When a thin periodic layer meets corners : asymptotic analysis of a singular Helmholtz problem.

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In this presentation, we are interested in the solution $u^\delta \in \mathrm{H}^1(\Omega^\delta)$, where Ω^δ is a domain containing a finite periodic layer represented on Fig. 1, of the Helmholtz equation

$$-\Delta u^\delta - (k^\delta)^2 u^\delta = f \quad \text{in } \Omega^\delta, \quad f \in \mathrm{L}^2(\Omega) \tag{1}$$

where the wavenumber k^δ differs from a constant value k_0 on the domain $(-L, L) \times (-\delta, \delta)$ by a function $\hat{k}(\cdot/\delta)$, which is 1-periodic with respect to X_1. We complete problem (1) with homogeneous Neumann boundary conditions on the small holes $\partial\Omega^\delta_{\text{hole}}$, absorbing boundary conditions on $\Gamma_\pm$, and Neumann boundary conditions on the remaining part of the boundary. The main difficulty of this problem is the presence of re-entrant corners at the end of the periodic layer.

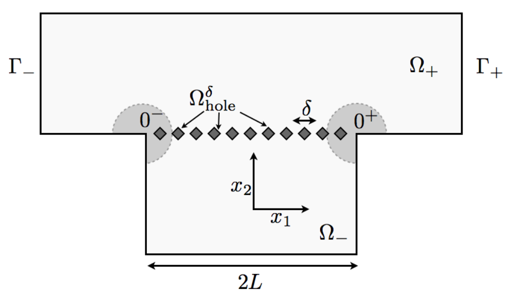

Figure 1: The domain Ω^δ.

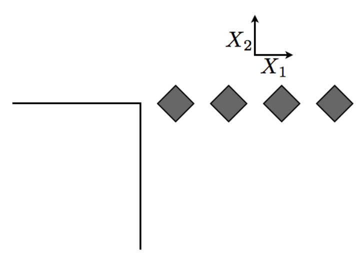

Figure 2: The normalized domain $\hat{\Omega}^-$.

We give here a construction of an asymptotic expansion of the solution u^δ as δ tends to 0. To do so, we combine the method of matched asymptotic expansion close to the corner and the method of surface homogenization close to the layer. In particular, we extend the Kondrat'ev theory, in the spirit of the works of Nazarov [1]. We give as well a justification of the asymptotic expansion (existence, uniqueness, stability) and an error estimate between the solution of problem (1) and its asymptotic expansion.

The presentation is a joint work [2,3] with Bérangère Delourme (Université Paris 13) and Kersten Schmidt (TU Berlin).

References:

[1] S. A. Nazarov. The Neumann problem in angular domains with periodic and parabolic perturbations of the boundary. Tr. Mosk. Mat. Obs., 69:182–241, 2008.

[2] B. Delourme, K. Schmidt, and A. Semin. When a thin periodic layer meets corners: asymptotic analysis of a singular Poisson problem. arXiv:1506.06964. Juni 2015.

[3] A. Semin, B. Delourme, and K. Schmidt. On the homogenization of the Helmholtz problem with thin perforated walls of finite length. *In preparation.*