Properties of minimal action solutions for a nonlinear Schrödinger system

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Consider the following nonlinear Schrödinger system:

$$\Delta u_i - \omega_i u_i + \sum_{j=1}^{M} k_{ij}|u_j|^{p+1}|u_i|^{p-1}u_i = 0, \quad u_i : \mathbb{R}^N \to \mathbb{C}, \quad i = 1, \dots, M.$$

Our aim in this talk will be to study properties of the solutions of (M-NLS) with minimal action, *i.e.* ground-states. These solutions are relevant both mathematically and physically. Moreover, we shall give special attention to the $p = 1$ case, which corresponds to a model arising in nonlinear optics.

On one hand, we shall give an explicit formula for ground-states in some special cases, by reducing an infinite-dimensional minimization problem to an M-dimensional one. On the other hand, we shall give several criteria on wether ground-states have all components nonzero (*i.e.*, if they are fully nontrivial). These results give a very comprehensive picture in the physical case $p = 1$. Furthermore, it will become clear that the case $M \geq 3$ has a much richer structure than that of $M = 2$, the most studied in the literature.

References:

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