The wave equation on domains with cracks growing on a prescribed path

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Given a bounded open set $\Omega \subset \mathbb{R}^d$ with Lipschitz boundary and an increasing family Γ_t, $t \in [0,T]$, of closed subsets of Ω, we analyze the scalar wave equation $\ddot{u} - \mathrm{div}(A\nabla u) = f$ in the time varying cracked domains $\Omega \setminus \Gamma_t$. Here we assume that the sets Γ_t are contained into a *prescribed* $(d-1)$-manifold of class C^2. Our approach relies on a change of variables: recasting the problem on the reference configuration $\Omega \setminus \Gamma_0$, we are led to consider a hyperbolic problem of the form $\ddot{v} - \mathrm{div}(B\nabla v) + a \cdot \nabla v - 2b \cdot \nabla \dot{v} = g$ in $\Omega \setminus \Gamma_0$. Under suitable assumptions on the regularity of the change of variables that transforms $\Omega \setminus \Gamma_t$ into $\Omega \setminus \Gamma_0$, we prove existence and uniqueness of weak solutions for both formulations. Moreover, we provide an energy equality, which gives, as a by-product, the continuous dependence of the solutions with respect to the cracks.

The talk is based on joint work with Gianni Dal Maso.