# Hyperbolic boundary condition for a simplified model of perfect plasticity

Clément MIFSUD. Jean-François BABADJIAN

Affiliation: Sorbonne Universités, UPMC Univ Paris 06, CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, 4, place Jussieu 75005, Paris, France

email: mifsud@ljll.math.upmc.fr

This presentation is devoted to study a simplified scalar model of perfect plasticity focussing on the time/space hyperbolic structure of the equations. It is well known that the system of dynamical linearized elasticity can be written as a boundary value symmetric linear hyperbolic system, called Friedrichs' system. Using recent works on Friedrichs' systems under convex constraints [1] or with boundary conditions [2], we reformulate the model of dynamical elasto-plasticity as a constrained boundary value hyperbolic system, and show that the natural boundary conditions emerging from this new formulation are of Robin type. More precisely, our purpose is to investigate the following problem: given $\Omega \subset \mathbb{R}^N$ a bounded smooth open set, we look for functions $u : (0,T) \times \Omega \to \mathbb{R}$ and $\sigma$, $p : (0,T) \times \Omega \to \mathbb{R}^N$ such that

$$\begin{cases} \partial_{tt} u - \operatorname{div} \sigma = f, & \text{and} \quad \nabla u = \sigma + p, & \text{in } (0,T) \times \Omega, \\ \sigma \in K, & \text{and} \quad \sigma \cdot \partial_t p = |\partial_t p|, & \text{in } (0,T) \times \Omega, \\ (u, \partial_t u, \sigma, p)(0) = (u_0, v_0, \sigma_0, p_0), & \text{in } \Omega, \quad \text{“}\partial_t u + \frac{1}{\lambda}\sigma \cdot \nu = 0\text{”}, & \text{in } (0,T) \times \partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0$, and $K$ is the closed unit ball of $\mathbb{R}^N$. The variable $u$ stands for a displacement field, $\sigma$ is a stress and $p$ is an internal variable describing the evolution of plasticity. The resolution of (1) is performed by means of a visco-elasto-plastic regularization which consists in adding up a diffusive term in $\partial_t u$, and relaxing the constraint in $\sigma$. For each $\epsilon > 0$, we consider the problem

$$\begin{cases} \partial_{tt} u_\epsilon - \operatorname{div}\ (\sigma_\epsilon + \epsilon \nabla \partial_t u_\epsilon) = f_\epsilon, & \text{in } (0,T) \times \Omega, \\ \nabla u_\epsilon = \sigma_\epsilon + p_\epsilon, \quad \text{and} \quad \partial_t p_\epsilon = -\left(P_K(\sigma_\epsilon) - \sigma_\epsilon\right)/\epsilon, & \text{in } (0,T) \times \Omega, \\ (u, \partial_t u, \sigma, p)(0) = (u_0, v_0, \sigma_0, p_0), & \text{in } \Omega, \\ \partial_t u_\epsilon + \frac{1}{\lambda}\left(\sigma_\epsilon + \epsilon \nabla \partial_t u_\epsilon\right) \cdot \nu = g_\epsilon, & \text{in } (0,T) \times \partial\Omega. \end{cases} \tag{2}$$

We will then explain how one can get a solution to (1) when when passing to the limit as $\epsilon$ tends to zero in (2). In particular, a concentration phenomena, classical in plasticity, in the variable $u$ forces one to relax the boundary condition into a nonlinear one, compatible with the constraint on $\sigma$. This work is a first insight into the comprehension of the interaction between convex constraints and boundary conditions in the framework of linear hyperbolic systems.

The talk is based on joint work with Bruno DESPRÉS and Nicolas SEGUIN.

**References:**

[1] B. Després, F. Lagoutière and N. Seguin: Weak solutions to Friedrichs systems with convex constraints, *Nonlinearity*, 24 (2011), 3055–3081.

[2] C. Mifsud, B. Després and N. Seguin: Dissipative formulation of initial boundary value problems for Friedrichs' systems, preprint hal-01074542, 2014.