Boundary value problems for the infinity Laplacian: regularity and geometric results

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We discuss boundary value problems where the operator is the infinity Laplacian, or its normalized version. In particular, we focus our attention on the homogeneous Dirichlet problem with constant source term, and on a related Serrin-type overdetemined problem.

For the unique solution to the Dirichlet problem, we give a power-concavity result, valid on bounded convex subsets of \mathbb{R}^n . It is obtained by the convex envelope method introduced by Alvarez, Lasry and Lions, and yields as a consequence the C^1 regularity of the solution.

For the Serrin-type problem, we provide a complete characterization of convex sets where a solution exists, as "stadium-like domains". This result is closely related to a purely geometric topic, which is the characterization of sets with positive reach and empty interior in \mathbb{R}^n .

The talk is based on some recent joint works with GRAZIANO CRASTA (Università di Roma "La Sapienza").

References:

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