Fracture models for elasto-plastic materials as limits of gradient damage models coupled with plasticity: the antiplane case

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- energy dissipated by the damage process:

$$\mathcal{W}(\alpha) := b\int_\Omega W(\alpha) \, dx + \ell \int_\Omega |\nabla \alpha|^2 dx$$

$b, \ell > 0$, $W \colon [0, 1] \to \mathbb{R}$ is a continuous decreasing function with $W(1) = 0$.

The minimum problem

The model proposed by Alessi, Marigo, and Vidoli is based on the minimization of the total energy

$$\mathcal{E}(e, p, \alpha) := \mathcal{Q}(e, \alpha) + \mathcal{H}(p, \alpha) + \mathcal{W}(\alpha)\,,$$

defined for $e \in L^2(\Omega; \mathbb{R}^n)$, $p \in L^1(\Omega; \mathbb{R}^n)$, and $\alpha \in H^1(\Omega; [\alpha_{min}, 1])$, with the constraint $e + p = \nabla u$ for some $u \in W^{1,1}(\Omega)$ satisfying prescribed boundary conditions.

Dependence on ε

- We study the asymptotic behavior of the functional $\mathcal{E}(e, p, \alpha)$ when the minimum problem forces the damage to be concentrated along sets of codimension one. This means that α must be close to 1 on great part of the domain and can be close to 0 just on sets of codimension one.

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- To force this behavior we assume that the three constants α_{min}, b, ℓ depend on a small parameter $\varepsilon > 0$ in a precise way:
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- Then $\mathcal{E}(e, p, \alpha)$ becomes $\mathcal{E}_\varepsilon(e, p, \alpha) := \mathcal{Q}(e, \alpha) + \mathcal{H}(p, \alpha) + \mathcal{W}_\varepsilon(\alpha)$ where $\mathcal{Q}(e, \alpha) := \frac{1}{2}\int_\Omega \alpha|e|^2 dx, \quad \mathcal{H}(p, \alpha) = \int_\Omega \kappa(\alpha)|p|dx,$ and $\mathcal{W}_\varepsilon(\alpha) := \frac{1}{\varepsilon}\int_\Omega W(\alpha)\, dx + \varepsilon\int_\Omega |\nabla\alpha|^2 dx.$

The reduced functional

- We extend $\mathcal{H}$ to $p \in \mathcal{M}_b(\Omega;\mathbb{R}^n)$ which has better compactness properties, by setting $\mathcal{H}(p,\alpha) := \int_\Omega \kappa(\alpha)\,d|p|$.

  Therefore $u \in BV(\Omega)$ and $e + p = Du = \nabla u \mathcal{L}^n + D^s u$.

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- We consider the functional $\mathcal{F}_\varepsilon(u,\alpha)$ defined for $u \in BV(\Omega)$ and $\alpha \in H^1(\Omega)$ by

$$\mathcal{F}_\varepsilon(u,\alpha) := \min_{e,p} \{\mathcal{E}_\varepsilon(e,p,\alpha) : e \in L^2(\Omega;\mathbb{R}^n),\ p \in \mathcal{M}_b(\Omega;\mathbb{R}^n),\ e+p = Du\}$$

if $\delta_\varepsilon \le \alpha \le 1$ and

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$\mathcal{F}_\varepsilon$ represents the energy of the optimal additive decomposition of the displacement gradient (the minimum is achieved at a unique pair (e,p)).

The reduced functional

- The functional $\mathcal{F}_\varepsilon(u, \alpha)$ can be written in an integral form as

$$\mathcal{F}_\varepsilon(u, \alpha) := \int_\Omega f_\varepsilon(\alpha, |\nabla u|)\, dx + \int_\Omega \kappa(\alpha) d|D^s u| + \mathcal{W}_\varepsilon(\alpha)\,,$$

where $f(\alpha, t) := \min_{0 \le s \le t} \left\{ \frac{1}{2}\alpha s^2 + \kappa(\alpha)(t - s) \right\}$ and

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- The asymptotic behavior of the functionals $\mathcal{F}_\varepsilon(u, \alpha)$ is described by their Γ-limit as $\varepsilon \to 0$ in the space $L^1(\Omega) \times L^1(\Omega)$.

  We set $\mathcal{F}_\varepsilon(u, \alpha) = +\infty$ if $u \notin BV(\Omega)$ or $\alpha \notin H^1(\Omega)$.

The limit problem

The limit problem is given by the functional

$$\mathcal{F}(u) := \int_\Omega f(1, |\nabla u|)\, dx + \kappa(1)|D^c u|(\Omega) + \int_{J_u} \Psi(|[u]|)\, d\mathcal{H}^{n-1},$$

defined for u in the space $GBV(\Omega)$ of generalized functions of bounded variation, where

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$$\Psi(t) := \min\left\{\gamma_W(0), \min_{0<\alpha\le 1}\left[\kappa(\alpha)t + \gamma_W(\alpha)\right]\right\},$$

with $\gamma_W(\alpha)$ defined by $\gamma_W(\alpha) := 4\int_\alpha^1 \sqrt{W(s)}\, ds$ for every $\alpha \in [0,1]$.

The purely elastic problem

- Under the constraint $p = 0$ (i.e., $e = \nabla u$), which corresponds formally to $\kappa(\alpha) = +\infty$ for every $\alpha_{min} \leq \alpha \leq 1$, this problem has been studied by Ambrosio and Tortorelli.

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  if $u \in GSBV(\Omega)$ (i.e., $D^c u = 0$), while $\mathcal{F}(u) = +\infty$ if $D^c u \neq 0$.

The fracture term

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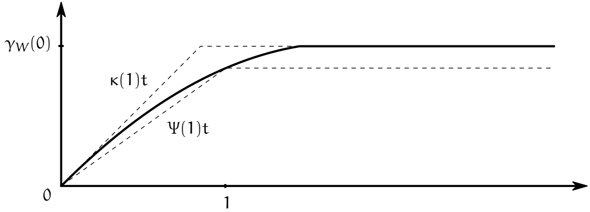

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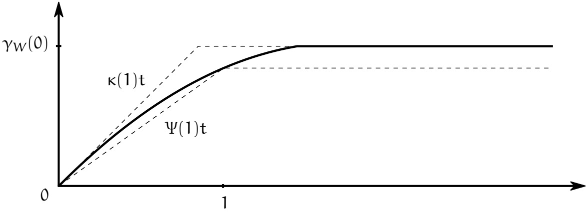

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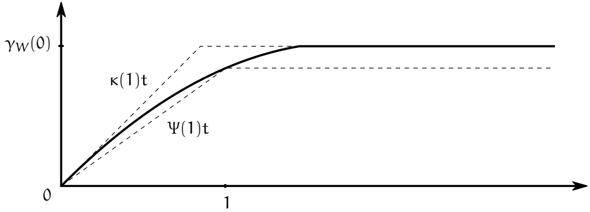

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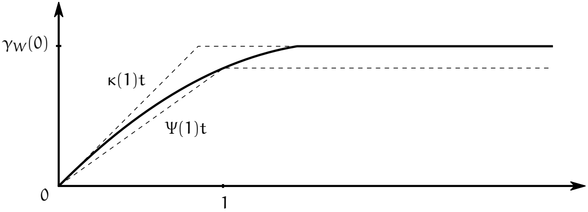

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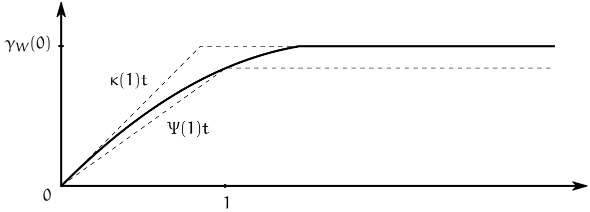

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- $\lim_{t\to+\infty} \Psi(t) = \gamma_W(0)$.

Theorem [Dal Maso-Orlando-T. 2015]

The functionals $\mathcal{F}_\varepsilon$ Γ-converge in $L^1(\Omega)\times L^1(\Omega)$ to the functional $\mathcal{F}_0 : L^1(\Omega)\times L^1(\Omega) \to [0, +\infty]$ defined by

$$\mathcal{F}_0(u, \alpha) = \begin{cases} \mathcal{F}(u) & \text{if } u \in GBV(\Omega) \text{ and } \alpha = 1 \ \mathcal{L}^n\text{-a.e. in } \Omega\,, \\ +\infty & \text{otherwise.} \end{cases}$$

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