# Hierarchy of pre-strained behaviors for thin materials

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A hierarchy of four models for thin structures was first obtained by means of a systematic, but formal, approach in [5]. Namely, the nonlinear membrane model, the nonlinear bending model, the slightly nonlinear von Kàrmàn model and the linear model were recovered. Emphasis was put on the loading order of magnitude and on the induced magnitude of the deformations. A rigorous variational derivation of the nonlinear membrane model was then given in [7]. In particular, the degeneracy under compression was exhibited. Then, [6] provided a rigidity result that allowed to rigorously justify the other models.

Some recent papers by physicists [4] suggest models where thin bodies (leaves, gel disks) try to reach an elastic equilibrium state after some given non Euclidean metrics has been imposed and, possibly, fail when considered as 3d bodies (case when the 3d metric tensor has no realization); see also [3]. Which thin limit models should be used? It turns out, as shown in [2], [8], [9], that the driving mechanism is the structure of the 3d Riemann curvature tensor $\mathcal{R}$. Depending on the vanishing of separate entries of $\mathcal{R}$, models that share several features with the Eucidean hierarchy are derived.

## References:

[1] P. Bella & R.V. Kohn, *Metric-induced wrinkling of a thin elastic sheet*, J. Nonlinear Sci. **24** (2014), 1147–1176.

[2] K. Bhattacharya, M. Lewicka & M. Schaffner, *Plates with incompatible prestrain*, to appear in Arch. Rational Mech. Anal.

[3] J. Dervaux & M. Ben Amar, *Morphogenesis of growing soft tissues*, Phys. Rev. Lett. **101** (2008), 068101.

[4] E. Efrati, E. Sharon, & R. Kupferman, *Elastic theory of unconstrained non-Euclidean plates* J. Mech. Phys. Solids, **57** (2009), 762–775.

[5] D.D. Fox, A. Raoult & J.C. Simo, *A justification of nonlinear properly invariant plate theories*, Arch. Rational Mech. Anal. **124** (1993), 157–199.

[6] G. Friesecke, R. James & S. Müller, *A hierarchy of plate models derived from nonlinear elasticity by $\Gamma$-convergence*, Arch. Ration. Mech. Anal. **180** (2006), 183–236.

[7] H. Le Dret & A. Raoult, *The nonlinear membrane model as a variational limit of nonlinear three-dimensional elasticity*, J. Math. Pures Appl. **73** (1995), 549–578.

[8] M. Lewicka & R. Pakzad, *Scaling laws for non-Euclidean plates and the $W^{2,2}$ isometric immersions of Riemannian metrics*, ESAIM: COCV, **17** (2011), 1158–1173.

[9] M. Lewicka & A. Raoult, *Plates with incompatible prestrain of higher order*, submitted.