Fibred logics: characterizing mixed reasoning and applications *

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Abstract Fibring is a powerful mechanism for combining logics, and an essential tool for designing and understanding complex logical systems. Given two logic systems $\mathcal{L}_1$ and $\mathcal{L}_2$, their fibring $\mathcal{L}_1 \bullet \mathcal{L}_2$ is the smallest logical system for the combined language which contains both $\mathcal{L}_1$ and $\mathcal{L}_2$. That is, given consequence relations for $\mathcal{L}_1$ and $\mathcal{L}_2$, their fibring is the smallest consequence relation over the combined language that extends the two. From the point of view of the Hilbert systems, this corresponds precisely to putting together the Hilbert systems of the two logics, allowing instantiations with formulas in the joint signature along the derivations. In this presentation we give a full characterization of the consequence relation that emerges from fibring in the particular case where the logics being combined do not share any connectives. We show the power of this tool by presenting various applications on important problems regarding combined logics: (1) conservativity, (2) decidability and complexity, and (3) the preservation of many-valuedness.

Regarding (1), we provide a full characterization of the conditions under which the fibring of two logics without shared connectives is a conservative extension of both logics. Although this result does not apply to fibring with shared connectives, we have also developed a method based on translations whose scope goes well beyond.

Concerning (2), we take advantage of the description of the mixed patterns of reasoning to extract a decision procedure for the fibred logic that uses only the decision procedures for the component logics, and analyze its complexity. A full characterization of the decidability of fibred logics without shared connectives follows. In particular, the existence of this algorithm implies that fibring of logics with disjoint signatures preserves decidability. Again, although the results are limited to logics obtained by fibring syntactically disjoint fragments, we discuss how having this result may still be helpful regarding logics that can be presented as the fibring of disjoint fragments plus (a finite number of) interaction axioms.

Finally, with respect to (3), we show that our result can be useful in understanding the semantics of fibring. Using our characterization, we can show that fibring (even without shared connectives) does not preserve finite-valuedness nor many-valuedness.

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