GBT-based finite element to assess the buckling behaviour of steel–concrete composite beams

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2. Formulation
3. Examples
4. Concluding remarks
1. Introduction

Main features of GBT:

• It is a thin-walled beam theory that includes cross-section deformation.

• The displacement field is approximated using pre-determined cross-section deformation modes.

• Powerful tool to analyse linear/non linear behaviour, buckling loads and dynamic behaviour.

• Constitutes a computationally efficient alternative to shell finite elements and finite strips.
1. Introduction

GBT modal decomposition

\[
\text{Bending} + \text{Torsion} + \text{Distortion} + \text{Local}
\]
1. Introduction

General goal (on-going work):
• Develop GBT-based finite elements for steel-concrete composite beams.

Problem statement:
• GBT was developed for linear elastic materials and is very efficient, due to its modal nature and the use of constraints to the stress and strain fields.
• The efficiency is partially lost with material non-linearity, thus formulations for steel-concrete beams must be carefully “fine-tuned”.
1. Introduction

Achievements:
• A geometrically linear GBT finite element was developed, accounting for concrete cracking/crushing, steel plasticity and shear lag.
• GBT-FE for calculating buckling (bifurcation) loads accounting for concrete cracking and shear lag. Compliance with Eurocode 4 (not exclusively).

Present goal:
• Include creep, shrinkage, together with cracking and shear lag.
1. Introduction

Buckling in steel-concrete composite beams

Distortional  Local  The inverted U-frame
1. Introduction

EC4:

6.4.3 Simplified verification for buildings without direct calculation

(1) A continuous beam (or a beam within a frame that is composite throughout its length) with Class 1, 2 or 3 cross-sections may be designed without additional lateral bracing when the following conditions are satisfied:

a) Adjacent spans do not differ in length by more than 20% of the shorter span. Where there is a cantilever, its length does not exceed 15% of that of the adjacent span.

b) The loading on each span is uniformly distributed, and the design permanent load exceeds 40% of the total design load.

c) The top flange of the steel member is attached to a reinforced concrete or composite slab by shear connectors in accordance with 6.6.

d) The same slab is also attached to another supporting member approximately parallel to the composite beam considered, to form an inverted-U frame as illustrated in Figure 6.11.

e) If the slab is composite, it spans between the two supporting members of the inverted-U frame considered.

f) At each support of the steel member, its bottom flange is laterally restrained and its web is stiffened. Elsewhere, the web may be un-stiffened.

g) If the steel member is an IPE section or an HE section that is not partially encased, its depth \( h \) does not exceed the limit given in Table 6.1.

h) If the steel member is partially encased in concrete according to 5.5.3(2), its depth \( h \) does not exceed the limit given in Table 6.1 by more than 200 mm for steel grades up to S355 and by 150 mm for grades S420 and S460.

Note: Provisions for other types of steel section may be given in the National Annex.

Table 6.1: Maximum depth \( h \) (mm) of unencased steel member for which clause 6.4.3 is applicable

<table>
<thead>
<tr>
<th>Steel member</th>
<th>Nominal steel grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S 235</td>
</tr>
<tr>
<td>IPE</td>
<td>600</td>
</tr>
<tr>
<td>HE</td>
<td>800</td>
</tr>
</tbody>
</table>

6.4.2 Verification of lateral-torsional buckling of continuous composite beams with cross-sections in Class 1, 2 and 3 for buildings

\[
M_{b,Rd} = \lambda_{LT} M_{Rd}
\]

\[
\lambda_{LT} = \sqrt{\frac{M_{Rk}}{M_{cr}}}
\]

Must account for:
- concrete cracking
- shear lag
- local buckling (effective widths)
- construction sequence
- concrete creep and shrinkage

How about a FE which takes into account all relevant effects?
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2. Formulation

Cross-section analysis

Pre-buckling analysis

Additional modes for the buckling analysis

Rebars smeared along x and y
2. Formulation

Pre-buckling analysis

• Cracking is the only source of material non-linearity.
• For the steel beam, $S_{yy} = 0$ is assumed, together with elastic behavior.
• The rebars behave uniaxially and elastically.
• For concrete, $S_{yy} = 0$ is also assumed and cracking inevitably occurs. A shear reduction factor is used for the tangential stresses $S_{xy} = \beta G\gamma_{xy}$. An uniaxial law is adopted for $S_{xx}$, with null tensile strength and a linear compressive branch.
• A standard displacement-based finite element formulation is implemented.
• Gauss quadrature is employed, with 3 points along $x$ and an arbitrary number of points along $y$ and $z$ (5 points proved to be sufficient).
2. Formulation

Buckling analysis

• Only the $S_{xx}$ membrane stresses are retained and $S_{yy} = E_{yy} = 0$ are assumed.

• The discretized buckling eigenvalue problem reads

\[
(K_t + \lambda G) \Delta d = 0
\]

\[
K_t = \int_{\Omega} \begin{bmatrix} \psi & \psi_{,x} \\ \psi_{,xx} & \psi_{,xx} \end{bmatrix}^T \left( \Xi_e^M \tilde{C}_t^M \Xi_e^M + (\Xi_e^B)^T \tilde{C}_t^B \Xi_e^B \right) \begin{bmatrix} \psi \\ \psi_{,x} \end{bmatrix} d\Omega,
\]

\[
G = \int_{\Omega} t S_{xx}^M \psi_{,x}^T \left( \bar{v}^T \bar{v}^T + \bar{w} \bar{w}^T \right) \psi_{,x} d\Omega,
\]

Steel beam

\[
\tilde{C}_t^M = \begin{bmatrix} E_s t & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_s t \end{bmatrix}, \quad \tilde{C}_t^B = \frac{E_s t^3}{12(1-\nu_d^2)} \begin{bmatrix} 1 & \nu_d & 0 \\ \nu_d & 1 & 0 \\ 0 & 0 & \frac{1-\nu_d}{2} \end{bmatrix}
\]

Rebars

\[
\tilde{C}_s^M = \begin{bmatrix} E_s A_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{C}_c^M = \begin{bmatrix} E_{tec} h_c^3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \beta G_c h_c \end{bmatrix}, \quad \tilde{C}_c^B = \frac{E_{tec} h_c^3}{12(1-\nu_c^2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\beta G_c h_c}{12} \end{bmatrix}
\]

Concrete

zero if cracked
includes transverse rebars
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3. Examples

3.1 Linear Elastic Shear lag

GBT: 7 modes / 8 elements
$L = 6 \text{ m}$

$\sigma_{xy}$ (kPa)

$L = 8 \text{ m}$

$\sigma_{xy}$ (kPa)

Key

- GBT
- Brick
3. Examples

3.2 Creep analysis
3. Examples

3.2 Collapse of composite beams

Steel beam
- $E_a = 210$ GPa
- $\nu_a = 0.3$
- $f_{ay} = 235$ MPa

Concrete
- $E_c = 31$ GPa
- $\nu_c = 0.2$
- $f_c = 33$ MPa
- $\varepsilon_{c1} = 0.0021$

Reinforcement
- $E_s = 200$ GPa
- $f_{sy} = 500$ MPa
- $A_s/A_c = 0.4\%$

Dimensions:
- $h_c = 100$
- $t_r = 10$
- $t_w = 7$
- $h_w = 180$

(mm)

Graph: Load (kN/m) vs. Mid-span vertical displacement (m)
3. Examples

3.3 Buckling of simply supported beams under negative moment

Steel beam
- $E_s = 210$ GPa
- $f_s = 0.3$
- $h_w = 800, 1200$ mm
- $b_w = 15$ mm
- $b_r = 300$ mm
- $t_r = 30$ mm

Reinforcement
- $E_s = 200$ GPa
- $A_{s, long} = 7.854$ cm$^2$/m

Concrete
- $E_c = 37$ GPa
- $f_c = 0.1$
- $h_c = 0.20$ m
- $b_{c1} = 1.50$ m
- $b_{c2} = 2.00$ m

GBT:
- 7 modes
- 10 elements
3. Examples

3.4 Buckling of two-span beams subjected to uniformly distributed load

Uncracked analysis

Steel beam
\( E_s = 210 \text{ GPa} \)
\( v_s = 0.3 \)
\( h_s = 800 \text{ mm} \)
\( t_w = 15, 30 \text{ mm} \)
\( b_s = 300 \text{ mm} \)
\( t = 30 \text{ mm} \)

Reinforcement
\( E_s = 200 \text{ GPa} \)
\( A_{s,long} = 1.5 \% \)
\( A_{s,transv} = 1.5 \% \)

Concrete
\( E_c = 37 \text{ GPa} \)
\( v_c = 0 \)
\( h_c = 0.20 \text{ m} \)
\( b_{2,1} = 1.50 \text{ m} \)
\( b_{2,2} = 2.00 \text{ m} \)

Intermediate support

Shell
15 mm 5.4%
30 mm 1.7%
3. Examples

3.4 Buckling of two-span beams subjected to uniformly distributed load

Steel beam
- $E_s = 210$ GPa
- $\nu_s = 0.3$
- $h_w = 800$ mm
- $b_w = 15, 30$ mm
- $b_r = 300$ mm
- $t_r = 30$ mm

Reinforcement
- $E_r = 200$ GPa
- $A_{s, long} = 1.5\%$
- $A_{s, transv} = 1.5\%$

Concrete
- $E_c = 37$ GPa
- $\nu_c = 0$
- $h_c = 0.20$ m
- $b_{c, 1} = 1.50$ m
- $b_{c, 2} = 2.00$ m

Cracked analysis

Shell

2.6 - 7.5%

$t_w = 15$ mm
transversally cracked
transversally uncracked

$t_w = 30$ mm
transversally cracked
transversally uncracked

Modal participation

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3. Examples

3.5 Other features

- Non Uniform Cross Section.

<table>
<thead>
<tr>
<th></th>
<th>( p_{cr} ) (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>shell</td>
<td>324.7</td>
</tr>
<tr>
<td>GBT, ( S_{xy} + S_{xy} )</td>
<td>355.3 (9.4%)</td>
</tr>
<tr>
<td>GBT, ( S_{xx} )</td>
<td>331.0 (1.9%)</td>
</tr>
</tbody>
</table>

- Influence of the long (I)-to-short (II) term loading ratio
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4. Concluding remarks

- A computationally efficient GBT-FE for calculating bifurcation loads of steel-concrete beams was proposed. It accounts for shear lag, concrete cracking and distortional/local buckling effects.

- The pre-buckling analysis is carried out with only 7 deformation modes (5 for symmetric sections) and includes concrete cracking and shear lag. The influence of creep and construction stages may be taken into account.

- The buckling analysis involves 8 additional deformation modes (1 D, 7 LP). Through-thickness integration is avoided and it is possible to define, individually, the various bending/membrane stiffness terms for concrete (e.g. we can prescribe a transversally cracked slab as specified in EC4).

- The efficiency of the FE stems from a combination of the GBT versatility to model the behavior of thin-walled members and the use of assumptions concerning stresses and strains. This way, a reduced number of modes leads to accurate results.