Definable logical operations in the proof-theory of classical logic

José Espírito Santo
Centro de Matemática
Universidade do Minho
Portugal
jes@math.uminho.pt

January 11, 2016

Abstract

In his monograph on natural deduction [5], when treating classical logic, Prawitz studies a natural deduction system where disjunction and the existential quantifier are omitted (let us call this system *restricted*), and judges the restricted system as "adequate" because, when taking disjunction and the existential quantifier as defined operations in the usual way, the inference rules of these operations become derivable in the restricted system.

However, this adequacy is only evident at the level of provability, but not so at the level of proof theory; that is, it is not evident that such reduction of provability to the restricted system also works for properties of the system. For instance, can strong normalization for the full system (where all operations are taken as primitive) be inferred from strong normalization for the restricted system? Indeed, observations of this kind were done by Stålmarck [4] before giving a direct proof of strong normalization for the full system.

In this talk we report about a closer investigation of the translation of the full system into the restricted system induced by taking disjunction and the existential quantifier as defined operations. The main message is that, while keeping the translation of formulas fixed, other aspects may be fine-tuned with noteworthy effects. Two examples:

- With an appropriate definition of proof conversions, strong normalization does lift from the restricted system to the full system.
- With an appropriate definition of the translation of proofs, commutative conversions vanish in the restricted system (cf. [1]).

In order to achieve a compact notation of proofs and reveal the underlying computational meaning of results, we follow the propositions-as-types principle [3, 2], and define the natural deduction system along the lines of [6].

References

[1] F. Ferreira and G. Ferreira. Commuting conversions vs. the standard coversions of the "good" connectives. *Studia Logica*, 92:63–84, 2009.

[2] T. Griffin. A formulae-as-types notion of control. In *ACM Conf. Principles of Programming Languages*. ACM Press, 1990.

[3] W. A. Howard. The formulae-as-types notion of construction. In J. P. Seldin and J. R. Hindley, editor, *To H. B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism*, pages 480–490. Academic Press, New York, 1980.

[4] G. Stålmarck. Normalization theorems for full first order classical natural deduction. *The Journal of Symbolic Logic*, 56(1):129–149, 1991.

[5] D. Prawitz. *Natural Deduction. A Proof-Theoretical Study.* Almquist and Wiksell, Stockholm, 1965.

[6] N. Rehof and M. Sørensen. The λ_Δ-calculus. In *TACS'94*, volume 789 of *Lecture Notes in Computer Science*. Springer Verlag, 1994.