An evolutionary variational problem for image decomposition

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We consider the following image decomposition model involving a variational (minimization) problem and an evolutionary PDE:

$$u(t,x) = \mathbf{u}(x) : \min_{\mathbf{u}:\Omega\to\mathbb{R}} \left\{ \int_\Omega g(w(t,x))|\nabla \mathbf{u}(x)|\, dx + \int_\Omega \mu(x)|\mathbf{u}(x) - f(x)|\, dx \right\},$$

with the diffusion constraint

$$\begin{aligned} \frac{\partial w(t,x)}{\partial t} = \Delta_{p,\lambda} w(t,x) + (1-\lambda(x))(|\nabla u(t,x)| - w(t,x)), \\ w(t,x) = 0,\ x \in \partial\Omega, \\ w(0,x) = F(x), \end{aligned}$$

where $p \geq 2$, and $f : \Omega \to \mathbb{R}$, $F : \Omega \to [0,+\infty)$, $\lambda : \Omega \to (0,1]$, $\mu : \Omega \to (0,+\infty)$, $g : [0,+\infty) \to (0,+\infty)$ are prescribed functions, and the operator $\Delta_{p,\lambda}$ is the following weighted p-Laplacian:

$$\Delta_{p,\lambda} v = \lambda \mathrm{div}(|\nabla v|^{p-2}\nabla v) - (1 - |\nabla v|^{p-2})\nabla v \cdot \nabla \lambda.$$

In particular, for $p = 2$ the second term vanishes and we recover the linear diffusion case.

We study the conditions which secure existence of weak solutions to this coupled problem. One of the difficulties is that in the imaging framework g can decay at infinity.

The talk is based on joint work with J.C. Moreno, V. B. Surya Prasath, K. Palaniappan and H. Proença.