Crystalline curvature flow of a graph-like curve and corresponding obstacle problems

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In this presentation we consider a motion of planar graphs Γ_t by the crystalline curvature flow

$$V = \gamma(\mathbf{n})[\kappa_\gamma + \sigma] \quad \text{on } \Gamma_t.$$

Here, γ is an anisotropic norm with singularities, e.g., $\gamma(p,q) = |p| + |q|$, and σ is a smooth function depending on the spatial variable. The symbols V, $\mathbf{n}$ and κ_γ denote normal velocity, unit normal vector and anisotropic curvature of Γ_t, respectively. The study of this kind of equations is motivated by crystal growth problems.

Let us restrict ourselves to the case when Γ_t is given by the graph of a function $u = u(t,x) : (0,T) \times \mathbf{T}^1 \to \mathbf{R}^1$ and study a parabolic equation of u reflecting the singularity of γ and a corresponding obstacle problem. We point out that the authors of [1] and [2] introduce a notion of a solution by combining the theory of viscosity solutions and subdifferentials. In this presentation a general existence result of a solution will be established as a consequence of a new careful study on the obstacle problem.

The presentation is based on joint work with Mi-Ho Giga and Yoshikazu Giga (Univ. Tokyo) [3].

References:

[1] M.-H. Giga, Y. Giga: Evolving graphs by singular weighted curvature, *Arch. Rational Mech. Anal.*, 141 (1998), 117-198.

[2] M.-H. Giga, Y. Giga, P. Rybka: A comparison principle for singular diffusion equations with spatially inhomogeneous driving force for graphs, *Arch. Ration. Mech. Anal.*, 211 (2014), 419-453.

[3] M.-H. Giga, Y. Giga, A. Nakayasu: On general existence results for one-dimensional singular diffusion equations with spatially inhomogeneous driving force, *Geometric partial differential equations*, 145-170, Ed. Norm., Pisa, 2013.