematik eine hervorragende Rolle spielt; dies sind in erster Linie die Wahrscheinlichkeitsrechnung und die Mechanik.

The solution of philosophical problems

- it has often been suggested that philosophers are just sloppy and confused: clear and distinct formulation is enough to solve all their problems
 - no more than appropriate axiomatisation, formalisation would be needed to deal with the difficulties of quantum mechanics, for instance

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Regressus in infinitum

just as the *primum mobile* is invoked to cut off a *causal* regress which would otherwise be infinite, axioms are used to cut off a similar *logical* regress

- if all we're interested in is probability, for instance, why bother regressing all the way back to the fundamentals of real analysis, set theory *etc.*; why not take all that for granted, and start with what really characterizes probability theory?
- but there's a difference between *taking for granted* and *wiping out*; once we've axiomatised, nothing prevents us from looking back beyond the axioms to consider what's been assumed and why

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Incompatible observables

we can always choose to blame the failure of

- distributivity (quantum logic)
- joint distributions (quantum probability)

on incompatible observables

- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

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