Gamma-expansion for an adhesive obstacle problem arising from materials science

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In this presentation we consider the following one-dimensional obstacle problem as proposed in [3]:

$$\underset{u\geq\psi}{\text{Minimize}}: \ E_\varepsilon[u] = \varepsilon^2 \int \kappa^2 ds + \int ds - \int_{\{u=\psi\}} (1-\alpha)\, ds. \tag{1}$$

Here a smooth obstacle function $\psi : [a,b] \to \mathbb{R}$, a constant coefficient $\varepsilon > 0$ and a continuous function $\alpha : [a,b] \to (0,1)$ are given. Admissible functions $u : [a,b] \to \mathbb{R}$ are constrained above the obstacle. The symbols κ and ds denote the curvature and the arclength element of the graph of u respectively.

This minimizing problem is motivated to perceive shape formation of adhesive membranes on substrates. For simplicity, we consider the one-dimensional case as in [3]. The third term of our energy corresponds to the effect of adhesion.

It is difficult to know the shape of minimizers of E_ε. A main cause is the first term of E_ε, which is the higher order term called bending energy. If the bending energy is deleted ($\varepsilon = 0$) then we can know some properties of minimizers, for example that "edge" singularities of the solutions (membranes) occur at the free boundary as the Alt-Caffarelli problem [1]. However, the energy E_0 is non-convex and may admit multiple minimizers.

The main result in this presentation is to give the first order term of a gamma-expansion of E_ε with respect to ε [2]. The obtained energy only depends on the state of "edge" singularities. This result reveals a selection principle for minimizers of the original problem as usually seen in phase transition models.

References:

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