

Hypoelliptic Laplacian and the calculus of variations

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If X is a compact Riemannian manifold, the hypoelliptic Laplacian $L_b|_{b>0}$ is a family of operators acting on the total space \mathcal{X} of the tangent bundle TX , that interpolates between the Laplace-Beltrami operator and the geodesic flow. The associated diffusion is a geometric Langevin process on X , whose dynamics interpolates between Brownian motion and the geodesic flow.

This interpolation preserves certain spectral quantities. On very rigid manifolds like tori or on locally symmetric spaces, the spectrum of the original elliptic Laplacian remains rigidly embedded in the spectrum of the hypoelliptic deformation. Generalized Poisson formulas like Selberg's trace formula can be obtained as a consequence of this interpolation.

In the lecture, I will explain the construction of the hypoelliptic Laplacian, and emphasize the role of the calculus of variations in the construction. The spectral properties of the hypoelliptic deformation will be described.