Taut String Algorithms and the ROF Model

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Minimization of the total variation-regularized least squares functional

$$E(u) = \lambda \int_{\Omega} |\nabla u| \, dx + \frac{1}{2} \|u - f\|^2_{L^2(\Omega)} \tag{1}$$

over functions $u \in \mathrm{BV}(\Omega)$ is known, in the planar case when $\Omega \subset \mathbf{R}^2$, as the Rudin-Osher-Fatemi (ROF) model. It is used in image processing for edge-preserving denoising of an input image f. The corresponding one-dimensional minimization problem is used for similar purposes in signal processing.

The taut string algorithm is another procedure for denoising one-dimensional signals while preserving jump-discontinuities. For discrete signals, it has been known for some time that this algorithm is equivalent to (nonparametric) total variation-regularized least square regression (E. Mammen and S. van de Geer, Annals of Statistics, vol. 25 no. 1, 1998.) This result was later extended to the continuous case by M. Grasmair (J. Math. Imaging Vis. vol 27, 2007).

We consider the taut string algorithm and one-dimensional version of (1) in the continous setting and propose a simplified proof of Grasmair's result. The proof is based on duality and a minimax theorem and the methods used are powerful enough to suggest a derivation of the "lower convex envelope"-property of the solution to the isotonic regression problem, i.e. the minimization of $\|u - f\|^2_{L^2(\Omega)}$ over the closed convex cone of nondecreasing functions u.