CVA
WORKSHOP

December 17–19, 2015

INTERNATIONAL WORKSHOP ON

CALCULUS OF VARIATIONS AND ITS APPLICATIONS

ON THE OCCASION OF LUÍSA MASCARENHAS' 65TH BIRTHDAY

17-19 December, 2015 | Universidade Nova de Lisboa, Caparica, Portugal

WELCOME

The Organizing Organizing Committee would like to thank all the participants for their interest in attending the *International Workshop on Calculus of Variations and its Applications* on the occasion of Luísa Mascarenhas' 65th birthday.

VENUE

The workshop will take place in the Main Library of the Faculty of Science and Technology (FCT) of the New (NOVA) University of Lisbon (Campus of Caparica).

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The Organizing Committee would like to thank all our sponsors for the generous support allowing us to organize this event:
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MAPA DO CAMPUS *CAMPUS MAP*

1. Main Entrance
2. Bus Stops
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4. Department of Mathematics (E. VII)
5. Library
6. ATM Machines:
   - 6a. In building E. I
   - 6b. In building E. VII
   - 6c. In CGD's building
7. Bank (*CGD*)
8. Minimarket (*MiniNova*)
9. Food Services:
   - 9a. In Biblioteca's building
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   - 9c. In building E. VII
   - 9d. In Cantina's building (*C@mpus.come*, *My Spot Bar*, and *Snack Bar*)
   - 9e. In building E. Dep (*Espaço Mais*)

Abstracts

&

Schedule

Invited Talks

G. Allaire $\int_{15:15-15:55}^{\text{December 17, 2015}}$

Robust shape optimization with small uncertainties

Grégoire ALLAIRE
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Abstract: In this talk we propose two approaches for dealing with small uncertainties in geometry and topology optimization of structures. Uncertainties occur in the loadings, the material properties, the geometry or the imposed vibration frequency. A first approach, in a worst-case scenario, amounts to linearize the considered cost function with respect to the uncertain parameters, then to consider the supremum function of the obtained linear approximation, which can be rewritten as a more 'classical' function of the design, owing to standard adjoint techniques from optimal control theory. The resulting 'linearized worst-case' objective function turns out to be the sum of the initial cost function and of a norm of an adjoint state function, which is dual with respect to the considered norm over perturbations.

A second approach considers objective functions which are mean values, variances or failure probabilities of standard cost functions under random uncertainties. By assuming that the uncertainties are small and generated by a finite number N of random variables, and using first- or second-order Taylor expansions, we propose a deterministic approach to optimize approximate objective functions. The computational cost is similar to that of a multiple load problems where the number of loads is N.

We demonstrate the effectiveness of both approaches on various parametric and geometric optimization problems for elastic structures in two space dimensions.

The talk is based on joint work with Charles Dapogny (LJK, Grenoble).

References:

[1] G. Allaire, Ch. Dapogny: A linearized approach to worst-case design in parametric and geometric shape optimization, *M3AS* Vol. 24, No. 11 (2014) 2199-2257. HAL preprint: hal-00918896, version 1 (December 2013).

[2] G. Allaire, Ch. Dapogny, A deterministic approximation method in shape optimization under random uncertainties, *submitted to SMAI J. Comp. Math.* HAL preprint: hal-01160256v1 (June 2015).

H. Beirão da Veiga $\int_{15:15-15:55}^{\text{December 18, 2015}}$

On some regularity results for elliptic boundary value problems

Hugo BEIRÃO da VEIGA
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Abstract: About thirty years ago, in reference "On the solutions in the large of the two-dimensional flow of a nonviscous incompressible fluid", J. Diff. Eq., vol.54, 1984, we looked for "minimal assumptions" on the data which guarantee that solutions to the two dimensional evolution Euler equations in a bounded domain are classical. Classical means here that all the derivatives appearing in the equations and boundary conditions are continuous up to the boundary. Following a well known device, the above problem led to consider this same regularity problem for the Poisson equation under homogeneous Dirichlet boundary conditions. At this point, one was naturally led to consider the extension of this last problem to more general linear elliptic boundary value problems, and also to try to extend the results to more general data spaces. This leads to new problems and functional spaces. We want to describe the route followed by us in studying this kind of problems.

J.-M. Bismut $\int_{12:20-13:00}^{\text{December 19, 2015}}$

Hypoelliptic Laplacian and the calculus of variations

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Abstract: If X is a compact Riemannian manifold, the hypoelliptic Laplacian $L_b|_{b>0}$ is a family of operators acting on the total space $\mathcal{X}$ of the tangent bundle TX, that interpolates between the Laplace-Beltrami operator and the geodesic flow. The associated diffusion is a geometric Langevin process on X, whose dynamics interpolates between Brownian motion and the geodesic flow.

This interpolation preserves certain spectral quantities. On very rigid manifolds like tori or on locally symmetric spaces, the spectrum of the original elliptic Laplacian remains rigidly embedded in the spectrum of the hypoelliptic deformation. Generalized Poisson formulas like Selberg's trace formula can be obtained as a consequence of this interpolation.

In the lecture, I will explain the construction of the hypoelliptic Laplacian, and emphasize the role of the calculus of variations in the construction. The spectral properties of the hypoelliptic deformation will be described.

G. Bouchitté

$\int_{14:30-15:10}^{\text{December 17, 2015}}$

A new class of costs in optimal transport planning

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ABSTRACT: In optimal mass transport theory, many problems can be written in the Monge-Kantorovich form

$$\inf\{\int_{X\times Y} c(x,y)\,d\gamma \;:\; \gamma\in\Pi(\mu,\nu)\}\;, \tag{1}$$

where μ,ν are given probability measures on X,Y and $c: X\times Y\to[0,+\infty[$ is a cost function. Here the competitors are probability measures γ on $X\times Y$ with marginals μ and ν respectively (transport plans). Let us recall that if an optimal transport plan $\gamma\in\Pi(\mu,\nu)$ is carried by the graph of a map $T: X\to Y$ i.e. if

$$<\gamma,\varphi(x,y)>=\int_X \varphi(x,Tx)\,d\mu \quad , \quad T^\sharp\mu=\nu\;,$$

then T solves the original Monge problem: $\inf\{\int_X c(x,Tx)\,d\mu \;:\; T^\sharp\mu=\nu\}$.

Here we are interested in a different case. Indeed in some applications to economy or in probability theory, it can be interesting to favour optimal plans which are non associated to a single valued transport map $T(x)$. The idea is then to consider, instead of $T(x)$, the family of conditional probabilities γ^x such that

$$<\gamma,\varphi(x,y)>=\int_X(\int_X \varphi(x,y)d\gamma^x(y))\,d\mu\;,$$

and to incorporate in problem (1) an additional cost over γ^x as follows

$$\inf\left\{\int_{X\times X} c(x,y)\,d\gamma+\int_X G(x,\gamma^x)\,d\mu \;:\; \gamma\in\Pi(\mu,\nu)\right\}\;, \tag{2}$$

being $G:(x,p)\in X\times\mathcal{P}(X)\to[0,+\infty]$ a given non linear function.

In this talk I will describe some recent results concerning problem (2) (existence, duality principle, optimality conditions) and focus on specific examples where $X=Y$ and X is a convex compact subset of $\mathbb{R}^d$.

This is a joined work with Thierry Champion and J.J. Alibert (University of Toulon).

G. Buttazzo

$\int_{09:45-10:25}^{\text{December 18, 2015}}$

Dirichlet-Neumann shape optimization problems

GIUSEPPE BUTTAZZO
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ABSTRACT: We consider spectral optimization problems of the form

$$\min\Big\{\lambda_1(\Omega;D):\;\Omega\subset D,\;|\Omega|=1\Big\},$$

where D is a given subset of the Euclidean space $\mathbb{R}^d$. Here $\lambda_1(\Omega; D)$ is the first eigenvalue of the Laplace operator $-\Delta$ with Dirichlet conditions on $\partial\Omega \cap D$ and Neumann or Robin conditions on $\partial\Omega \cap \partial D$. The equivalent variational formulation

$$\lambda_1(\Omega; D) = \min\Big\{ \int_\Omega |\nabla u|^2\,dx + k\int_{\partial D} u^2\,d\mathcal{H}^{d-1}\ : \\ u \in H^1(D),\ u = 0 \text{ on } \partial\Omega \cap D,\ \|u\|_{L^2(\Omega)} = 1\Big\}$$

reminds the classical drop problems, where the first eigenvalue replaces the total variation functional. We prove an existence result for general shape cost functionals and we show some qualitative properties of the optimal domains. The case of Dirichlet condition on a *fixed* part and of Neumann condition on the *free* part of the boundary is also considered.

References:

[1] D. Bucur, G. Buttazzo, B. Velichkov: *paper in preparation.*

D. Cioranescu December 18, 2015 10:50−11:30

Homogenization for inner boundary conditions with equi-valued surfaces via unfolding

Doina CIORANESCU

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Abstract: We consider two examples of the general setting of equi-valued surfaces with corresponding assigned total fluxes.

Problem 1. [1] *The linear elastic torsion of an infinite three dimensional rod* with a multiply-connected two-dimensional cross-section Ω^* obtained from a bounded open set $\Omega \subset \mathbb{R}^2$ perforated by a finite number of regular closed subsets, $S^1, S^2, \ldots$.

Problem 2. [4] *The electro-conductivity problem* in presence of isolated perfect conductors (arising in resistivity well-logging), set in any dimension with the same type of geometry.

In both situations one looks for $\varphi \in H_0^1(\Omega)$ with $\varphi_{|S^j}$ unknown constant for each j, satisfying

$$-\mathrm{div}(A(x)\nabla\varphi(x)) = f$$

in Ω^* and the corresponding boundary conditions,

$$\text{Problem 1} \ \int_{\partial S^j} \frac{\partial\varphi}{\partial n} d\sigma(x) = |S^j|, \ \ \text{Problem 2} \ \int_{\partial S^j} \frac{\partial\varphi}{\partial \nu_A} d\sigma(x) = g^j, \ \ g^j \text{ given numbers.}$$

We are interested in the periodic homogenization by the unfolding method ([5]) of these two problems. We refer to [2] and [3] for the first proof for the elastic torsion problem (via extension operators and oscillating test functions), where regularity assumptions are made for the boundary of the inclusions.

One of the advantages of the unfolding method is that it requires no regularity for the boundary of the inclusions. Another one is that a corrector result, completely general, is obtained as an immediate consequence.

The talk is based on joint work [6] with Alain Damlamian (Université Paris-Est, Créteil) and Li Tatsien (Fudan University, Shanghai).

References:

[1] H. Lanchon, Torsion élastoplastique d'un arbre cylindrique de section simplement ou multiplement connexe, *J. Mécanique* , 13 (1974), 267-320.

[2] L. Tartar, Problèmes d'homogénéisation dans des équations aux dérivées partielles. *Cours Peccot, Collège de France*,1977.

[3] D. Cioranescu and J. Saint Jean Paulin: Homogenization in open sets with holes, *J. Mathematical Analysis and Applications*, 71 (1979), 590-607.

[4] Li Tatsien, Zheng Songmu, Tan Yongji and Shen Weixi: *Boundary value problems with equi-valued surface and resistivity well-logging.* Longman, 1998.

[5] D. Cioranescu, A. Damlamian and G. Griso, The periodic unfolding method in homogenization, *SIAM J. of Math. Anal.*, 40 (2008), 1585-1620.

[6] D. Cioranescu, A. Damlamian and Li Tatsien: Periodic homogenization for inner boundary conditions with equi-valued surfaces:the unfolding approach. *Chin. Ann. Math.*, 34 (2013), 213-236.

C. Conca $\int_{09:45-10:25}^{\text{December 19, 2015}}$

Detection of a moving rigid solid in a perfect fluid

CARLOS CONCA

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ABSTRACT: In this lecture we will consider a moving rigid solid immersed in a potential fluid. The fluid-solid system fills the whole two-dimensional space and the fluid is assumed to be at rest at infinity. Our aim is to study the inverse problem that consists in recovering the position and the velocity of the solid assuming that the potential function is known at a given time.

We will see that this problem is in general ill-posed by providing counterexamples for which the same potential corresponds to different positions and velocities of a same solid. However, it is also possible to find solids having a specific shape, such as ellipses for instance, for which the problem of detection admits a unique solution.

Using complex analysis, we prove that the well-posedness of the inverse problem is equivalent to the solvability of an infinite set of nonlinear equations. This result allows us to show that when the solid enjoys some symmetry properties, it can be partially detected. Besides, for any solid, the velocity can always be recovered when both the potential function and the position are supposed to be known.

The talk is based on joint work with co-authors Muslim MALIK and Alexandre MUNNIER.

J.-P. Dias $\int_{14:30-15:10}^{\text{December 18, 2015}}$

Coupled nonlinear Schrödinger equations with a gauge potential

JOÃO-PAULO DIAS

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ABSTRACT: We study the solutions of the Cauchy problem for nonlinear Schrödinger system in R^3 with nonlinear coupling and linear coupling modeling synthetic magnetic field in spin-orbit coupled Bose-Einstein condensates. Three main results are presented: a proof of the local existence, a proof of the

sufficient condition for the blowup result in finite time for some solutions, and the persistence of the nonlinear dynamics in the limit where the spin-orbit coupling converges to zero. This talk is based in a joint paper with Mário Figueira (CMAF-CIO and DM/FCUL) and Vladimir V. Konotop (CFTC and DF/FCUL), submitted for publication, with title: *Coupled nonlinear Schrödinger equations with a gauge potential: existence and blowup.*

P. Donato $\int_{11:35-12:15}^{\text{December 19, 2015}}$

Asymptotic Analysis of the Approximate Control for Parabolic Equations with Periodic Interface

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Abstract: We present here some results concerning the asymptotic behavior of the approximate control for some parabolic equations with periodic rapidly oscillating coefficients in a two-component composite with a periodic interfacial resistance.

We first show the approximate controllability of the problem, as well as that of the homogenized one, which is more difficult to study since, as proved in]4], it is a coupled system P.D.E.-O.D.E. Following an idea introduced by J.-L. Lions in [5], for each problem the approximate control is constructed as the solution of related transposed problem having as final data the (unique) minimum point of a suitable functional. In the second part of the talk, we show that the control and the corresponding solution of the periodic problem converge respectively to the control and to the solution of the homogenized problem. The corrector results given in [1] play an important role in the proofs.

The approximate controllability behaviour of a linear parabolic equations with rapidly oscillating coefficients was studied by E. Zuazua in [6] for a fixed domain with Dirichlet conditions, and in [3] for periodically perforated domains with Neumann conditions.

One of the main difficulties here is the construction of the appropriate functionals in order to obtain non only the approximate controllability, but also the convergence results.

The talk is based on the joint work with Editha C. Jose (UP-Los Baños) appeared in [2].

References:

[1] P. Donato and E. Jose: Corrector Results for a Parabolic Problem with a Memory Effect, *ESAIM: M2AN*, 44 (2010), 421-454.

[2] P. Donato and E.C. Jose: Asymptotic behavior of the approximate controls for parabolic equations with interfacial contact resistance, *ESAIM: Control, Optimisation and Calculus of Variations*, 21 (1), (2015), 138-164, DOI 10.1051/cocv/2014029.

[3] P. Donato and A. Nabil: Approximate Controllability of Linear Parabolic Equations in Perforated Domains, *ESAIM: COCV*, 6 (2001), 21-38.

[4] E. Jose: Homogenization of a Parabolic Problem with an Imperfect Interface, *Rev. Roumaine Math. Pures Appl.*, 54 (2009) (3), 189-222.

[5] J.-L. Lions: Remarques sur la controlabilité approchée, in *Jornadas Hispano-Francesas sobre Control de Sistemas Distribuidos, octubre 1990, Grupo de Análisis Matemático Aplicado de la University of Malaga*, 77-87, 1991.

[6] E. Zuazua: Approximate Controllability for Linear Parabolic Equations with Rapidly Oscillating Coefficients, *Control Cybernet.*, 4 (1994), 793-801.

I. Fonseca December 19, 2015 17:30−18:10

A Chromaticity-Brightness Model for Denoising of Color Images

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Abstract: A variational model for imaging denoising aimed at restoring color images is proposed. The model combines Meyers $u + v$ decomposition with a chromaticity-brightness framework, and is expressed in terms of a minimization of energy integral functionals depending on a small parameter $\varepsilon > 0$. The asymptotic behavior as $\varepsilon \to 0^+$ is characterized, and convergence of infima, almost minimizers, and energies are established. In particular, an integral representation of the lower semicontinuous envelope, with respect to the L^1-norm, of functionals with linear growth and defined for maps taking values on a certain manifold is provided.

I. Fragalà December 17, 2015 12:20−13:00

Boundary value problems for the infinity Laplacian: regularity and geometric results

Ilaria FRAGALÀ
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Abstract: We discuss boundary value problems where the operator is the infinity Laplacian, or its normalized version. In particular, we focus our attention on the homogeneous Dirichlet problem with constant source term, and on a related Serrin-type overdetemined problem. For the unique solution to the Dirichlet problem, we give a power-concavity result, valid on bounded convex subsets of $\mathbb{R}^n$. It is obtained by the convex envelope method introduced by Alvarez, Lasry and Lions, and yields as a consequence the C^1 regularity of the solution.

For the Serrin-type problem, we provide a complete characterization of convex sets where a solution exists, as "stadium-like domains". This result is closely related to a purely geometric topic, which is the characterization of sets with positive reach and empty interior in $\mathbb{R}^n$.

The talk is based on some recent joint works with Graziano CRASTA (Università di Roma "La Sapienza").

References:

[1] G. Crasta, I. Fragalà: Characterization of stadium-like domains via boundary value problems for the infinity Laplacian, preprint (2015).

[2] G. Crasta, I. Fragalà: A C^1 regularity result for the inhomogeneous normalized infinity Laplacian, to appear on *Proc. Amer. Math. Soc.*

[3] G. Crasta, I. Fragalà: On the Dirichlet and Serrin problems for the inhomogeneous infinity Laplacian in convex domains: Regularity and geometric results, *Arch. Rat. Mech. Anal.* (2015)

[4] G. Crasta, I. Fragalà: On the characterization of some classes of proximally smooth sets, *ESAIM: Control Optim. Calc. Var.* (2015)

[5] G. Crasta, I. Fragalà: A symmetry problem for the infinity Laplacian, *Int. Mat. Res. Not. IMRN* (2014)

G. Francfort $\int_{16{:}00-16{:}40}^{\text{December 17, 2015}}$

Periodic and stochastic homogenization in linear elasticity

Gilles FRANCFORT
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Abstract: In 1993, Giuseppe Geymonat, Stefan Müller and Nicolas Triantafyllidis demonstrated that, in the setting of linearized elasticity, a Γ-convergence result holds for highly oscillating sequences of elastic energies whose functional coercivity constant over the whole space is zero while the corresponding coercivity constant on the torus remains positive. We find sufficient conditions for such a situation to occur through a rigorous revisiting of a laminate construction given by Gutiérrez in 1999. We further demonstrate that isotropy prohibits such an occurrence.

The results apply to both the periodic and the stochastic setting. They were obtained in part with Marc Briane (Rennes), and in part with Antoine Gloria (Brussels) and Scott Armstrong (Paris).

A. Gaudiello $\int_{11{:}35-12{:}15}^{\text{December 18, 2015}}$

Homogenization of the brush problem with a source term in L^1

Antonio GAUDIELLO, Olivier GUIBÉ, François MURAT
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Abstract: We consider a brush composed of cylindrical vertical teeth distributed over a fixed basis. All the teeth have a similar fixed height and a cross section of diameter less than a small parameter ε, and their asymptotic density is bounded from below away from zero. Let us note that the diameters of the teeth can be of different orders, and that their distribution is not assumed to satisfy any type of periodicity. In this domain we study the asymptotic behavior, as ε tends to zero, of a second order elliptic equation with a zeroth order term which is bounded from below away from zero, with homogeneous Neumann boundary condition, and with an L^1 source term. Working in the framework of renormalized solutions, and introducing a notion of renormalized solution for some type of degenerated elliptic equations, we identify the limit problem and prove a corrector result.

D. Kinderlehrer $\int_{09{:}00-09{:}40}^{\text{December 19, 2015}}$

Coarsening in microstructure and the mass transport paradigm

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Abstract: During the coarsening process, in a typical polycrystalline material, some cells or grains grow while others contract or are deleted. Historically many statistical descriptions of the process have been studied emphasizing, primarily, geometric properties such as cell size and number of facets of cells. Although very useful, these are not robust. During coarsening an orientational texture, characterized by the Grain Boundary Character Distribution (GBCD), also emerges. We discuss this from the viewpoint of a gradient flow for a Wasserstein metric and illustrate its entropic nature. We shall also attempt to give some attention to other gradient flows found 'in the wild.' This is joint work with P. Bardsley, K. Barmak, E. Eggeling, M. Emelianenko, Y. Epshteyn, D. Kinderlehrer, X.Y. Lu, R. Sharp, and S. Ta'asan.

G. Leoni $\int_{14{:}30-15{:}10}^{\text{December 19, 2015}}$

Asymptotic Development by Gamma Convergence

Giovanni LEONI
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Abstract: In this talk we present some recent results on the asympotic development by Γ convergence of order two of the Cahn–Hilliard functional with mass constraint. We also give some applications to slow motion for the Cahn-Hilliard equation and the nonlocal Allen–Cahn equation.

The talk is based on joint work with G. Dal Maso, I. Fonseca, and R.Murray.

References:

[1] G. Dal Maso, I. Fonseca, and G. Leoni: Second order asymptotic development for the anisotropic Cahn-Hilliard functional. *Calculus of Variations and Partial Differential Equations*, to appear.

[2] G.Leoni and R. Murray: Second-order Γ-limit for the Cahn–Hilliard functional, *Archive for Rational Mechanics and Analysis*, to appear.

[3] R. Murray and M. Rinaldi: Slow Motion for the nonlocal Allen–Cahn equation in n-dimensions. In preparation.

P. Oliveira $\int_{12:20-13:00}^{\text{December 18, 2015}}$

Viscoelastic biodegradable materials: modelling, mathematical analysis and medical applications

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Abstract: Viscoelastic properties of materials represent a compromise between viscous and elastic responses, under mechanical stress. Biodegradation is the erosion of materials by the action of biological processes that cause a progressive breakdown of the material. The degradation and the unique viscoelastic properties of polymers give them a central role in controlled drug delivery to provide sustained release of therapeutic agents while avoiding removal surgery.

The release of drug is governed by an instantaneous swelling, a nonlinear diffusion, a stress driven convection and a decrease of the polymer molecular weight. The interaction of these phenomena is represented by a system of partial integro-differential equations, coupled with initial and boundary conditions. The qualitative properties of the solution are studied and the stability of the system is analyzed.

Medical applications are addressed. The evolution of the concentration of drug, released from a biodegradable viscoelastic implant inserted into an unhealthy eye, is presented. Numerical simulations illustrate how to tune polymeric material properties that give rise to predefined release profiles.

The talk is based on joint work with E. Azhdari, J. Ferreira and P. Silva.

A. Piatnitski $\int_{09:00-09:40}^{\text{December 18, 2015}}$

Ground state asymptotics for a singularly perturbed convection-diffusion operator in a thin cylinder

Andrey PIATNITSKI
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Abstract: The talk focuses on the limit behaviour of a non-trivial solution of the equation

$$-\varepsilon \mathrm{div}\Big(a\Big(\frac{x}{\varepsilon}\Big)\nabla u^\varepsilon(x)\Big) - \mathrm{div}\Big(b\Big(\frac{x}{\varepsilon}\Big)u^\varepsilon(x)\Big) = 0 \quad \text{in } G_\varepsilon,$$
$$\varepsilon a\Big(\frac{x}{\varepsilon}\Big)\nabla u^\varepsilon \cdot \nu^\varepsilon + b\Big(\frac{x}{\varepsilon}\Big)\cdot \nu^\varepsilon u^\varepsilon = 0 \quad \text{on } \partial G_\varepsilon$$

with a small positive parameter ε. Here G_ε is a thin cylinder in $\mathbb{R}^d$:

$$G_\varepsilon = (0,1)\times(\varepsilon Q),$$

Q is a smooth bounded domain in $\mathbb{R}^{d-1}$; ν is the unit exterior normal on ∂G_ε.

We assume that the coefficients $a_{ij} = a_{ij}(y)$ and $b_j = b_j(y)$ are periodic in y_1 and satisfy the uniform ellipticity conditions,

$$\Lambda^{-1}|\xi|^2 \le a(y)\xi\cdot\xi \le \Lambda|\xi|^2, \qquad \Lambda > 0,\ \xi\in\mathbb{R}^d,\ y \in G,$$

$a \in L^\infty(G;\mathbb{R}^{d^2})$, $b\in L^\infty(G;\mathbb{R}^d)$ with $G = (0,\infty)\times Q$.

Under these assumptions we study the limit behaviour of u^ε. We show that the leading terms of its asymptotic expansion take the form

$$u^\varepsilon(x) = p_\theta\left(\frac{x}{\varepsilon}\right)\exp\left(\frac{\theta x_1}{\varepsilon}\right) + u^{-,\varepsilon}(x) + u^{+,\varepsilon}(x),$$

where $\theta \in \mathbb{R}$, $p_\theta(y)$ is a periodic in y_1 positive function, and $u^{-,\varepsilon}(x)$ and $u^{+,\varepsilon}(x)$ are boundary layer functions such that

$$|u^{-,\varepsilon}(x)| \le C\exp\left(\frac{\theta_- x_1}{\varepsilon}\right), \quad |u^{+,\varepsilon}(x)| \le C\exp\left(\frac{\theta_+(1-x_1)}{\varepsilon}\right),$$

with $\theta_\pm < \min(0,\theta)$. The sign of θ depends on the sign of the so-called axial effective drift of the convection-diffusion operator

$$Av = -\mathrm{div}\big(a(y)\nabla v(y)\big) + b(y)\nabla v(y).$$

The talk is based on joint work with Gregoire Allaire.

A. Raoult $\int_{10:50-11:30}^{\text{December 17, 2015}}$

Hierarchy of pre-strained behaviors for thin materials

Marta LEWICKA, Annie RAOULT

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Abstract: A hierarchy of four models for thin structures was first obtained by means of a systematic, but formal, approach in [5]. Namely, the nonlinear membrane model, the nonlinear bending model, the slightly nonlinear von Kàrmàn model and the linear model were recovered. Emphasis was put on the loading order of magnitude and on the induced magnitude of the deformations. A rigorous variational derivation of the nonlinear membrane model was then given in [7]. In particular, the degeneracy under compression was exhibited. Then, [6] provided a rigidity result that allowed to rigorously justify the other models.

Some recent papers by physicists [4] suggest models where thin bodies (leaves, gel disks) try to reach an elastic equilibrium state after some given non Euclidean metrics has been imposed and, possibly, fail when considered as 3d bodies (case when the 3d metric tensor has no realization); see also [3]. Which thin limit models should be used? It turns out, as shown in [2], [8], [9], that the driving mechanism is the structure of the 3d Riemann curvature tensor $\mathcal{R}$. Depending on the vanishing of separate entries of $\mathcal{R}$, models that share several features with the Eucidean hierarchy are derived.

References:

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[3] J. Dervaux & M. Ben Amar, *Morphogenesis of growing soft tissues*, Phys. Rev. Lett. **101** (2008), 068101.

[4] E. Efrati, E. Sharon, & R. Kupferman, *Elastic theory of unconstrained non-Euclidean plates* J. Mech. Phys. Solids, **57** (2009), 762–775.

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J.F. Rodrigues December 18, 2015 16:00–16:40

On the free boundary in heterogeneous problems of obstacle-type with two phases

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Abstract: We consider some properties of the solutions of free boundary problems of obstacle-type with two phases for a class of heterogeneous quasilinear elliptic operators, including the p-Laplacian operator with $1 < p < \infty$. Under a natural non-degeneracy assumption on the interface, when the level set of the change of phase has null Lebesgue measure, we prove a continuous dependence result for the characteristic functions of each phase and we establish sharp estimates on the variation of its Lebesgue measure with respect to the L^1-variation of the data, in a rather general framework. For elliptic quasilinear equations which heterogeneities have appropriate integrable derivatives, we show that the characteristic functions of both phases are of bounded variation for general data with bounded variation. This extends recent results for the obstacle problem and is a first result on the regularity of the free boundary of the heterogeneous two phases problem, which is therefore an interface locally of class C^1 up to a possible singular set of null perimeter.

L. Sanchez December 19, 2015 15:15–15:55

Travelling waves and critical speed in the presence of nonlinear diffusion

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Abstract: We study some features of the parametric boundary value problem

$$(P(u'))' - cu' + g(u) = 0 \tag{1}$$

$$u(-\infty) = 0, \quad u(+\infty) = 1,$$

where $D > 0$ and $g(0) = g(1) = 0$, $g > 0$ in $]0, 1[$.

The problem arises when one looks for travelling waves to

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(P(\frac{\partial u}{\partial x})\right) + g(u)$$

c being the wave speed.

Possible models for P are $P(v) = \frac{v}{\sqrt{1-v^2}}, \quad P(v) = \frac{v}{\sqrt{1+v^2}}$.

We shall be interested in similarities and differences between those models. Another interesting type of nonlinear diffusion, involving the one-dimensional p-Laplacian, corresponds to the choice $P(v) = |v|^{p-2}v$ in (1). In some instances an advection term may be considered as well.

As in the classical Fisher-Kolmogorov-Petrovskii-Piskounov equations, there exists an interval of admissible speeds $[c^*, \infty)$ and characterizations of the *critical speed* c^* can be obtained.

In particular, we present a variational characterization of the critical speed in the p-Laplacian setting.

The talk is based on papers by Enguiça, Gavioli and Sanchez [1], and joint work with S. Correia [2], Isabel Coelho [3], Maurizio Garrione [4] and A. Gavioli [5].

References:

[1] R. Enguiça, A. Gavioli, L. Sanchez, *Discrete and Continuous Dynamical Systems - Series A*, 33 (2013), 173-191.

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P. Seppecher $\int_{11:35-12:15}^{\text{December 17, 2015}}$

Homogenization of highly contrasted elastic checkerboards

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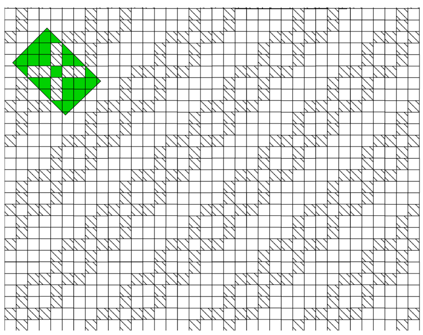
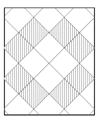
Abstract: We are interested in the homogenization of a soft elastic matrix containing very stiff inclusions. In the framework of two dimensional linear elasticity, we consider irregular checkered patterns: the stiff inclusions are rectangles arranged periodically and joining at corners. Such a geometry may lead to interesting phenomena. Indeed, stiff inclusions touching each other at corners may behave like rigid bodies linked together with pivot joints. In this way complex mechanisms can be simulated.

Here we will focus on the following geometry



in which the following period can be identified



Considering, for sake of simplicity a vanishing Poisson coefficient, the elastic energy is

$$E_\varepsilon(u) := \int_\Omega \mu_\varepsilon(x) \|e(u)\|^2 dx$$

where the elastic stiffness takes the value $\mu_e(x) = a_\varepsilon$ or $\mu_e = b_\varepsilon$ respectively in the soft (white) and hard (black) part of the domain. We will study the limit of this energy when all the quantities a_ε, $(b_\varepsilon)^{-1}$ and ℓ_ε tend to zero. We show that these limits do not commute and we describe a critical case where the effective medium is intermediate between the classical elastic one and the pseudo-rigid mo del.

A. Sequeira $\int_{10:50-11:30}^{\text{December 19, 2015}}$

Toward a novel approach to the mathematical modeling of atherosclerosis

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Abstract: Cardiovascular diseases, such as heart attack and stroke are responsible for more deaths than cancer in developed countries, and one of the major underlying causes for these events is atherosclerosis, a slow and complex disease that leads to the formation and eventual rupture of atherosclerotic plaques affecting large and medium sized arteries in the systemic circulation. Atherosclerosis is a chronic in inflammation that starts when low-density proteins cholesterol (LDL) enter the intima of the blood vessel where they are oxidized. The anti-inflammatory response of oxLDL triggers the response of monocytes that are transformed into macrophages and foam cells, leading to the production of inflammatory cytokins and further recruitment of monocytes. This complex process generates the formation of an atherosclerotic plaque and possibly its rupture (vulnerable plaques). Clots can be formed, they are carried in the bloodstream and can block the coronary vessels, the cerebral arteries, or even reduce or block blood supply to the legs. This is a silent disease with a long preclinical period that only produces symptoms when the artery is harshly narrowed and the obstruction occurs, resulting in severe complications.

Mathematical modeling and numerical simulations are important tools for a better understanding of atherosclerosis and subsequent development of more effective treatment and prevention strategies. Mathematical models of the atherosclerosis processes lead to complex systems of nonlinear partial differential equations of flow, transport, chemical reactions, interactions of fluid and elastic structures, movement of cells, coagulation and growth processes and additional complex dynamics of the vessel walls. Several theories have been developed to describe the pathogenesis of atherosclerosis but none of them can explain the whole process due to the large number of factors involved. In the recent times some simplified mathematical models have been used to study certain aspects of this complex process but more realistic and comprehensive mathematical models still need to be derived to describe the complete process of atherosclerosis.

In this talk we present a new mathematical model that captures essential features of the early stage of atherosclerosis development.

This is based on a joint work with T. Silva, J. Tiago, W. Jäger, M. Neuss-Radu.

References:

[1] T. Silva, J. Tiago, A. Sequeira, R. F. Santos. Existence, uniqueness, stability and asymptotic behavior of solutions for a mathematical model of atherosclerosis, Discrete and Continuous Dynamical Systems, in press, 2015.

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L. Tartar $\int_{09:45-10:25}^{\text{December 17, 2015}}$

Developing mathematical tools for real continuum mechanics and physics

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Abstract: About 10 years ago, I was asked to write an article for the 70th birthday of the founding of Portugaliae Mathematica, and my reason for writing it in French (*Qu'est-ce que l'homogénéisation ?* = What is homogenization?, Vol 64, Fasc. 4, 2007, 389–444) was to recall the time where good Portuguese students in mathematics were trying to study in France (usually Paris) for their thesis: my advisor (Jacques-Louis LIONS) had once told me that (since only Moscow had a similar concentration of good mathematicians around) Paris was one of the two best places in the world for studying mathematics. Later, a less challenging choice developed, to go study for a PhD elsewhere, for example in United States.

I met João-Paulo CARVALHO DIAS and Hugo BEIRÃO DA VEIGA during my studies, and when they invited me to Lisboa in January 1974 (a few months before the end of the dictatorship) I talked for the first time of trying to understand the continuum mechanics behind the partial differential equations which I studied.

The reason was that the theory of homogenization which I started developing with François MURAT in the early 1970s (extending the late 1960s work of Sergio SPAGNOLO, helped with the insight of Ennio DE GIORGI) had given me a way to understand effective properties of mixtures without any probabilistic language, as I had guessed from the work of Évariste SANCHEZ-PALENCIA, who restricted his attention to (partly formal) expansions in a periodically modulated context. I had learned classical mechanics and continuum mechanics, as well as a few approaches to physics (classical, quantic, relativistic, statistical) during my 1965–1967 studies at École Polytechnique (then in Paris), and homogenization gave me a way to attack a crucial question for me: how much of what I was taught made sense from a mathematical point of view?

With the Div–Curl lemma, found (in April or May 1974) with François MURAT, which we extended into the compensated compactness theory in 1976, I devised an improved (and unifying) approach for the question of convergence of approximate solutions to the partial differential equations of continuum mechanics, and when I lectured at EPFL (École Polytechnique Fédérale de Lausanne) in the Spring of 1977, I still had a technical obstacle to overcome with "entropy conditions": at some basic level realistic evolution models are hyperbolic, i.e. before one adds dissipative effects in a dogmatic (and probably wrong) way, and it was clear to me at the time that (apart from 1st principle, i.e. conservation of energy) thermodynamics is flawed.

I understood in the Fall of 1977 how to treat "entropy conditions" for a scalar equation, and in the Summer of 1978 I lectured on my compensated compactness method at Heriot-Watt University, while Ron DI PERNA lectured on his work: he believed enough in my method to work a few years for extending its application to some systems, while I thought that some important new ideas were missing, and moreover many equations from physics are flawed, so that I needed to understand that for deriving better equations.

Around 1979–1980, I guessed that the reason for the spontaneous absorption and emission rules imagined by physicists is just their way to describe nonlocal effects induced by homogenization, in a different way than what Évariste SANCHEZ-PALENCIA had done, since one has to work in an hyperbolic setting.

I also developed something which I now call compensated integrability, in the academic framework of discrete velocity models (first introduced by MAXWELL), corresponding to an idea different from the semi-group approach for evolution problems.

This was when Luísa MASCARENHAS came to work with me in Orsay, so that I asked her questions related to what my program of research looked at the time.

It was a difficult period, which led me to leave the French university system where one was denying me the right of vote. For 5 years (1982–1987), I worked at CEA (Commissariat à l'Énergie Atomique), improving my understanding of physics and finding other defects of classical equations which one should fix. Unable to come back where I had met with a curious form of racism, I exiled myself on the other side of the Atlantic.

I shall describe a few flaws of classical models, with intuition about what one could do, and the tools which I introduced for improving the situation, H-measures and variants, like multi-scales H-measures.

Contributed Talks

S. Almi $\int_{\text{17:35--17:50 (A)}}^{\text{December 18, 2015}}$

A lower semicontinuity result for a free discontinuity functional with a boundary term

Stefano ALMI, Gianni DAL MASO, Rodica TOADER
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Abstract: We study the lower semicontinuity in $GSBV^p(\Omega;\mathbb{R}^m)$ of a free discontinuity functional $\mathcal{F}(u)$ that can be written as the sum of a crack term, depending only on the jump set S_u, and of a boundary term, depending on the trace of u on $\partial\Omega$. We give sufficient conditions on the integrands for the lower semicontinuity of $\mathcal{F}$. Moreover, we prove a relaxation result, which shows that, if these conditions are not satisfied, the lower semicontinuous envelope of $\mathcal{F}$ can be represented by the sum of two integrals on S_u and $\partial\Omega$, respectively.

N. Antonic $\int_{\text{17:35--17:50 (B)}}^{\text{December 18, 2015}}$

Localisation principles for variant H-measures

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Abstract: Most of the known applications of H-measures, such as the compactness by compensation for differential relations with variable coefficients, are consequences of the localisation principle for H-measures [9].

During the last decade, a number of variant H-measures were introduced, such as parabolic H-measures [3], ultraparabolic H-measures [7], fractional H-measures [5], one-scale H-measures [10,1], H-distributions [4,6], one-scale H-distributions [2] and microlocal compactness forms [8]. The localisation principles for some of these variants will be discussed, together with their first applications.

The talk is based on joint work with Marko Erceg, Martin Lazar, Marin Mišur and Darko Mitrović.

References:

[1] Nenad Antonić, Marko Erceg, Martin Lazar: *Localisation principle for one-scale H-measures,* submitted, 35 pp. [`http://arxiv.org/abs/1504.03956`]

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[4] Nenad Antonić, Darko Mitrović: *H-distributions - an extension of H-measures to an* $\mathrm{L}^p - \mathrm{L}^q$ *setting*, *Abstract Appl. Analysis* **2011** (2011) Article ID 901084, 12 p.

[5] Marko Erceg, Ivan Ivec: *On a generalisation of H-measures*, submitted, 19 pp.

[6] Marin Mišur, Darko Mitrović: *On a generalization of compensated compactness in the* $\mathrm{L}^p - \mathrm{L}^q$ *setting*, *J. Functional Analysis* **268** (2015) 1904–1927.

[7] Evgenij Jurjevič Panov: *Ultra-parabolic H-measures and compensated compactness*, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **28** (2011) 47–62.

[8] Filip Rindler: *Directional oscillations, concentrations, and compensated compactness via microlocal compactness forms*, *Arch. Ration. Mech. Anal.* **215** (2015) 1–63.

[9] Luc Tartar: *H-measures, a new approach for studying homogenisation, oscillations and concentration effects in partial differential equations*, *Proc. Roy. Soc. Edinburgh* **115A** (1990) 193–230.

[10] Luc Tartar: *Multi-scale H-measures*, *Discrete and Continuous Dynamical Systems, S* **8** (2015) 77–90.

C. Barbarosie December 17, 2015 18:15–18:30 (A)

Smoothness of eigenvalues and eigenvectors of a matrix depending on parameters

Cristian Barbarosie

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Abstract: The study of eigenvalues and corresponding eigenvectors is very important; these eigensolutions characterize the normal modes of vibration for structures modelled by finite element methods. Based on the derivatives of these modes, an engineer is able to optimize a structure's design. Repeated eigenvalues, or nearly equal eigenvalues can exist in structures due to structural symmetries, but can also occur as parameters change, for instance, as the length of a beam varies, the torsion and bending mode frequencies may cross. We shall discuss the smoothness of eigenvalues and eigenvectors with respect to structural parameters, with focus on the situation where two eigenvalues cross. Special attention will be given to the labelling, or ordering, of eigenvalues. An algorithm will be presented which "restores" smoothness by relabelling eigenvalues. Also, analytical formulas for computing the derivatives of the eigensolutions will be discussed.

K. Burazin $\int_{17:55-18:10\,(A)}^{\text{December 18, 2015}}$

Exact solutions in multiple state optimal design problems

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Abstract: We consider multiple state optimal design problems for stationary diffusion in the case of two isotropic phases, aiming to minimize a weighted sum of compliances. It is well-known that such problems do not have *classical* solutions, and thus a relaxation is needed by introducing generalized materials. We consider (proper) relaxation by the homogenization method which consists in introducing *composite materials*, which are mixtures of original materials on the micro-scale.

It is well known that for problems with one state equation, there exist relaxed solutions which correspond to simple laminates at each point of the domain. As a consequence, one can write down a simpler relaxation, ending by a convex minimization problem.

For multiple state optimal design problems this is not the case in general, but we derive analogous result in the spherically symmetric case. To be precise, we consider the simpler relaxation and prove that there exists corresponding optimal (relaxed) design which is a radial function, and which is also a solution for proper relaxation of original problem. Since this simpler relaxation is convex optimization problem, one can easily derive the necessary and sufficient conditions of optimality and use them to calculate the optimal design. We demonstrate this procedure on some examples of optimal design problems, where the presented method enables us to explicitly calculate the solution.

I. Coelho $\int_{17:55-18:10\,(B)}^{\text{December 17, 2015}}$

Heteroclinics for a mean curvature problem in Lorentz-Minkowski space

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Abstract: We consider the singular action functional

$$\int_{\mathbb{R}\times\omega} 1-\sqrt{1-|\nabla u|^2}+W(u)\ dx,$$

which plays a role in special relativity. Here, we assume that $\omega\subset\mathbb{R}^{N-1}$ is a bounded domain and $W\colon\mathbb{R}\to[0,+\infty[$ is a double-well potential, i.e., W is of class C^1 and satisfies $W(-1)=W(1)=0$ and $W(u)>0$ if $u\neq\pm1$.

Using variational arguments and a rearrangement technique, we prove the existence, one-dimensionality and uniqueness (up to translations) of a smooth minimising phase transition between the stable states -1 and 1.

We also discuss the existence of minimising heteroclinic connections for the nonautonomous model

$$\int_{\mathbb{R}} 1-\sqrt{1-|u'|^2}+a(t)W(u)\ dt,$$

where $a\in L^\infty(\mathbb{R})$ is a positive function that can have a constant asymptotic behaviour at infinity or can be periodic.

The talk is based on joint work with Denis Bonheure and Manon Nys.

F. Giannetti

$\int_{17:35-17:50\,(A)}^{\text{December 17, 2015}}$

Some regularity results for integral functionals with variable growth conditions

Flavia Giannetti
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Abstract: We deal with the regularity properties of local minimizers of integral functionals of the type

$$\mathcal{F}(u;\Omega) := \int_\Omega F(x, Du(x))\,dx$$

where the integrand $F$is convex and satisfies the following so-called $p(x)$-growth conditions

$$|\xi|^{p(x)} \leq F(x,\xi) \leq C(1 + |\xi|^{p(x)})$$

for an exponent function $p(\cdot)$ belonging to a suitable Orlicz-Sobolev class. More precisely, we show an higher differentiability result for the minimizers and a dimension free higher integrability result for their gradients which have been object of a recent paper in collaboration with Antonia Passarelli di Napoli ([1]). Moreover, we discuss about a $C^{1,\alpha}$-partial regularity result for the local minimizers of functionals of the form

$$\mathcal{E}(u;\Omega) := \int_\Omega (1 + |Du|^2)^{\frac{p(x)}{2}}\ dx,$$

$p(\cdot) \geq 2$, which has been recently established ([2]).

References:

[1] F.Giannetti-A.Passarelli di Napoli: Higher differentiability of minimizers of variational integrals with variable exponents, *Math. Zeit.*, 280,3 (2015), 873–892.

[2] F.Giannetti: A $C^{1,\alpha}$ partial regularity result for integral functionals with $p(x)$-growth condition, *Adv. Calc. Var.* (to appear).

C. Kreisbeck

$\int_{17:55-18:10\,(B)}^{\text{December 18, 2015}}$

Homogenization of layered materials with rigid components in single-slip finite plasticity

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Abstract: This talk reports on first progress toward a better quantitative understanding of the effective behavior of polycrystals in the framework of geometrically nonlinear plasticity. Precisely, we study a variational model for plastic material composed of fine parallel layers of two types. While one

component is completely rigid in the sense that it admits only local rotations, the other one is softer featuring a single active slip system with linear self-hardening. As a main result, explicit homogenization formulas are determined by means of Γ-convergence. Due to the anisotropic nature of the problem the findings depend critically on the orientation of the slip direction relative to the layers. We observe three qualitatively different regimes involving macroscopic shearing and blocking effects. Technical difficulties in the proofs are rooted in the intrinsic rigidity of the model, which calls for new rigidity estimates as well as a careful analysis of the admissible microstructures restricted by differential inclusions.

L. Monsaingeon December 18, 2015, 17:15–17:30 (B)

A new Optimal Transport distance between nonnegative Radon measures

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Abstract: In this talk, I will introduce a new distance between nonnegative finite Borel measures in $\mathbb{R}^d$ with arbitrary masses. The distance is constructed by a Lagrangian variational approach (minimization of an action functional), which is similar to the celebrated Benamou-Brenier formula for the quadratic Kantorovich-Rubinstein-Wasserstein distance between probability measures. In contrast with the classical theory of optimal transportation of probability measures, we allow for mass variations and do not require decay at infinity. I will present several topological and geometrical properties of the resulting metric space. If time permits, I will discuss the application to a fitness-driven model of population dynamics: once suitably interpreted as a gradient flow with respect to our metric, we show that the model satisfies exponential convergence to the unique steady state with explicit rates.

This is joint with D. Vorotnikov and S. Kondratyev (Univ. Coimbra).

M. Morandotti December 17, 2015, 17:55–18:10 (A)

Formulas for Relaxed Disarrangement Densities

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Abstract: Structured deformations provide a multiscale geometry that captures the contributions at the macrolevel of both smooth geometrical changes and non-smooth geometrical changes (disarrangements) at submacroscopic levels. For each (first-order) structured deformation (g, G) of a continuous body, the tensor field G is known to be a measure of deformations without disarrangements, and $M := \nabla g - G$ is known to be a measure of deformations due to disarrangements. The tensor fields G and M together deliver not only standard notions of plastic deformation, but M and its curl deliver the Burgers vector field associated with closed curves in the body and the dislocation density field used in describing geometrical changes in bodies with defects. Recently, Owen and Paroni [4] evaluated explicitly some relaxed energy densities arising in Choksi and Fonseca's energetics of structured deformations [3] and thereby showed: (1) $(trM)^+$, the positive part of trM, is a volume density of disarrangements due to submacroscopic separations, (2) $(trM)^-$, the negative part of trM, is a volume density of disarrangements due to submacroscopic switches and interpenetrations, and (3) $|trM|$, the absolute value of trM, is a volume

density of all three of these non-tangential disarrangements: separations, switches, and interpenetrations.

In this talk we will show that a different approach to the energetics of structured deformations, that due to Baía, Matias, and Santos [1], confirms the roles of $(trM)^+$, $(trM)^-$, and $|trM|$ established by Owen and Paroni. In doing so, we give an alternative, shorter proof of Owen and Paroni's results, and we establish additional explicit formulas for other measures of disarrangements.

This is joint work with A. C. Barroso, J. Matias, and D. R. Owen [2].

References

[1] M. Baía, J. Matias, and P. M. Santos: A relaxation result in the framework of structured deformations in a bounded variation setting. Proc. Royal Society Edinburgh, **142A** (2012), 239-271.

[2] A. C. Barroso, J. Matias, M. Morandotti, and D. R. Owen: Explicit Formulas for Relaxed Disarrangement Densities Arising from Structured Deformations. arXiv:1508.06908 (2015). *Submitted.*

[3] R. Choksi and I. Fonseca: Bulk and interfacial energy densities for structured deformations of continua. Arch. Rational Mech. Anal., **138** (1997), 37-103.

[4] D. R. Owen and R. Paroni: Optimal flux densities for linear mappings and the multiscale geometry of structured deformations. Arch. Rational Mech. Anal., (2015) DOI 10.1007/s00205-015-0890-x.

V. Quítalo $\int_{17:35-17:50\,(B)}^{\text{December 17, 2015}}$

On a long range segregation model

Veronica QUITALO

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Abstract: In this talk we will present the main results about a free boundary problem arising from a long range segregation process. In this model, the growth of a population u_i at x is inhibited by the populations u_j in a full area surrounding x. This will force the populations to stay at distance 1 from each other in the limit configuration, and so the free boundary will be a strip along the support of the population with size exactly one. The talk is based on joint work with Luis Caffarelli and Stefania Patrizi.

References:

[1] L. Caffarelli and S. Patrizi and V. Quitalo: On a long range segregation model, *arXiv preprint arXiv:1505.05433*, submitted (2015).

L. Santos December 17, 2015 17:15–17:30 (B)

Existence and regularity of solutions of elliptic equations defined by the operator $\nabla \cdot (a(x)\nabla\ )$

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Abstract: In this talk we study the problem $\nabla \cdot (a(x)\nabla u) = f$ defined in a bounded open subset Ω of $\mathbb{R}^d$, $d \geq 2$, with $\partial\Omega$ smooth, considering nonhomogeneous Dirichlet or Neumann boundary condition. The function a satisfies $0 < a_* \leq a \leq a^*$.

Assuming that

$$f \in L^p(\Omega) \quad \text{and} \quad a \in W^{1,r}(\Omega), \text{ with } \quad r > d,$$

and imposing natural assumptions on the boundary data, we prove existence and uniqueness of strong solution.

If

$$f \in W^{-1,p}(\Omega) \quad \text{and} \quad a \in \mathcal{C}(\overline{\Omega}),$$

with weaker natural assumptions on the boundary data, we prove existence and uniqueness of weak solution.

Partial results concerning very weak solutions will also be presented.

We will show the importance of the previous results in the study of an electromagnetic induction heating problem.

H. Tavares December 17, 2015 18:15–18:30 (B)

Existence and regularity of solutions to optimal partition problems involving Laplacian eigenvalues

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Abstract: Let $\Omega \subset \mathbb{R}^N$ be an open bounded domain and $m \in \mathbb{N}$. Given $k_1, \ldots, k_m \in \mathbb{N}$, we consider a wide class of optimal partition problems involving Dirichlet eigenvalues of elliptic operators, including the following

$$\inf\left\{\Phi(\omega_1, \ldots, \omega_m) := \sum_{i=1}^{m} \lambda_{k_i}(\omega_i) : \ (\omega_1, \ldots, \omega_m) \in \mathcal{P}_m(\Omega)\right\},$$

where $\lambda_{k_i}(\omega_i)$ denotes the k_i–th eigenvalue of $(-\Delta, H_0^1(\omega_i))$ counting multiplicities, and $\mathcal{P}_m(\Omega)$ is the set of all open partitions of Ω, namely

$$\mathcal{P}_m(\Omega) = \{(\omega_1, \ldots, \omega_m) : \ \omega_i \subset \Omega \text{ open}, \ \omega_i \cap \omega_j = \emptyset \ \forall i \neq j\}.$$

We prove the existence of an open optimal partition $(\omega_1, \ldots, \omega_m) \in \mathcal{P}_m(\Omega)$, proving as well its regularity in the sense that the free boundary $\cup_{i=1}^m \partial\omega_i \cap \Omega$ is, up to a residual set, locally a $C^{1,\alpha}$ hypersurface.

The proof involves a careful study of an associate Schrödinger system with competition terms, as well as several free boundary techniques.

References:

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R. Toader December 17, 2015 17:15–17:30 (A)

Fracture models for elasto-plastic materials as limits of gradient damage models coupled with plasticity: the antiplane case

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Abstract: We study the asymptotic behavior of a variational model for damaged elasto-plastic materials, when the coefficients of the problem depend on a small parameter ε in such a way that the convergence of ε to 0 forces damage concentration on a set of codimension one. The starting point is a gradient damage model coupled with plasticity introduced by Alessi, Marigo, and Vidoli in the small strain regime, based on the minimisation, under suitable boundary conditions, of the total energy which is the sum of the stored elastic energy, of the energy dissipated by the plastic strain, and of the energy dissipated by the damage process. The last summand contains a gradient term which has a regularising effect and prevents sharp transitions of the damage.

In this context it is convenient to consider the reduced energy, i.e., the energy of the optimal additive decomposition of the displacement gradient into elastic strain and plastic strain, thus reducing to a functional which depends only on the displacement and on the damage variable. In the antiplane shear case we determine the Γ-limit of the reduced energy as ε tends to zero. In this case the displacements belong to the space of generalized functions of bounded variation. We show that the limit functional contains a surface energy term involving the crack opening. The surface energy density has an explicit formula, and it satisfies the following properties: it is concave, nondecreasing, in zero it equals zero and it has a finite derivative, while it is constant for large enough values of the crack opening. Therefore the Γ-limit functional can be interpreted as the total energy of an elasto-plastic material with a cohesive fracture.

The talk is based on joint work with G. Dal Maso and G. Orlando (SISSA, Trieste, Italy).

References:

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I. Velcic December 18, 2015 18:15–18:30 (B)

Homogenization of nonlinear shell models in elasticity

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Abstract: We discuss the derivation of the shell models (bending and von Karman) from three-dimensional nonlinear elasticity by doing simultaneous homogenization and dimensional reduction and by means of Γ-convergence. We discuss different models depending on the ratio between the thickness of the body and the oscillations of the material. In the case of bending shell we are able to obtain the limit model only for the convex shells in the following regimes: $\varepsilon(h) \ll h$, $\varepsilon(h) \sim h$, $\varepsilon(h)^2 \ll h \ll \varepsilon(h)$. In the case of von Kármán shell we obtain the models for the generic shell additionally in the regime $h \sim \varepsilon(h)^2$, while for the convex shell we completely recover the case $h \ll \varepsilon(h)$. Here h is the thickness of the body and $\varepsilon(h)$ is the period of the oscillations of the material.

References:

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Preprint: http://arxiv.org/pdf/1506.02571.pdf

D. Vorotnikov December 18, 2015 18:15–18:30 (A)

An evolutionary variational problem for image decomposition

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Abstract: We consider the following image decomposition model involving a variational (minimization) problem and an evolutionary PDE:

$$u(t,x) = \mathbf{u}(x) : \min_{\mathbf{u}:\Omega\to\mathbb{R}} \left\{ \int_\Omega g(w(t,x))|\nabla \mathbf{u}(x)|\,dx + \int_\Omega \mu(x)|\mathbf{u}(x) - f(x)|\,dx \right\},$$

with the diffusion constraint

$$\begin{aligned} \frac{\partial w(t,x)}{\partial t} = \Delta_{p,\lambda} w(t,x) + (1-\lambda(x))(|\nabla u(t,x)| - w(t,x)), \\ w(t,x) = 0,\ x \in \partial\Omega, \\ w(0,x) = F(x), \end{aligned}$$

where $p \geq 2$, and $f : \Omega \to \mathbb{R}$, $F : \Omega \to [0, +\infty)$, $\lambda : \Omega \to (0, 1]$, $\mu : \Omega \to (0, +\infty)$, $g : [0, +\infty) \to (0, +\infty)$ are prescribed functions, and the operator $\Delta_{p,\lambda}$ is the following weighted p-Laplacian:

$$\Delta_{p,\lambda} v = \lambda \mathrm{div}(|\nabla v|^{p-2} \nabla v) - (1 - |\nabla v|^{p-2}) \nabla v \cdot \nabla \lambda.$$

In particular, for $p = 2$ the second term vanishes and we recover the linear diffusion case.

We study the conditions which secure existence of weak solutions to this coupled problem. One of the difficulties is that in the imaging framework g can decay at infinity.

The talk is based on joint work with J.C. Moreno, V. B. Surya Prasath, K. Palaniappan and H. Proença.

E. Zappale $\int_{17:15-17:30\,(A)}^{\text{December 18, 2015}}$

Integral representation results in $BV \times L^p$

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Abstract: Integral representation results are obtained for the relaxation of a class of energy functionals depending on two vector fields with different behaviors, which find application in the context of elasticity, image decomposition and thermochemical equilibrium problems.

The talk is based on joint works with Graça Carita and Ana Margarida Ribeiro.

Posters

O. Boussaid

$\int_{16:00-17:30}^{\text{December 19, 2015}}$

Quasiconvexificaion of fucntions depending on two polynomials

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Abstract: In this note we are interested in the explicit computation of the quasiconvex envelope (see [1]) of some functional depending on two different polynomials having the property of changing sign on the cone of rank one matrices. This study is a continuation of a previous work dealing with functions depending on one polynomial (see [2]).

References:

[1] O. Boussaid, *Relaxation results for functions depending on polynomials changing sign on rank-one matrices*, J. Math. Anal. Appl. **349** (2009), 526 – 543

[2] B. Dacorogna, *Direct methods in the calculus of variations*. Second edition. Applied Mathematical Sciences **78**. Springer. New York, 2008.

J. Campos Cordero

$\int_{16:00-17:30}^{\text{December 19, 2015}}$

Regularity and sufficient conditions for strong local minimality

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Abstract: An outstanding question in the Calculus of Variations is that of finding sufficient conditions on extremals to ensure that they furnish strong local minimizers. Grabovsky and Mengesha [1] showed that, for quasiconvex integrands, assuming an priori C^1-regularity on the extremals is enough. We present here an alternative proof of this fact and we discuss a regularity result up to the boundary that further enables us to establish the sufficiency result for Lipschitz extremals whose gradient is in VMO. See also [2].

References:

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[2] J. Kristensen and A. Taheri. Partial Regularity of Strong Local Minimizers in the Multi-Dimensional Calculus of Variations. *Arch. Ration. Mech. Anal.,* 170(1):63-89, 2003.

S. Correia $\int_{16{:}00-17{:}30}^{\text{December 19, 2015}}$

Properties of minimal action solutions for a nonlinear Schrödinger system

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ABSTRACT: Consider the following nonlinear Schrödinger system:

$$\Delta u_i - \omega_i u_i + \sum_{j=1}^{M} k_{ij}|u_j|^{p+1}|u_i|^{p-1}u_i = 0, \quad u_i : \mathbb{R}^N \to \mathbb{C}, \quad i = 1, \dots, M.$$

Our aim in this talk will be to study properties of the solutions of (M-NLS) with minimal action, *i.e.* ground-states. These solutions are relevant both mathematically and physically. Moreover, we shall give special attention to the $p = 1$ case, which corresponds to a model arising in nonlinear optics.

On one hand, we shall give an explicit formula for ground-states in some special cases, by reducing an infinite-dimensional minimization problem to an M-dimensional one. On the other hand, we shall give several criteria on wether ground-states have all components nonzero (*i.e.*, if they are fully nontrivial). These results give a very comprehensive picture in the physical case $p = 1$. Furthermore, it will become clear that the case $M \geq 3$ has a much richer structure than that of $M = 2$, the most studied in the literature.

References:

[1] S. Correia, Characterization of ground-states for a system of M coupled semilinear Schrödinger equations, pre-print, arXiv:1410.7993

[2] S. Correia, Ground-states for systems of M coupled semilinear SchrÃűdinger equations with attraction-repulsion effects: characterization and perturbation results, pre-print, arXiv:1506.07758

[3] S. Correia, F. Oliveira, H. Tavares, Semitrivial vs. fully nontrivial ground states in cooperative cubic Schrödinger systems with $d \geq 3$ equations, pre-print, arXiv: 1508.01783

M. Friedrich $\int_{16{:}00-17{:}30}^{\text{December 19, 2015}}$

A quantitative geometric rigidity result in SBD and the derivation of linearized models from nonlinear Griffith energies in fracture mechanics

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ABSTRACT: We derive Griffith functionals in the framework of linearized elasticity from nonlinear and frame indifferent energies via Gamma-convergence. The convergence is given in terms of rescaled

configurations measuring the displacement of the deformations from piecewise rigid motions which are constant on each connected component of the cracked body. The key ingredient to establish a compactness result is a quantitative geometric rigidity result for special functions of bounded deformation (SBD). This estimate generalizes the result of Friesecke, James, Müller in nonlinear elasticity theory and the piecewise rigidity result of Chambolle, Giacomini, Ponsiglione for SBV functions which do not store elastic energy. The results are in part with Bernd Schmidt (Augsburg).

M. Jesenko $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Closure and commutability results for Γ-limits

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Abstract: Under a suitable notion of equivalence of integral densities we prove a Γ-closure theorem for integral functionals with standard p-growth: The limit of a sequence of Γ-convergent families is again a Γ-convergent family. Its Γ-limit can be recovered from Γ-limits of the original problems. This result not only provides a common basic principle for a number of linearization and homogenization results in elasticity theory. It also allows for new applications as we exemplify by proving that geometric linearization and homogenization of multi-well energy functionals commute. We then also address the case $p = 1$ with its difficulties.

Schematically:

$$\begin{tikzcd} \mathcal{F}_\varepsilon^{(j)} \arrow[d, "\Gamma"] \arrow[r, phantom, "\approx"] & \mathcal{F}_\varepsilon^{(\infty)} \arrow[d, "?"'] \\ \mathcal{F}_0^{(j)} \arrow[r, "?"] & \mathcal{F}_0^{(\infty)} \end{tikzcd}$$

The poster is based on joint work with Bernd Schmidt (Augsburg University).

References:

[1] M. Jesenko, B. Schmidt: Closure and commutability results for Γ-limits and the geometric linearization and homogenization of multiwell energy functionals. *SIAM J. Math. Anal.* 46 (2014), no. 4, 2525–2553.

S. Kondratyev $\int_{16:00-17:30}^{\text{December 19, 2015}}$

On the bulk velocity of Brownian ratchets

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Abstract: Brownian ratchet is a generic term for a few micro-level mechanisms in physics and biology that are capable of producing unidirectional transport of matter in systems without apparent bias to a particular direction. In this talk we study the transport phenomenon for rachets modeled by a

Fokker–Planck-type equation on the real axis. We establish a relation between the bulk transport velocity and a bi-periodic solution of the equation. We use this relation to characterise the transport for a few specific models such as adiabatic and semiadiabatic limits for tilting ratchets, generic ratchets with small diffusion, and the multistate chemical ratchets. We obtain qualitative results concerning the direction of transport as well as explicit asymptotic formulas for the bulk velocity.

I. Lucardesi December 19, 2015 16:00–17:30

The wave equation on domains with cracks growing on a prescribed path

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Abstract: Given a bounded open set $\Omega \subset \mathbb{R}^d$ with Lipschitz boundary and an increasing family Γ_t, $t \in [0,T]$, of closed subsets of Ω, we analyze the scalar wave equation $\ddot{u} - \mathrm{div}(A\nabla u) = f$ in the time varying cracked domains $\Omega \setminus \Gamma_t$. Here we assume that the sets Γ_t are contained into a *prescribed* $(d-1)$-manifold of class C^2. Our approach relies on a change of variables: recasting the problem on the reference configuration $\Omega \setminus \Gamma_0$, we are led to consider a hyperbolic problem of the form $\ddot{v} - \mathrm{div}(B\nabla v) + a \cdot \nabla v - 2b \cdot \nabla \dot{v} = g$ in $\Omega \setminus \Gamma_0$. Under suitable assumptions on the regularity of the change of variables that transforms $\Omega \setminus \Gamma_t$ into $\Omega \setminus \Gamma_0$, we prove existence and uniqueness of weak solutions for both formulations. Moreover, we provide an energy equality, which gives, as a by-product, the continuous dependence of the solutions with respect to the cracks.

The talk is based on joint work with Gianni Dal Maso.

G. Maggiani December 19, 2015 16:00–17:30

A dynamic evolution model for perfectly plastic plates

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Abstract: We consider the dynamic evolution of a linearly elastic-perfectly plastic plate subject to a purely vertical body load. As the thickness of the plate goes to zero, we prove that the three-dimensional evolutions converge to a solution of a certain reduced model. In this limit model admissible displacements are of Kirchhoff-Love type. Moreover, the motion of the body is governed by an equilibrium equation for the stretching stress, a hyperbolic equation involving the vertical displacement and the bending stress, and a rate-independent plastic flow rule. Some further properties of the reduced model are also discussed. This a joint work with Maria Giovanna Mora.

References:

[1] G.B. Maggiani, M.G. Mora: A dynamic evolution model for perfectly plastic plates. Preprint, (2015).

M. Mohamed $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Successive Approximation Methods for the Solution of the Fractional Euler-Lagrange Equation

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ABSTRACT: In this study a numerical scheme using the successive approximation method is proposed to solve the Fractional Euler-Lagrange equation. The left and right Riemann-Liouville derivatives are involved in this equation. After transforming this equation to an integral equation of a second kind, we discretize it by the successive approximations method.
In the final part, we apply our scheme to the fractional oscillator equation.

References:

[1] Agrawal O.P., Formulation of Euler-Lagrange equations for fractional variational problems, J. Math. Anal. Appl. 2002, 272, 368-379.

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C. Mifsud $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Hyperbolic boundary condition for a simplified model of perfect plasticity

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ABSTRACT: This presentation is devoted to study a simplified scalar model of perfect plasticity focussing on the time/space hyperbolic structure of the equations. It is well known that the system of dynamical linearized elasticity can be written as a boundary value symmetric linear hyperbolic system, called Friedrichs' system. Using recent works on Friedrichs' systems under convex constraints [1] or with boundary conditions [2], we reformulate the model of dynamical elasto-plasticity as a constrained boundary value hyperbolic system, and show that the natural boundary conditions emerging from this new

formulation are of Robin type. More precisely, our purpose is to investigate the following problem: given $\Omega \subset \mathbb{R}^N$ a bounded smooth open set, we look for functions $u : (0,T)\times\Omega \to \mathbb{R}$ and σ, $p : (0,T)\times\Omega \to \mathbb{R}^N$ such that

$$\begin{cases} \partial_{tt}u - \operatorname{div}\,\sigma = f, & \text{and} \quad \nabla u = \sigma + p, & \text{in } (0,T)\times\Omega, \\ \sigma \in K, & \text{and} \quad \sigma\cdot\partial_t p = |\partial_t p|, & \text{in } (0,T)\times\Omega, \\ (u,\partial_t u,\sigma,p)(0) = (u_0,v_0,\sigma_0,p_0), & \text{in } \Omega, \quad \text{“}\partial_t u + \frac{1}{\lambda}\sigma\cdot\nu = 0\text{”}, & \text{in } (0,T)\times\partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0$, and K is the closed unit ball of $\mathbb{R}^N$. The variable u stands for a displacement field, σ is a stress and p is an internal variable describing the evolution of plasticity. The resolution of (1) is performed by means of a visco-elasto-plastic regularization which consists in adding up a diffusive term in $\partial_t u$, and relaxing the constraint in σ. For each $\epsilon > 0$, we consider the problem

$$\begin{cases} \partial_{tt}u_\epsilon - \operatorname{div}\,(\sigma_\epsilon + \epsilon\nabla\partial_t u_\epsilon) = f_\epsilon, & \text{in } (0,T)\times\Omega, \\ \nabla u_\epsilon = \sigma_\epsilon + p_\epsilon, \quad \text{and} \quad \partial_t p_\epsilon = -\left(P_K(\sigma_\epsilon) - \sigma_\epsilon\right)/\epsilon, & \text{in } (0,T)\times\Omega, \\ (u,\partial_t u,\sigma,p)(0) = (u_0,v_0,\sigma_0,p_0), & \text{in } \Omega, \\ \partial_t u_\epsilon + \frac{1}{\lambda}\left(\sigma_\epsilon + \epsilon\nabla\partial_t u_\epsilon\right)\cdot\nu = g_\epsilon, & \text{in } (0,T)\times\partial\Omega. \end{cases} \tag{2}$$

We will then explain how one can get a solution to (1) when when passing to the limit as ϵ tends to zero in (2). In particular, a concentration phenomena, classical in plasticity, in the variable u forces one to relax the boundary condition into a nonlinear one, compatible with the constraint on σ. This work is a first insight into the comprehension of the interaction between convex constraints and boundary conditions in the framework of linear hyperbolic systems.

The talk is based on joint work with Bruno DESPRÉS and Nicolas SEGUIN.

References:

[1] B. Després, F. Lagoutière and N. Seguin: Weak solutions to Friedrichs systems with convex constraints, *Nonlinearity*, 24 (2011), 3055–3081.

[2] C. Mifsud, B. Després and N. Seguin: Dissipative formulation of initial boundary value problems for Friedrichs' systems, preprint hal-01074542, 2014.

T. Miura December 19, 2015, 16:00–17:30

Gamma-expansion for an adhesive obstacle problem arising from materials science

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[a] Supported by a Grant-in-Aid for JSPS Fellows 15J05166 and the Program for Leading Graduate Schools, MEXT, Japan.

Abstract: In this presentation we consider the following one-dimensional obstacle problem as proposed in [3]:

$$\underset{u\geq\psi}{\text{Minimize}}: \; E_\varepsilon[u] = \varepsilon^2 \int \kappa^2 ds + \int ds - \int_{\{u=\psi\}} (1-\alpha)\, ds. \tag{1}$$

Here a smooth obstacle function $\psi : [a,b] \to \mathbb{R}$, a constant coefficient $\varepsilon > 0$ and a continuous function $\alpha : [a,b] \to (0,1)$ are given. Admissible functions $u : [a,b] \to \mathbb{R}$ are constrained above the obstacle. The symbols κ and ds denote the curvature and the arclength element of the graph of u respectively.

This minimizing problem is motivated to perceive shape formation of adhesive membranes on substrates. For simplicity, we consider the one-dimensional case as in [3]. The third term of our energy corresponds to the effect of adhesion.

It is difficult to know the shape of minimizers of E_ε. A main cause is the first term of E_ε, which is the higher order term called bending energy. If the bending energy is deleted ($\varepsilon = 0$) then we can know some properties of minimizers, for example that "edge" singularities of the solutions (membranes) occur at the free boundary as the Alt-Caffarelli problem [1]. However, the energy E_0 is non-convex and may admit multiple minimizers.

The main result in this presentation is to give the first order term of a gamma-expansion of E_ε with respect to ε [2]. The obtained energy only depends on the state of "edge" singularities. This result reveals a selection principle for minimizers of the original problem as usually seen in phase transition models.

References:

[1] H. W. Alt, L. A. Caffarelli: Existence and regularity for a minimum problem with free boundary, *J. Reine Angew. Math.*, 325 (1981), 105–144.

[2] T. Miura: Singular perturbation by bending for an adhesive obstacle problem, preprint (2015), submitted.

[3] O. Pierre-Louis: Adhesion of membranes and filaments on rippled surfaces, *Phys. Rev. E*, 78 (2008) pp. 021603–021614.

A. Nakayasu $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Crystalline curvature flow of a graph-like curve and corresponding obstacle problems

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Abstract: In this presentation we consider a motion of planar graphs Γ_t by the crystalline curvature flow

$$V = \gamma(\mathbf{n})[\kappa_\gamma + \sigma] \quad \text{on } \Gamma_t.$$

Here, γ is an anisotropic norm with singularities, e.g., $\gamma(p,q) = |p| + |q|$, and σ is a smooth function depending on the spatial variable. The symbols V, $\mathbf{n}$ and κ_γ denote normal velocity, unit normal vector and anisotropic curvature of Γ_t, respectively. The study of this kind of equations is motivated by crystal growth problems.

Let us restrict ourselves to the case when Γ_t is given by the graph of a function $u = u(t,x) : (0,T) \times \mathbf{T}^1 \to \mathbf{R}^1$ and study a parabolic equation of u reflecting the singularity of γ and a corresponding obstacle problem. We point out that the authors of [1] and [2] introduce a notion of a solution by combining the theory of viscosity solutions and subdifferentials. In this presentation a general existence result of a solution will be established as a consequence of a new careful study on the obstacle problem.

The presentation is based on joint work with Mi-Ho Giga and Yoshikazu Giga (Univ. Tokyo) [3].

References:

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G. Orlando $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Lower semicontinuity of a class of integral functionals on the space of functions of bounded deformation

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ABSTRACT: We study the lower semicontinuity of some free discontinuity functionals, whose volume term depends on the Euclidean norm of the symmetrized gradient. This poster is based on a joint work with G. Dal Maso and R. Toader.

N.C. Overgaard $\int_{16:00-17:30}^{\text{December 19, 2015}}$

Taut String Algorithms and the ROF Model

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ABSTRACT: Minimization of the total variation-regularized least squares functional

$$E(u) = \lambda \int_\Omega |\nabla u|\, dx + \frac{1}{2}\|u - f\|^2_{L^2(\Omega)} \tag{1}$$

over functions $u \in \mathrm{BV}(\Omega)$ is known, in the planar case when $\Omega \subset \mathbf{R}^2$, as the Rudin-Osher-Fatemi (ROF) model. It is used in image processing for edge-preserving denoising of an input image f. The corresponding one-dimensional minimization problem is used for similar purposes in signal processing.

The taut string algorithm is another procedure for denoising one-dimensional signals while preserving jump-discontinuities. For discrete signals, it has been known for some time that this algorithm is equivalent to (nonparametric) total variation-regularized least square regression (E. Mammen and S. van de Geer, Annals of Statistics, vol. 25 no. 1, 1998.) This result was later extended to the continuous case by M. Grasmair (J. Math. Imaging Vis. vol 27, 2007).

We consider the taut string algorithm and one-dimensional version of (1) in the continous setting and propose a simplified proof of Grasmair's result. The proof is based on duality and a minimax theorem and the methods used are powerful enough to suggest a derivation of the "lower convex envelope"-property of the solution to the isotonic regression problem, i.e. the minimization of $\|u - f\|^2_{L^2(\Omega)}$ over the closed convex cone of nondecreasing functions u.

A. Semin $\int_{16:00-17:30}^{\text{December 19, 2015}}$

When a thin periodic layer meets corners: asymptotic analysis of a singular Helmholtz problem.

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ABSTRACT: In this presentation, we are interested in the solution $u^\delta \in \mathrm{H}^1(\Omega^\delta)$, where Ω^δ is a domain containing a finite periodic layer represented on Fig. 1, of the Helmholtz equation

$$-\Delta u^\delta - (k^\delta)^2 u^\delta = f \quad \text{in } \Omega^\delta, \quad f \in \mathrm{L}^2(\Omega) \tag{1}$$

where the wavenumber k^δ differs from a constant value k_0 on the domain $(-L, L) \times (-\delta, \delta)$ by a function $\hat{k}(\cdot/\delta)$, which is 1-periodic with respect to X_1. We complete problem (1) with homogeneous Neumann boundary conditions on the small holes $\partial\Omega^\delta_{\text{hole}}$, absorbing boundary conditions on $\Gamma_\pm$, and Neumann boundary conditions on the remaining part of the boundary. The main difficulty of this problem is the presence of re-entrant corners at the end of the periodic layer.

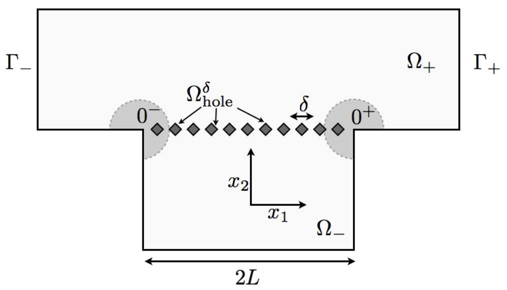

Figure 1: The domain Ω^δ.

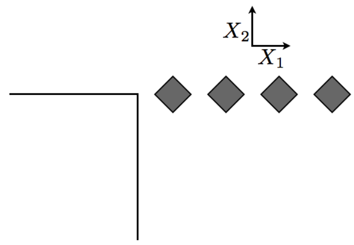

Figure 2: The normalized domain $\hat{\Omega}^-$.

We give here a construction of an asymptotic expansion of the solution u^δ as δ tends to 0. To do so, we combine the method of matched asymptotic expansion close to the corner and the method of surface homogenization close to the layer. In particular, we extend the Kondrat'ev theory, in the spirit of the works of Nazarov [1]. We give as well a justification of the asymptotic expansion (existence, uniqueness, stability) and an error estimate between the solution of problem (1) and its asymptotic expansion.

The presentation is a joint work [2,3] with Bérangère Delourme (Université Paris 13) and Kersten Schmidt (TU Berlin).

References:

[1] S. A. Nazarov. The Neumann problem in angular domains with periodic and parabolic perturbations of the boundary. Tr. Mosk. Mat. Obs., 69:182–241, 2008.

[2] B. Delourme, K. Schmidt, and A. Semin. When a thin periodic layer meets corners: asymptotic analysis of a singular Poisson problem. arXiv:1506.06964. Juni 2015.

[3] A. Semin, B. Delourme, and K. Schmidt. On the homogenization of the Helmholtz problem with thin perforated walls of finite length. *In preparation.*

R. M. Velho December 19, 2015 16:00–17:30

Some Numerical Aspects on Crowd Motion - The Hughes' Model

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Abstract: Here, we study a crowd model proposed by Roger Hughes in [1] and describe a numerical approach to solve it. This model comprises a Fokker-Planck equation coupled with an Eikonal in a 2-dimensional domain Ω with Dirichlet or Neumann data:

$$\begin{cases} \rho_t(x,t) + \operatorname{div}(\rho(1-\rho)^2 Du) = \Delta\rho, \\ |Du(x)|^2 = \dfrac{1}{(1-\rho)^2}. \end{cases} \tag{2}$$

The Fokker-Planck equation gives the evolution of the crowd density ρ. The Eikonal equation determines the optimal direction of movement for each individual if the rest of the population remains frozen.

We study periodic and Dirichlet/Neumann boundary conditions and special cases such as stationary and radial solutions. Open topics include qualitative properties of this system, for example, the decay of L^p norms in time or special solutions, and a qualitative description of its dynamics. Although significant progress has been achieved in [2], even 1-dimensional models are not completely understood.

We propose a numerical method that explores the adjoint structure present in this system, and we compare it to classical schemes of discretization - for 1-D models. One feature of our method is the conservation of the mass of agents.

These models can give important clues to the optimal design of routes. For example, answering the question on whether adding barriers at specific places can help the traffic of people assists in determining the ideal number of exits and their size - ensuring a given evacuation time. Variations in this model include agents with different mobilities and nonlinearities [3].

References:

[1] R. L. Hughes *A continuum theory for the flow of pedestrians* - 2002: Journal of Differential Equations - Transportation Research Part B - 507-535.

[2] M. Di Francesco, P. Markowich, J.-F. Pietschmann and M.-T. Wolfram *On the Hughes' model for pedestrian flow: the one-dimensional case* - 2011: Journal of Differential Equations - Vol. 250 - 1334-1362.

[3] M. Burger, M. Di Francesco, P. Markowich and M.-T. Wolfram *Mean field games with nonlinear mobilities in pedestrian dynamics* - 2013.

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Program

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