Developing mathematical tools for real continuum mechanics and physics

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About 10 years ago, I was asked to write an article for the 70th birthday of the founding of Portugaliae Mathematica, and my reason for writing it in French (*Qu'est-ce que l'homogénéisation ?* = What is homogenization?, Vol 64, Fasc. 4, 2007, 389–444) was to recall the time where good Portuguese students in mathematics were trying to study in France (usually Paris) for their thesis: my advisor (Jacques-Louis LIONS) had once told me that (since only Moscow had a similar concentration of good mathematicians around) Paris was one of the two best places in the world for studying mathematics. Later, a less challenging choice developed, to go study for a PhD elsewhere, for example in United States.

I met João-Paulo CARVALHO DIAS and Hugo BEIRÃO DA VEIGA during my studies, and when they invited me to Lisboa in January 1974 (a few months before the end of the dictatorship) I talked for the first time of trying to understand the continuum mechanics behind the partial differential equations which I studied.

The reason was that the theory of homogenization which I started developing with François MURAT in the early 1970s (extending the late 1960s work of Sergio SPAGNOLO, helped with the insight of Ennio DE GIORGI) had given me a way to understand effective properties of mixtures without any probabilistic language, as I had guessed from the work of Évariste SANCHEZ-PALENCIA, who restricted his attention to (partly formal) expansions in a periodically modulated context. I had learned classical mechanics and continuum mechanics, as well as a few approaches to physics (classical, quantic, relativistic, statistical) during my 1965–1967 studies at École Polytechnique (then in Paris), and homogenization gave me a way to attack a crucial question for me: how much of what I was taught made sense from a mathematical point of view?

With the Div–Curl lemma, found (in April or May 1974) with François MURAT, which we extended into the compensated compactness theory in 1976, I devised an improved (and unifying) approach for the question of convergence of approximate solutions to the partial differential equations of continuum mechanics, and when I lectured at EPFL (École Polytechnique Fédérale de Lausanne) in the Spring of 1977, I still had a technical obstacle to overcome with "entropy conditions": at some basic level realistic evolution models are hyperbolic, i.e. before one adds dissipative effects in a dogmatic (and probably wrong) way, and it was clear to me at the time that (apart from 1st principle, i.e. conservation of energy) thermodynamics is flawed.

I understood in the Fall of 1977 how to treat "entropy conditions" for a scalar equation, and in the Summer of 1978 I lectured on my compensated compactness method at Heriot-Watt University, while Ron DI PERNA lectured on his work: he believed enough in my method to work a few years for extending its application to some systems, while I thought that some important new ideas were missing, and moreover many equations from physics are flawed, so that I needed to understand that for deriving better equations.

Around 1979–1980, I guessed that the reason for the spontaneous absorption and emission rules imagined by physicists is just their way to describe nonlocal effects induced by homogenization, in a different way than what Évariste SANCHEZ-PALENCIA had done, since one has to work in an hyperbolic setting.

I also developed something which I now call compensated integrability, in the academic framework of discrete velocity models (first introduced by MAXWELL), corresponding to an idea different from the semi-group approach for evolution problems.

This was when Luísa MASCARENHAS came to work with me in Orsay, so that I asked her questions related to what my program of research looked at the time.

It was a difficult period, which led me to leave the French university system where one was denying me the right of vote. For 5 years (1982–1987), I worked at CEA (Commissariat à l'Énergie Atomique), improving my understanding of physics and finding other defects of classical equations which one should fix. Unable to come back where I had met with a curious form of racism, I exiled myself on the other side of the Atlantic.

I shall describe a few flaws of classical models, with intuition about what one could do, and the tools which I introduced for improving the situation, H-measures and variants, like multi-scales H-measures.