Existence and regularity of solutions of elliptic equations defined by the operator $\nabla\cdot(a(x)\nabla\ )$

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In this talk we study the problem $\nabla\cdot(a(x)\nabla u) = f$ defined in a bounded open subset Ω of $\mathbb{R}^d$, $d \geq 2$, with $\partial\Omega$ smooth, considering nonhomogeneous Dirichlet or Neumann boundary condition. The function a satisfies $0 < a_* \leq a \leq a^*$.

Assuming that

$$f \in L^p(\Omega) \quad \text{and} \quad a \in W^{1,r}(\Omega), \text{ with } \quad r > d,$$

and imposing natural assumptions on the boundary data, we prove existence and uniqueness of strong solution.

If

$$f \in W^{-1,p}(\Omega) \quad \text{and} \quad a \in \mathcal{C}(\overline{\Omega}),$$

with weaker natural assumptions on the boundary data, we prove existence and uniqueness of weak solution.

Partial results concerning very weak solutions will also be presented.

We will show the importance of the previous results in the study of an electromagnetic induction heating problem.