Travelling waves and critical speed in the presence of nonlinear diffusion

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We study some features of the parametric boundary value problem

$$(P(u'))' - cu' + g(u) = 0 \tag{1}$$

$$u(-\infty) = 0, \quad u(+\infty) = 1,$$

where $D > 0$ and $g(0) = g(1) = 0$, $g > 0$ in $]0, 1[$.

The problem arises when one looks for travelling waves to

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(P(\frac{\partial u}{\partial x})\right) + g(u)$$

c being the wave speed.

Possible models for P are $P(v) = \frac{v}{\sqrt{1-v^2}}$, $\quad P(v) = \frac{v}{\sqrt{1+v^2}}$.

We shall be interested in similarities and differences between those models. Another interesting type of nonlinear diffusion, involving the one-dimensional p-Laplacian, corresponds to the choice $P(v) = |v|^{p-2}v$ in (1). In some instances an advection term may be considered as well.

As in the classical Fisher-Kolmogorov-Petrovskii-Piskounov equations, there exists an interval of admissible speeds $[c^*, \infty)$ and characterizations of the *critical speed* c^* can be obtained.

In particular, we present a variational characterization of the critical speed in the p-Laplacian setting.

The talk is based on papers by Enguiça, Gavioli and Sanchez [1], and joint work with S. Correia [2], Isabel Coelho [3], Maurizio Garrione [4] and A. Gavioli [5].

References:

[1] R. Enguiça, A. Gavioli, L. Sanchez, *Discrete and Continuous Dynamical Systems - Series A*, 33 (2013), 173 191.

[2] S. Correia, L. Sanchez, *Boletim Soc. Port. Mat.* 67 (2012), 165-184.

[3] I. Coelho, L. Sanchez, *Appl. Math. Comput.* 235 (2014), 469-481.

[4] M. Garrione, L. Sanchez, *Boundary Value Problems* 2015, 2015:45

[5] A. Gavioli, L. Sanchez, *Applied Mathematical Letters*, 48 (2015), 47-54.