A new class of costs in optimal transport planning

Guy BOUCHITTE

IMATH, Université de Toulon, FRANCE

email: bouchitte@univ-tln.fr

In optimal mass transport theory, many problems can be written in the Monge-Kantorovich form

$$\inf\{\int_{X\times Y} c(x,y)\, d\gamma \ :\ \gamma\in\Pi(\mu,\nu)\}\ , \tag{1}$$

where μ,ν are given probability measures on X,Y and $c : X\times Y \to [0,+\infty[$ is a cost function. Here the competitors are probability measures γ on $X\times Y$ with marginals μ and ν respectively (transport plans). Let us recall that if an optimal transport plan $\gamma\in\Pi(\mu,\nu)$ is carried by the graph of a map $T : X\to Y$ i.e. if

$$<\gamma,\varphi(x,y)>=\int_X \varphi(x,Tx)\, d\mu \quad , \quad T^\sharp\mu=\nu\ ,$$

then T solves the original Monge problem: $\inf\{\int_X c(x,Tx)\, d\mu\ :\ T^\sharp\mu=\nu\}$.

Here we are interested in a different case. Indeed in some applications to economy or in probability theory, it can be interesting to favour optimal plans which are non associated to a single valued transport map $T(x)$. The idea is then to consider, instead of $T(x)$, the family of conditional probabilities γ^x such that

$$<\gamma,\varphi(x,y)>=\int_X (\int_X \varphi(x,y) d\gamma^x(y))\, d\mu\ ,$$

and to incorporate in problem (1) an additional cost over γ^x as follows

$$\inf\left\{\int_{X\times X} c(x,y)\, d\gamma + \int_X G(x,\gamma^x)\, d\mu\ :\ \gamma\in\Pi(\mu,\nu)\right\}\ , \tag{2}$$

being $G : (x,p)\in X\times\mathcal{P}(X)\to[0,+\infty]$ a given non linear function.

In this talk I will describe some recent results concerning problem (2) (existence, duality principle, optimality conditions) and focus on specific examples where $X=Y$ and X is a convex compact subset of $\mathbb{R}^d$.

This is a joined work with Thierry Champion and J.J. Alibert (University of Toulon).