Quantum logic & probability: Formal solutions to quantum paradoxes?

Alexander Afriat (Université de Brest, SPHÈRE)
Gino Tarozzi (Università di Urbino Carlo Bo)

Axiomatic thinking

Académie Internationale de Philosophie des Sciences
Academia das Ciências de Lisboa
Lisbon, 11-14 October 2017

Outline

Outline

Outline

Outline

Outline

Outline

Abstract

For almost a century, impressive efforts have been devoted the foundational problems of quantum mechanics. The efforts have been made in different directions, giving rise to the formation of various foundational sectors of research. Each sector has developed its own jargon, idiosyncrasies, and especially certainties: devout votaries have even been known to insist that their approach, unlike the others, 'solves all the problems of quantum mechanics.' What is true is that the various approaches are not pointless, each offering its own perspective, which can shed light from a particular angle. Quantum logic and quantum probability in their effort to provide formal solutions of quantum paradoxes, have been represented as axiomatisations in the spirit of Hilbert's programme. We can consider them as such, to assess their contributions to the logical clarification of crucial questions in the foundational debate.

Outline

Ohio

- the propositions of a theory are not all on an equal footing, some are typically more 'primitive' than others
- a handful are often singled out as being 'logically primitive' and called *axioms*
- but there is much freedom in the choice of axioms (the very notion of 'logically upstream' being somewhat arbitrary)
- Quine aptly wondered "which points of Ohio are starting points?"

Ohio

- the propositions of a theory are not all on an equal footing, some are typically more ‘primitive’ than others
- a handful are often singled out as being ‘logically primitive’ and called *axioms*
- but there is much freedom in the choice of axioms (the very notion of ‘logically upstream’ being somewhat arbitrary)
- Quine aptly wondered “which points of Ohio are starting points?”

Ohio

- the propositions of a theory are not all on an equal footing, some are typically more ‘primitive’ than others
- a handful are often singled out as being ‘logically primitive’ and called *axioms*
- but there is much freedom in the choice of axioms (the very notion of ‘logically upstream’ being somewhat arbitrary)
- Quine aptly wondered “which points of Ohio are starting points?”

Ohio

- the propositions of a theory are not all on an equal footing, some are typically more ‘primitive’ than others
- a handful are often singled out as being ‘logically primitive’ and called *axioms*
- but there is much freedom in the choice of axioms (the very notion of ‘logically upstream’ being somewhat arbitrary)
- Quine aptly wondered “which points of Ohio are starting points?”

Quantum peculiarities

- complex-valued wavefunctions (in configuration space)
- minimum quantities related to Planck's *h*
- incompatible observables
- failure of distributivity
- trouble with joint distributions

Quantum peculiarities

- complex-valued wavefunctions (in configuration space)
- minimum quantities related to Planck's h
- incompatible observables
- failure of distributivity
- trouble with joint distributions

Quantum peculiarities

- complex-valued wavefunctions (in configuration space)
- minimum quantities related to Planck's h
- incompatible observables
- failure of distributivity
- trouble with joint distributions

Quantum peculiarities

- complex-valued wavefunctions (in configuration space)
- minimum quantities related to Planck's h
- incompatible observables
- failure of distributivity
- trouble with joint distributions

Quantum peculiarities

- complex-valued wavefunctions (in configuration space)
- minimum quantities related to Planck's h
- incompatible observables
- failure of distributivity
- trouble with joint distributions

Arrangement

- there is little agreement as to how they should be arranged; are some more primitive than others? do all derive from one in particular?
- quantum logicians emphasise the failure of distributivity, Rovelli the Planck-sized cell, quantum probablists the trouble with joint distributions and so on

Arrangement

- there is little agreement as to how they should be arranged; are some more primitive than others? do all derive from one in particular?
- quantum logicians emphasise the failure of distributivity, Rovelli the Planck-sized cell, quantum probablists the trouble with joint distributions and so on

From classical to quantum

- here we'll start with Boolean algebras and Kolmogorovian probability (with its joint distributions), and generalise with departures from commutativity, distributivity *etc*.
- h, configuration space, entanglement will be less conspicuous

From classical to quantum

- here we'll start with Boolean algebras and Kolmogorovian probability (with its joint distributions), and generalise with departures from commutativity, distributivity *etc*.
- h, configuration space, entanglement will be less conspicuous

Outline

Boolean algebras

- set theory and propositional logic are structurally similar, being both Boolean algebras
- we can begin with a (denumerable) set Ω on which we take unions, intersections and complements
- if we think of Ω as an orthonormal basis $\{\Omega_k\}$, operators

$$D = \sum_k \lambda_k |\Omega_k\rangle\langle\Omega_k|$$

diagonal in Ω will merely assign to its elements their various eigenvalues: $\lambda_k \mapsto \Omega_k$

- a partition of Ω determines a resolution of the identity

Boolean algebras

- set theory and propositional logic are structurally similar, being both Boolean algebras
- we can begin with a (denumerable) set Ω on which we take unions, intersections and complements
- if we think of Ω as an orthonormal basis $\{\Omega_k\}$, operators

$$D = \sum_k \lambda_k |\Omega_k\rangle\langle\Omega_k|$$

  diagonal in Ω will merely assign to its elements their various eigenvalues: $\lambda_k \mapsto \Omega_k$
- a partition of Ω determines a resolution of the identity

Boolean algebras

- set theory and propositional logic are structurally similar, being both Boolean algebras
- we can begin with a (denumerable) set Ω on which we take unions, intersections and complements
- if we think of Ω as an orthonormal basis $\{\Omega_k\}$, operators

$$D = \sum_k \lambda_k |\Omega_k\rangle\langle\Omega_k|$$

  diagonal in Ω will merely assign to its elements their various eigenvalues: $\lambda_k \mapsto \Omega_k$
- a partition of Ω determines a resolution of the identity

Boolean algebras

- set theory and propositional logic are structurally similar, being both Boolean algebras
- we can begin with a (denumerable) set Ω on which we take unions, intersections and complements
- if we think of Ω as an orthonormal basis $\{\Omega_k\}$, operators

$$D = \sum_k \lambda_k |\Omega_k\rangle\langle\Omega_k|$$

  diagonal in Ω will merely assign to its elements their various eigenvalues: $\lambda_k \mapsto \Omega_k$
- a partition of Ω determines a resolution of the identity

Kolmogorovian probabilities

- a quantum state ψ can be thought of as a (square-summable) assignment of complex numbers ψ_k to Ω, where the squared moduli $|\psi_k|^2$ are (Kolmogorovian!) probabilities
- self-adjoint operators partition their ‘eigenset’ (here Ω) and represent random variables
  - two or more determine finer partitions and joint distributions (which make perfect sense here)

Kolmogorovian probabilities

- a quantum state ψ can be thought of as a (square-summable) assignment of complex numbers ψ_k to Ω, where the squared moduli $|\psi_k|^2$ are (Kolmogorovian!) probabilities
- self-adjoint operators partition their 'eigenset' (here Ω) and represent random variables
  - two or more determine finer partitions and joint distributions (which make perfect sense here)

Kolmogorovian probabilities

- a quantum state ψ can be thought of as a (square-summable) assignment of complex numbers ψ_k to Ω, where the squared moduli $|\psi_k|^2$ are (Kolmogorovian!) probabilities
- self-adjoint operators partition their ‘eigenset’ (here Ω) and represent random variables
  - two or more determine finer partitions and joint distributions (which make perfect sense here)

(uncountable measure spaces)

- separable Hilbert spaces are typically used in quantum mechanics
- a space $\mathscr{L}_2(\Omega, \mathbb{C})$ of square-integrable functions can be separable even if the configuration space Ω is uncountable
- appropriate real-valued functions on Ω give self-adjoint operators with 'eigenset' Ω (in the sense that they're multiplication operators on Ω, 'diagonal' on Ω)
- even if such an operator has no eigenvectors $|\Omega_k\rangle \in \mathscr{L}_2(\Omega, \mathbb{C})$, there are the projectors χ_Δ, where Δ is a (measurable) subset of Ω

(uncountable measure spaces)

- separable Hilbert spaces are typically used in quantum mechanics
- a space $\mathscr{L}_2(\Omega, \mathbb{C})$ of square-integrable functions can be separable even if the configuration space Ω is uncountable
- appropriate real-valued functions on Ω give self-adjoint operators with 'eigenset' Ω (in the sense that they're multiplication operators on Ω, 'diagonal' on Ω)
- even if such an operator has no eigenvectors $|\Omega_k\rangle \in \mathscr{L}_2(\Omega, \mathbb{C})$, there are the projectors χ_Δ, where Δ is a (measurable) subset of Ω

(uncountable measure spaces)

- separable Hilbert spaces are typically used in quantum mechanics
- a space $\mathscr{L}_2(\Omega, \mathbb{C})$ of square-integrable functions can be separable even if the configuration space Ω is uncountable
- appropriate real-valued functions on Ω give self-adjoint operators with ‘eigenset’ Ω (in the sense that they’re multiplication operators on Ω, ‘diagonal’ on Ω)
- even if such an operator has no eigenvectors $|\Omega_k\rangle \in \mathscr{L}_2(\Omega, \mathbb{C})$, there are the projectors χ_Δ, where Δ is a (measurable) subset of Ω

(uncountable measure spaces)

- separable Hilbert spaces are typically used in quantum mechanics
- a space $\mathscr{L}_2(\Omega, \mathbb{C})$ of square-integrable functions can be separable even if the configuration space Ω is uncountable
- appropriate real-valued functions on Ω give self-adjoint operators with 'eigenset' Ω (in the sense that they're multiplication operators on Ω, 'diagonal' on Ω)
- even if such an operator has no eigenvectors $|\Omega_k\rangle \in \mathscr{L}_2(\Omega, \mathbb{C})$, there are the projectors χ_Δ, where Δ is a (measurable) subset of Ω

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Four Boolean algebras

- in fact there are four equivalent Boolean algebras
  1. the sets $\Delta \subset \Omega$
  2. the corresponding propositions
  3. the characteristic functions χ_Δ equal to one on Δ and zero elsewhere
  4. the subspaces onto which the characteristic functions project
- this gives us a *distributive, Boolean* (degenerate) quantum logic, with Boolean connectives; and *Kolmogorovian* (degenerate) quantum probability, with unproblematic joint distributions

Quantum case

- if we now allow the basis $\{\Omega_k\}$ to rotate, we obtain a similar classical, Kolmogorovian scheme for every $\{\Omega'_k\}, \{\Omega''_k\}, \ldots$
- but if we consider the whole vector space $\mathbb{V} = \operatorname{span} \Omega_k$, everything breaks down: commutativity, distributivity, joint distributions …

Quantum case

- if we now allow the basis $\{\Omega_k\}$ to rotate, we obtain a similar classical, Kolmogorovian scheme for every $\{\Omega_k'\}, \{\Omega_k''\}, \ldots$
- but if we consider the whole vector space $\mathbb{V} = \operatorname{span} \Omega_k$, everything breaks down: commutativity, distributivity, joint distributions ...

Outline

Incompleteness

- one can wonder about the exact nature of quantum logic & probability
  - are they merely *epistemic* schemes, somehow related to empirical limitations?
  - or are they downright *ontic*?
- if quantum theory were an incomplete description of an underlying empirically inaccessible classical domain (hidden variables?), quantum logic & probability could be the empirical schemes 'above' the inaccessible classical world somehow described by Boolean logic and Kolmogorovian probability

Incompleteness

- one can wonder about the exact nature of quantum logic & probability
  - are they merely *epistemic* schemes, somehow related to empirical limitations?
  - or are they downright *ontic*?
- if quantum theory were an incomplete description of an underlying empirically inaccessible classical domain (hidden variables?), quantum logic & probability could be the empirical schemes 'above' the inaccessible classical world somehow described by Boolean logic and Kolmogorovian probability

Incompleteness

- one can wonder about the exact nature of quantum logic & probability
  - are they merely *epistemic* schemes, somehow related to empirical limitations?
  - or are they downright *ontic*?
- if quantum theory were an incomplete description of an underlying empirically inaccessible classical domain (hidden variables?), quantum logic & probability could be the empirical schemes 'above' the inaccessible classical world somehow described by Boolean logic and Kolmogorovian probability

Incompleteness

- one can wonder about the exact nature of quantum logic & probability
  - are they merely *epistemic* schemes, somehow related to empirical limitations?
  - or are they downright *ontic*?
- if quantum theory were an incomplete description of an underlying empirically inaccessible classical domain (hidden variables?), quantum logic & probability could be the empirical schemes ‘above’ the inaccessible classical world somehow described by Boolean logic and Kolmogorovian probability

Completeness

- but if the formalism were complete, with hidden variables ruled out, quantum logic & probability would assume a more *ontic* character—there would be no underlying, empirically inaccessible, Boolean logic and Kolmogorovian probability

Joint distributions

- the trouble is that the spectra of operators diagonal on Ω label subsets of a common set
  - one operator just refines the partition of the other
  - a joint distribution corresponds to a finer partition
- but with incompatible operators one has to choose—each spectrum now labels subspaces of a common space $\mathbb{V}$, which changes everything

Joint distributions

- the trouble is that the spectra of operators diagonal on Ω label subsets of a common set
    - one operator just refines the partition of the other
    - a joint distribution corresponds to a finer partition
- but with incompatible operators one has to choose—each spectrum now labels subspaces of a common space $\mathbb{V}$, which changes everything

Joint distributions

- the trouble is that the spectra of operators diagonal on Ω label subsets of a common set
  - one operator just refines the partition of the other
  - a joint distribution corresponds to a finer partition
- but with incompatible operators one has to choose—each spectrum now labels subspaces of a common space $\mathbb{V}$, which changes everything

Joint distributions

- the trouble is that the spectra of operators diagonal on Ω label subsets of a common set
  - one operator just refines the partition of the other
  - a joint distribution corresponds to a finer partition
- but with incompatible operators one has to choose—each spectrum now labels subspaces of a common space $\mathbb{V}$, which changes everything

Outline

6. Mathematische Behandlung der Axiome der Physik

Durch die Untersuchungen über die Grundlagen der Geometrie wird uns die Aufgabe nahegelegt, *nach diesem Vorbilde diejenigen physikalischen Disciplinen axiomatisch zu behandeln, in denen schon heute die Mathematik eine hervorragende Rolle spielt; dies sind in erster Linie die Wahrscheinlichkeitsrechnung und die Mechanik*.

The solution of philosophical problems

- it has often been suggested that philosophers are just sloppy and confused: clear and distinct formulation is enough to solve all their problems
    - *no more than appropriate axiomatisation, formalisation would be needed to deal with the difficulties of quantum mechanics*, for instance

The solution of philosophical problems

- it has often been suggested that philosophers are just sloppy and confused: clear and distinct formulation is enough to solve all their problems
  - *no more than appropriate axiomatisation, formalisation would be needed to deal with the difficulties of quantum mechanics*, for instance

Regressus in infinitum

- just as the *primum mobile* is invoked to cut off a *causal* regress which would otherwise be infinite, axioms are used to cut off a similar *logical* regress
    - if all we're interested in is probability, for instance, why bother regressing all the way back to the fundamentals of real analysis, set theory *etc.*; why not take all that for granted, and start with what really characterizes probability theory?
- but there's a difference between *taking for granted* and *wiping out*; once we've axiomatised, nothing prevents us from looking back beyond the axioms to consider what's been assumed and why

Regressus in infinitum

- just as the *primum mobile* is invoked to cut off a *causal* regress which would otherwise be infinite, axioms are used to cut off a similar *logical* regress
  - if all we're interested in is probability, for instance, why bother regressing all the way back to the fundamentals of real analysis, set theory *etc*.; why not take all that for granted, and start with what really characterizes probability theory?
- but there's a difference between *taking for granted* and *wiping out*; once we've axiomatised, nothing prevents us from looking back beyond the axioms to consider what's been assumed and why

Regressus in infinitum

- just as the *primum mobile* is invoked to cut off a *causal* regress which would otherwise be infinite, axioms are used to cut off a similar *logical* regress
    - if all we're interested in is probability, for instance, why bother regressing all the way back to the fundamentals of real analysis, set theory *etc*.; why not take all that for granted, and start with what really characterizes probability theory?
- but there's a difference between *taking for granted* and *wiping out*; once we've axiomatised, nothing prevents us from looking back beyond the axioms to consider what's been assumed and why

Incompatible observables

- we can always choose to blame the failure of
  - distributivity (quantum logic)
  - joint distributions (quantum probability)

  on incompatible observables
- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

Incompatible observables

- we can always choose to blame the failure of
  - distributivity (quantum logic)
  - joint distributions (quantum probability)

  on incompatible observables
- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

Incompatible observables

- we can always choose to blame the failure of
  - distributivity (quantum logic)
  - joint distributions (quantum probability)

  on incompatible observables
- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

Incompatible observables

- we can always choose to blame the failure of
  - distributivity (quantum logic)
  - joint distributions (quantum probability)

  on incompatible observables
- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

Incompatible observables

- we can always choose to blame the failure of
  - distributivity (quantum logic)
  - joint distributions (quantum probability)

  on incompatible observables
- but then we can wonder about *them* (and the corresponding uncertainty relations); again, are they related to a radical and fundamental *ontic* fuzziness, or to a merely *epistemic* limitation which may eventually be overcome?
- this brings us back to the whole debate about realism, completeness and hidden variables—which has therefore been swept under the carpet, not done away with

Circumlocution?

- are quantum logic & probability
  - just *parts* of the quantum-mechanical formalism, chosen by particular communities to dwell upon their predilections?
  - no more than misleading and elaborate forms of mathematical circumlocution, used to avoid calling a spade a spade?
- they don't seem to *solve* anything; but do they at least clarify?

Circumlocution?

- are quantum logic & probability
  - just *parts* of the quantum-mechanical formalism, chosen by particular communities to dwell upon their predilections?
  - no more than misleading and elaborate forms of mathematical circumlocution, used to avoid calling a spade a spade?
- they don't seem to *solve* anything; but do they at least clarify?

Circumlocution?

- are quantum logic & probability
    - just *parts* of the quantum-mechanical formalism, chosen by particular communities to dwell upon their predilections?
    - no more than misleading and elaborate forms of mathematical circumlocution, used to avoid calling a spade a spade?
- they don't seem to *solve* anything; but do they at least clarify?

Circumlocution?

- are quantum logic & probability
  - just *parts* of the quantum-mechanical formalism, chosen by particular communities to dwell upon their predilections?
  - no more than misleading and elaborate forms of mathematical circumlocution, used to avoid calling a spade a spade?
- they don't seem to *solve* anything; but do they at least clarify?