Formulas for Relaxed Disarrangement Densities

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Structured deformations provide a multiscale geometry that captures the contributions at the macrolevel of both smooth geometrical changes and non-smooth geometrical changes (disarrangements) at submacroscopic levels. For each (first-order) structured deformation (g, G) of a continuous body, the tensor field G is known to be a measure of deformations without disarrangements, and $M := \nabla g - G$ is known to be a measure of deformations due to disarrangements. The tensor fields G and M together deliver not only standard notions of plastic deformation, but M and its curl deliver the Burgers vector field associated with closed curves in the body and the dislocation density field used in describing geometrical changes in bodies with defects. Recently, Owen and Paroni [4] evaluated explicitly some relaxed energy densities arising in Choksi and Fonseca's energetics of structured deformations [3] and thereby showed: (1) $(trM)^+$, the positive part of trM, is a volume density of disarrangements due to submacroscopic separations, (2) $(trM)^-$, the negative part of trM, is a volume density of disarrangements due to submacroscopic switches and interpenetrations, and (3) $|trM|$, the absolute value of trM, is a volume density of all three of these non-tangential disarrangements: separations, switches, and interpenetrations.

In this talk we will show that a different approach to the energetics of structured deformations, that due to Baía, Matias, and Santos [1], confirms the roles of $(trM)^+$, $(trM)^-$, and $|trM|$ established by Owen and Paroni. In doing so, we give an alternative, shorter proof of Owen and Paroni's results, and we establish additional explicit formulas for other measures of disarrangements.

This is joint work with A. C. Barroso, J. Matias, and D. R. Owen [2].

References

[1] M. Baía, J. Matias, and P. M. Santos: A relaxation result in the framework of structured deformations in a bounded variation setting. Proc. Royal Society Edinburgh, **142A** (2012), 239-271.

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