Homogenization for inner boundary conditions with equi-valued surfaces via unfolding

Doina CIORANESCU

Université Pierre et Marie Curie, Laboratoire J. L. Lions, Paris, France

email: cioran@ann.jussieu.fr

We consider two examples of the general setting of equi-valued surfaces with corresponding assigned total fluxes.

Problem 1. [1] *The linear elastic torsion of an infinite three dimensional rod* with a multiply-connected two-dimensional cross-section Ω^* obtained from a bounded open set $\Omega \subset \mathbb{R}^2$ perforated by a finite number of regular closed subsets, $S^1, S^2, \ldots$.

Problem 2. [4] *The electro-conductivity problem* in presence of isolated perfect conductors (arising in resistivity well-logging), set in any dimension with the same type of geometry.

In both situations one looks for $\varphi \in H_0^1(\Omega)$ with $\varphi_{|S^j}$ unknown constant for each j, satisfying $-\mathrm{div}(A(x)\nabla\varphi(x)) = f$ in Ω^* and the corresponding boundary conditions,

$$\text{Problem 1} \quad \int_{\partial S^j} \frac{\partial \varphi}{\partial n} d\sigma(x) = |S^j|, \quad \text{Problem 2} \quad \int_{\partial S^j} \frac{\partial \varphi}{\partial \nu_A} d\sigma(x) = g^j, \quad g^j \text{ given numbers.}$$

We are interested in the periodic homogenization by the unfolding method ([5]) of these two problems. We refer to [2] and [3] for the first proof for the elastic torsion problem (via extension operators and oscillating test functions), where regularity assumptions are made for the boundary of the inclusions.

One of the advantages of the unfolding method is that it requires no regularity for the boundary of the inclusions. Another one is that a corrector result, completely general, is obtained as an immediate consequence.

The talk is based on joint work [6] with Alain Damlamian (Université Paris-Est, Créteil) and Li Tatsien (Fudan University, Shanghai).

References:

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