Exact solutions in multiple state optimal design problems

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We consider multiple state optimal design problems for stationary diffusion in the case of two isotropic phases, aiming to minimize a weighted sum of compliances. It is well-known that such problems do not have *classical* solutions, and thus a relaxation is needed by introducing generalized materials. We consider (proper) relaxation by the homogenization method which consists in introducing *composite materials*, which are mixtures of original materials on the micro-scale.

It is well known that for problems with one state equation, there exist relaxed solutions which correspond to simple laminates at each point of the domain. As a consequence, one can write down a simpler relaxation, ending by a convex minimization problem.

For multiple state optimal design problems this is not the case in general, but we derive analogous result in the spherically symmetric case. To be precise, we consider the simpler relaxation and prove that there exists corresponding optimal (relaxed) design which is a radial function, and which is also a solution for proper relaxation of original problem. Since this simpler relaxation is convex optimization problem, one can easily derive the necessary and sufficient conditions of optimality and use them to calculate the optimal design. We demonstrate this procedure on some examples of optimal design problems, where the presented method enables us to explicitly calculate the solution.