Some Numerical Aspects on Crowd Motion - The Hughes' Model

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Here, we study a crowd model proposed by Roger Hughes in [1] and describe a numerical approach to solve it. This model comprises a Fokker-Planck equation coupled with an Eikonal in a 2-dimensional domain Ω with Dirichlet or Neumann data:

$$\begin{cases} \rho_t(x,t) + \operatorname{div}(\rho(1-\rho)^2 Du) = \Delta\rho, \\ |Du(x)|^2 = \dfrac{1}{(1-\rho)^2}. \end{cases} \tag{1}$$

The Fokker-Planck equation gives the evolution of the crowd density ρ. The Eikonal equation determines the optimal direction of movement for each individual if the rest of the population remains frozen.

We study periodic and Dirichlet/Neumann boundary conditions and special cases such as stationary and radial solutions. Open topics include qualitative properties of this system, for example, the decay of L^p norms in time or special solutions, and a qualitative description of its dynamics. Although significant progress has been achieved in [2], even 1-dimensional models are not completely understood.

We propose a numerical method that explores the adjoint structure present in this system, and we compare it to classical schemes of discretization - for 1-D models. One feature of our method is the conservation of the mass of agents.

These models can give important clues to the optimal design of routes. For example, answering the question on whether adding barriers at specific places can help the traffic of people assists in determining the ideal number of exits and their size - ensuring a given evacuation time. Variations in this model include agents with different mobilities and nonlinearities [3].

References:

[1] R. L. Hughes *A continuum theory for the flow of pedestrians* - 2002: Journal of Differential Equations - Transportation Research Part B - 507-535.

[2] M. Di Francesco, P. Markowich, J.-F. Pietschmann and M.-T. Wolfram *On the Hughes' model for pedestrian flow: the one-dimensional case* - 2011: Journal of Differential Equations - Vol. 250 - 1334-1362.

[3] M. Burger, M. Di Francesco, P. Markowich and M.-T. Wolfram *Mean field games with nonlinear mobilities in pedestrian dynamics* - 2013.