Existence and regularity of solutions to optimal partition problems involving Laplacian eigenvalues

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Let $\Omega \subset \mathbb{R}^N$ be an open bounded domain and $m \in \mathbb{N}$. Given $k_1, \dots, k_m \in \mathbb{N}$, we consider a wide class of optimal partition problems involving Dirichlet eigenvalues of elliptic operators, including the following

$$\inf \left\{ \Phi(\omega_1, \dots, \omega_m) := \sum_{i=1}^{m} \lambda_{k_i}(\omega_i) : \ (\omega_1, \dots, \omega_m) \in \mathcal{P}_m(\Omega) \right\},$$

where $\lambda_{k_i}(\omega_i)$ denotes the k_i–th eigenvalue of $(-\Delta, H_0^1(\omega_i))$ counting multiplicities, and $\mathcal{P}_m(\Omega)$ is the set of all open partitions of Ω, namely

$$\mathcal{P}_m(\Omega) = \{(\omega_1, \dots, \omega_m) : \ \omega_i \subset \Omega \text{ open}, \ \omega_i \cap \omega_j = \emptyset \ \forall i \neq j\}.$$

We prove the existence of an open optimal partition $(\omega_1, \dots, \omega_m) \in \mathcal{P}_m(\Omega)$, proving as well its regularity in the sense that the free boundary $\cup_{i=1}^m \partial\omega_i \cap \Omega$ is, up to a residual set, locally a $C^{1,\alpha}$ hypersurface.

The proof involves a careful study of an associate Schrödinger system with competition terms, as well as several free boundary techniques.

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