A Nelson-Oppen combination theorem for many-sorted shiny theories

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Recently, the Automated Reasoning community has turned its attention to the area of Satisfiability Modulo Theories (SMT). Shortly, SMT deals with problems that are a generalization of boolean SAT problems to first-order formulas. Studying this problem is natural since SMT problems arise ever more frequently in fields such as Automated Theorem Proving and Software Verification.

Here, we deal with the problem of deciding formulas in the combination of two decidable first-order theories, by modularly combining their procedures. This method of combination, the Nelson-Oppen method, requires the theories to satisfy many properties.

Since Nelson and Oppen introduced this combination procedure in 1979 [4], the study of the classes of theories which decision procedures can be combined has been actively studied. In 2005, it was shown that shiny [6] and polite [5] theories could be combined with an arbitrary theory. Later, a stronger notion of polite theory was proposed, see [3], in order to overcome a subtle issue with a proof in [5]. In [2], we investigated the relationship between shiny and strongly polite theories in the one-sorted case. We showed that a shiny theory with a decidable quantifier-free satisfiability problem is strongly polite, and, for the other direction, we provided two different sets of conditions under which a polite theory is shiny.

Here, we show that the class of many-sorted shiny theories coincides, with respect to a set of sorts, with the class of strongly polite theories, when the theory is equipped with a quantifier-free satisfiability solver. We begin by adapting to the many-sorted case the notion of shininess. Then, we generalize the results in [2], by extending them to the many-sorted context and by proving the equivalence unconditionally. Capitalizing on this equivalence and on the Nelson-Oppen combination theorem for many-sorted strongly polite theories in [3], we establish a Nelson-Oppen combination theorem for many-sorted shiny theories.

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