Ground state asymptotics for a singularly perturbed convection-diffusion operator in a thin cylinder.

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The talk focuses on the limit behaviour of a non-trivial solution of the equation

$$-\varepsilon \mathrm{div}\Big(a\Big(\frac{x}{\varepsilon}\Big)\nabla u^\varepsilon(x)\Big) - \mathrm{div}\Big(b\Big(\frac{x}{\varepsilon}\Big)u^\varepsilon(x)\Big) = 0 \quad \text{in } G_\varepsilon,$$

$$\varepsilon a\Big(\frac{x}{\varepsilon}\Big)\nabla u^\varepsilon \cdot \nu^\varepsilon + b\Big(\frac{x}{\varepsilon}\Big)\cdot \nu^\varepsilon u^\varepsilon = 0 \quad \text{on } \partial G_\varepsilon$$

with a small positive parameter ε. Here G_ε is a thin cylinder in $\mathbb{R}^d$:

$$G_\varepsilon = (0,1) \times (\varepsilon Q),$$

Q is a smooth bounded domain in $\mathbb{R}^{d-1}$; ν is the unit exterior normal on ∂G_ε.

We assume that the coefficients $a_{ij} = a_{ij}(y)$ and $b_j = b_j(y)$ are periodic in y_1 and satisfy the uniform ellipticity conditions,

$$\Lambda^{-1}|\xi|^2 \le a(y)\xi\cdot\xi \le \Lambda|\xi|^2, \qquad \Lambda > 0,\ \xi \in \mathbb{R}^d,\ y \in G,$$

$a \in L^\infty(G;\mathbb{R}^{d^2})$, $b \in L^\infty(G;\mathbb{R}^d)$ with $G = (0,\infty)\times Q$.

Under these assumptions we study the limit behaviour of u^ε. We show that the leading terms of its asymptotic expansion take the form

$$u^\varepsilon(x) = p_\theta\Big(\frac{x}{\varepsilon}\Big)\exp\Big(\frac{\theta x_1}{\varepsilon}\Big) + u^{-,\varepsilon}(x) + u^{+,\varepsilon}(x),$$

where $\theta \in \mathbb{R}$, $p_\theta(y)$ is a periodic in y_1 positive function, and $u^{-,\varepsilon}(x)$ and $u^{+,\varepsilon}(x)$ are boundary layer functions such that

$$|u^{-,\varepsilon}(x)| \le C\exp\Big(\frac{\theta_- x_1}{\varepsilon}\Big), \quad |u^{+,\varepsilon}(x)| \le C\exp\Big(\frac{\theta_+(1-x_1)}{\varepsilon}\Big),$$

with $\theta_\pm < \min(0,\theta)$. The sign of θ depends on the sign of the so-called axial effective drift of the convection-diffusion operator

$$Av = -\mathrm{div}\big(a(y)\nabla v(y)\big) + b(y)\nabla v(y).$$

The talk is based on joint work with Gregoire Allaire.