Multi-physics variational methods for power and magnet applications

Enric Pardo, Milan Kapolka

Francesco Grilli

Thomas Reis, Simon Wolfstaedler

Mark Ainslie, Jan Srpcic, Difan Zhou, Anthony Dennis

The third dimension can kill your model

Your formulation does not take the third dimension into account

Too time-consuming numerical method

Physical model may not be good!

Force-free anisotropy in the **E**(**J**) relation

Multi-physics variational methods for power and magnet applications

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Acknowledgements

Horizon 2020 projects

Funded by the European Commission Grant no. 723119 and no. 721019

National projects

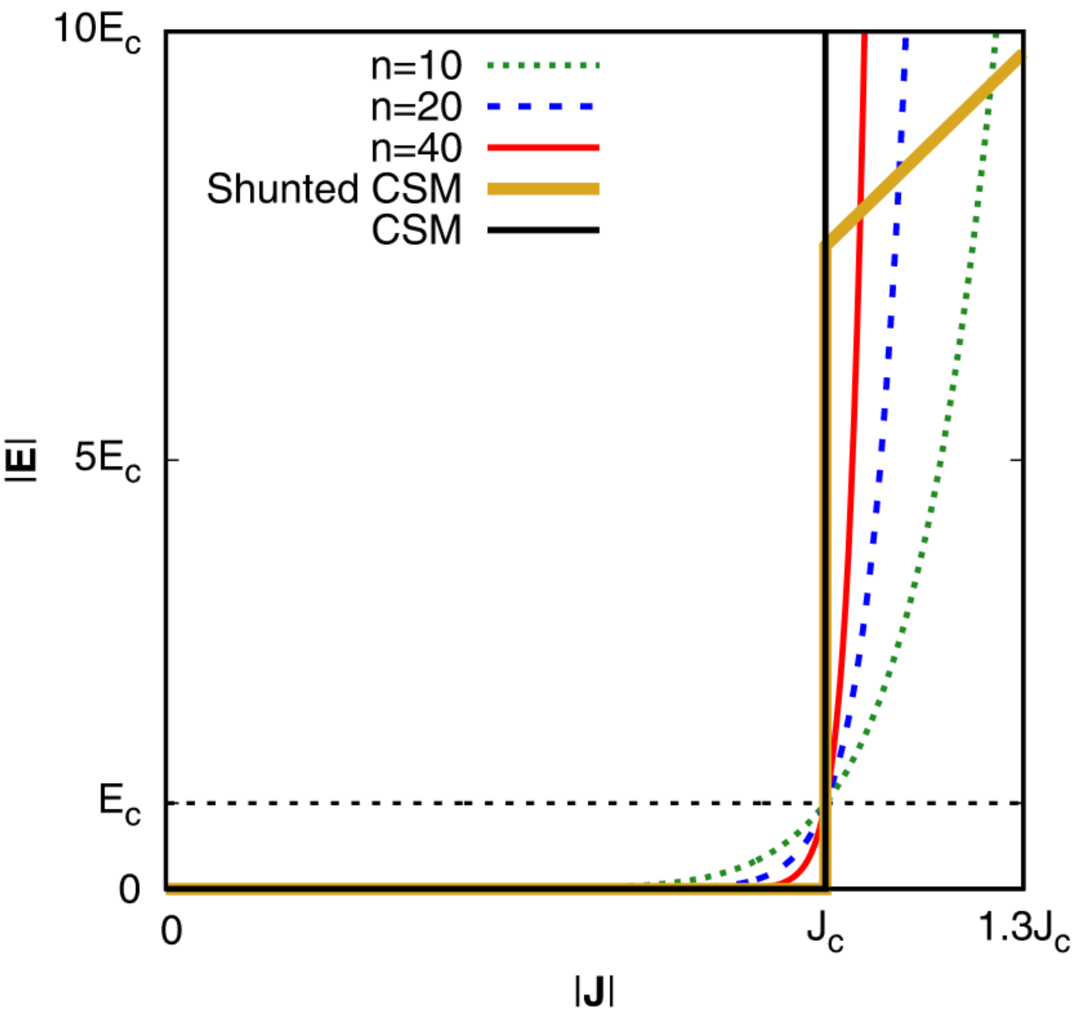
APVV Grant no. 89807
VEGA Grant no. 2/0097/18

Non-linear eddy currents

Interaction with ferromagnetic material

Electro-thermal modelling

Non-linear eddy current problem



electric field

current density

$$\mathbf{E}(\mathbf{J}) = E_c \left(\frac{|\mathbf{J}|}{J_c} \right)^n \frac{\mathbf{J}}{|\mathbf{J}|}$$

constants

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Non-linear eddy currents

Maths

Numerical method

Cross-field demagnetization

Force-free anisotropy

Interaction with ferromagnetic material

Electro-thermal modelling

Minimum Magnetic Entropy Production (MEMEP)

Equations

$$\mathbf{E}(\mathbf{J}) = -\frac{\Delta \mathbf{A}}{\Delta t} - \nabla \phi$$

for given **E(J)** relation

$$\nabla \cdot \mathbf{J} = 0$$

are the Euler equations of

J change between two time instants

A from Δ**J**

A from applied field

scalar potential

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_J}{\Delta t} + \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_a}{\Delta t} + U(\mathbf{J}) + \nabla \phi \cdot \mathbf{J} \right]$$

$$U(\mathbf{J}) = \int_0^{\mathbf{J}} \mathrm{d}\mathbf{J}' \cdot \mathbf{E}(\mathbf{J})'$$

E Pardo et al. DOI: 10.1088/0953-2048/28/4/044003

Minimum Magnetic Entropy Production (MEMEP)

You find **J** by minimizing the functional

scalar potential

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_J}{\Delta t} + \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_a}{\Delta t} + U(\mathbf{J}) + \nabla\phi \cdot \mathbf{J} \right]$$

$$U(\mathbf{J}) = \int_0^{\mathbf{J}} \mathrm{d}\mathbf{J}' \cdot \mathbf{E}(\mathbf{J})'$$

Cross-sectional models:

Good for voltage constrains

If you keep the current constrains,
you can ignore the scalar potential

E Pardo et al. DOI: 10.1088/0953-2048/28/4/044003

Novel 3D variational principle

M Kapolka, E Pardo DOI: 10.1016/j.jcp.2017.05.001

$$\mathbf{J} = \nabla \times \mathbf{T}$$ → effective magnetization

T is the minimization variable

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_J}{\Delta t} + \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_a}{\Delta t} + U(\mathbf{J}) \right]$$

or

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta \mathbf{T} \cdot \frac{\Delta \mathbf{B}_J}{\Delta t} + \Delta \mathbf{T} \cdot \frac{\Delta \mathbf{B}_a}{\Delta t} + U(\nabla \times \mathbf{T}) \right]$$

You can forget about scalar potential!

Still easy to take transport currents into account

The solution is a minimum
and it is unique

Because the second variation is always positive

$$\delta^2 L > 0$$

$$\delta^2 L[\Delta \mathbf{J}] = \frac{1}{2}\epsilon^2 \int_V \mathrm{d}V \int_V \mathrm{d}V' \frac{\mu_0}{4\pi\,\Delta t} \frac{\mathbf{g}(\mathbf{r})\cdot\mathbf{g}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \frac{1}{2}\epsilon^2 \int_V \mathrm{d}V\, \mathbf{g}(\mathbf{r}) \overline{\overline{\rho}}(\mathbf{J}_0 + \Delta\mathbf{J})\mathbf{g}(\mathbf{r})$$

We made this check with all functionals!

M Kapolka, E Pardo DOI: 10.1016/j.jcp.2017.05.001

Non-linear eddy currents

- Maths
- Numerical method
- Cross-field demagnetization
- Force-free anisotropy

Interaction with ferromagnetic material

Electro-thermal modelling

Self-programmed method in C++

Advantages:

Fast and paralel

No license

Expensive for super-computers

You can modify it at will

Efficient for shapes

with large air gaps

REBCO coils

Stack of tapes

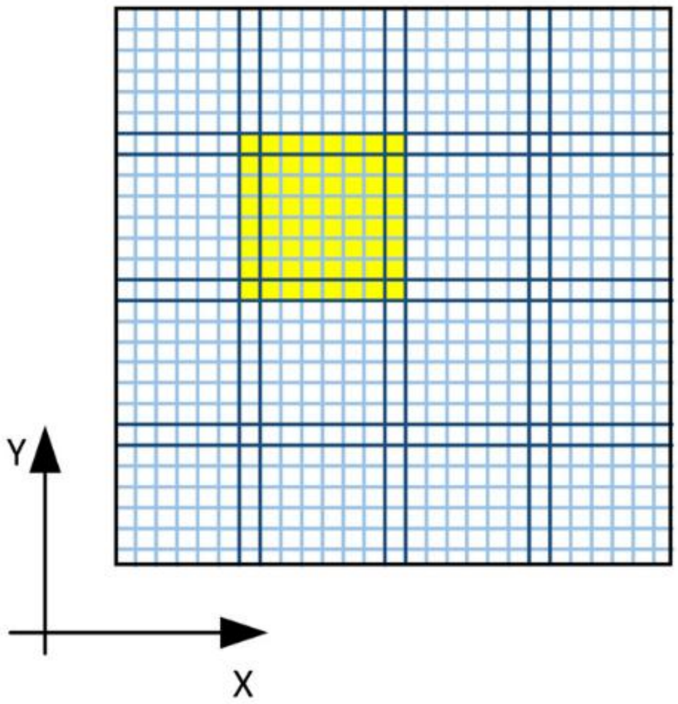
Self-programmed method in C++

Disadvantages:

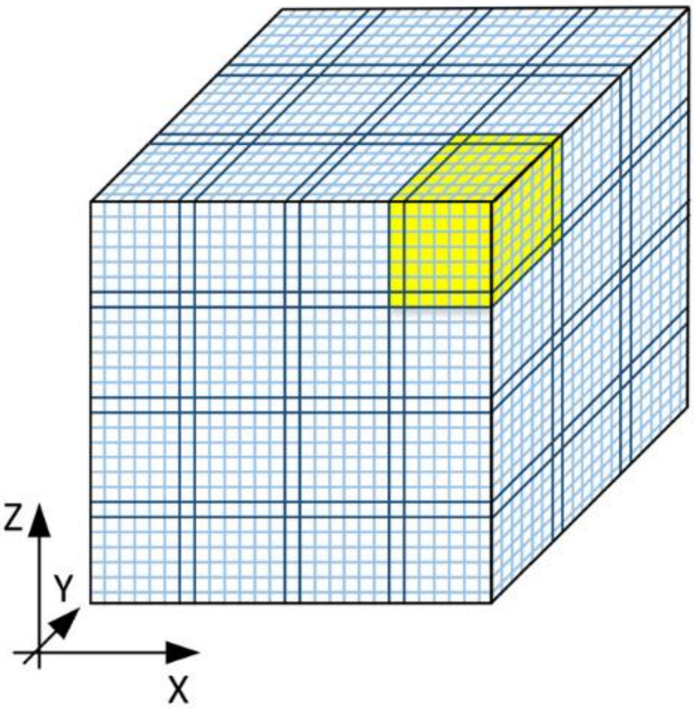
Not easy post-processing

Difficult to make complicated shapes

Dividing into sectors speeds up calculations



surface



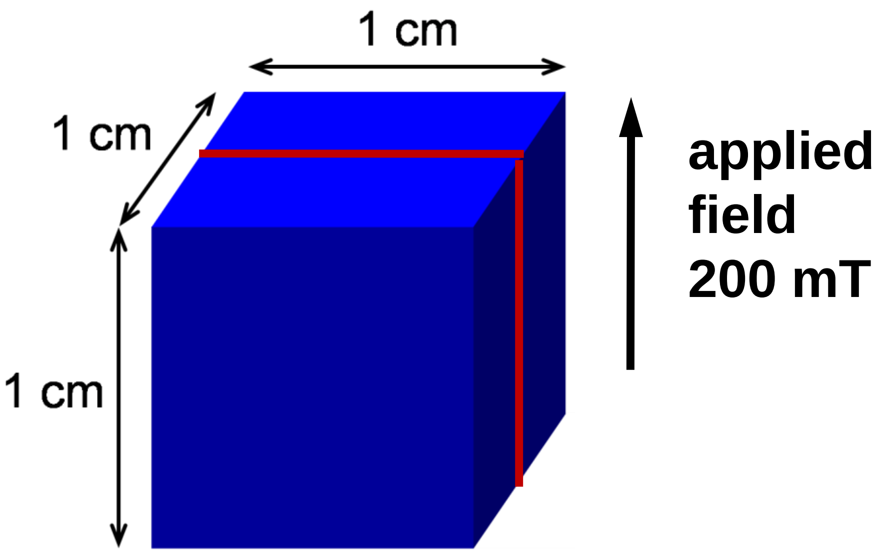
3D body

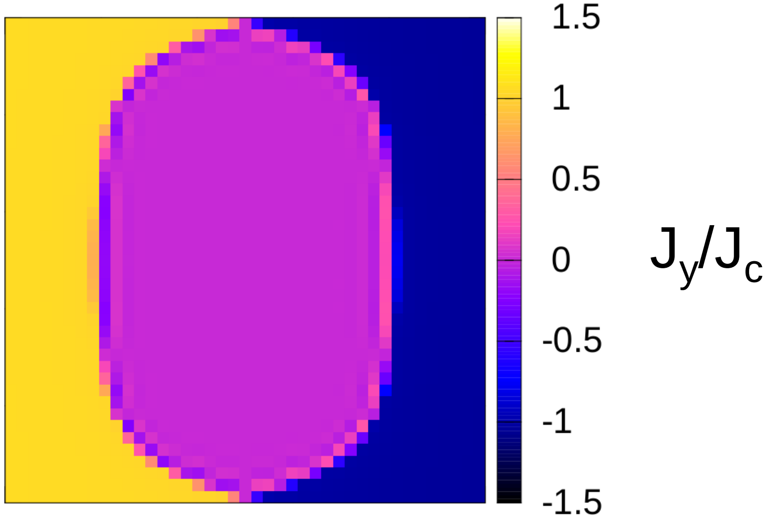
3D cube: benchmark 5

Frequency: **50 Hz** sinusoidal

Power-law exponent: **100**

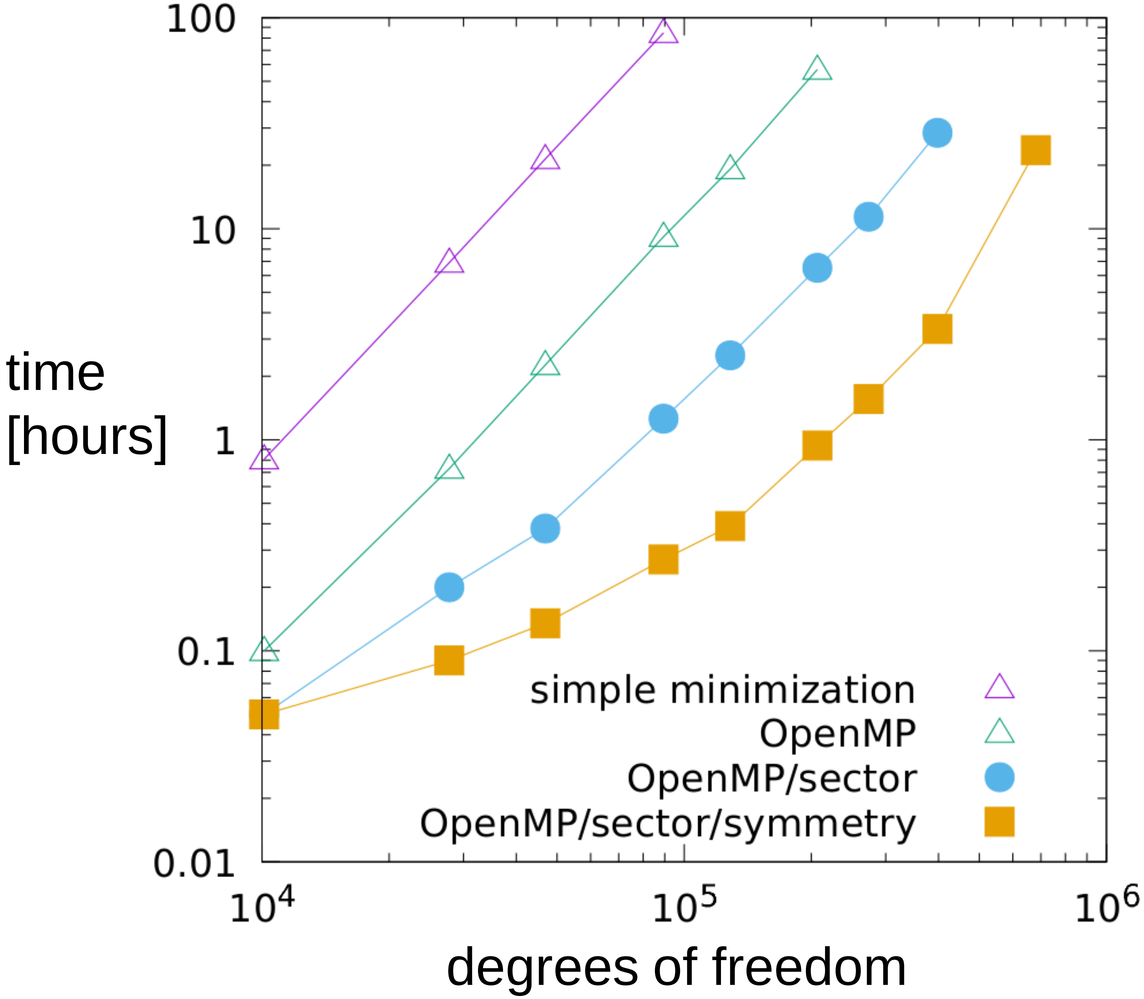
J_c= **10^8 A/m^2**





E Pardo et al. DOI: 10.1088/1361-6668/aa69ed

90% parallel efficiency in one computer



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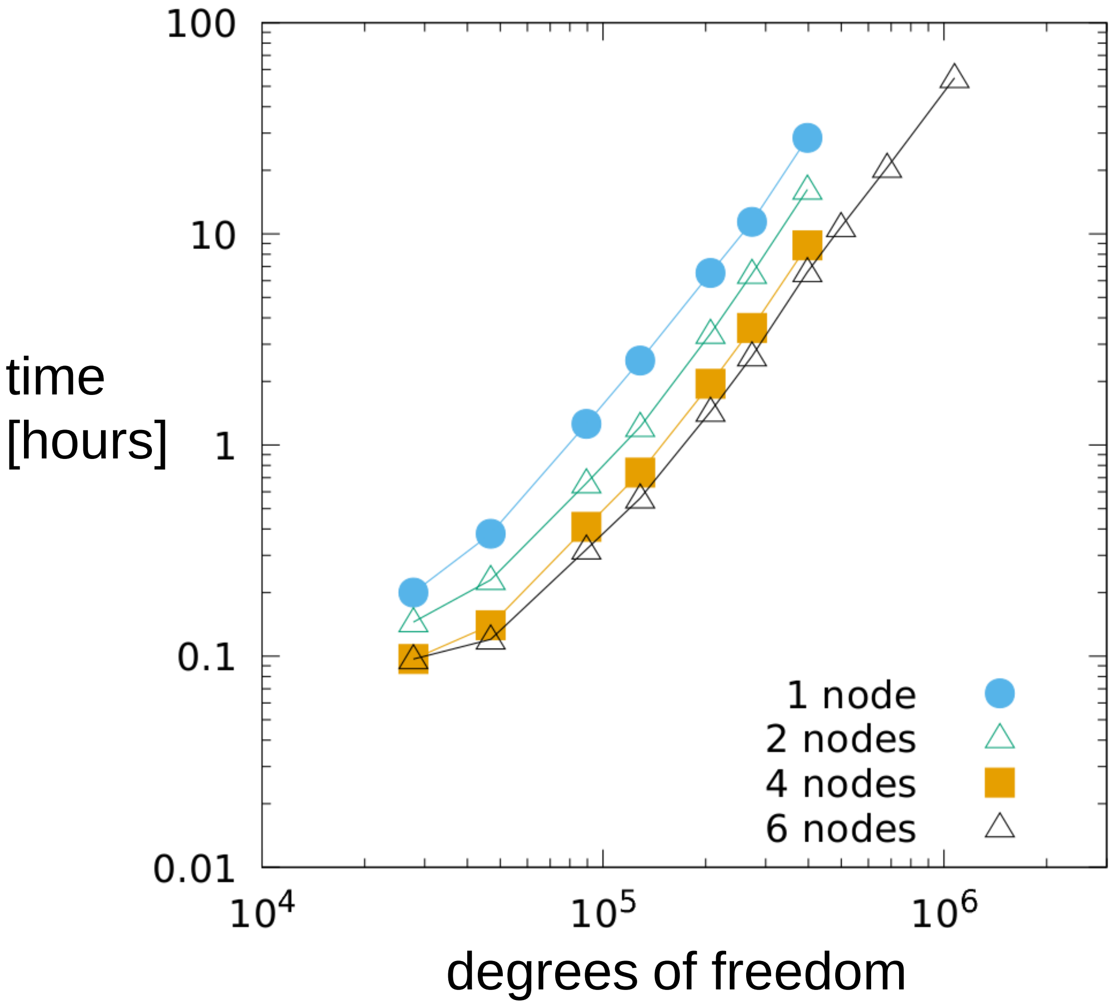
With computer clusters:
more than 1 million degrees of freedom



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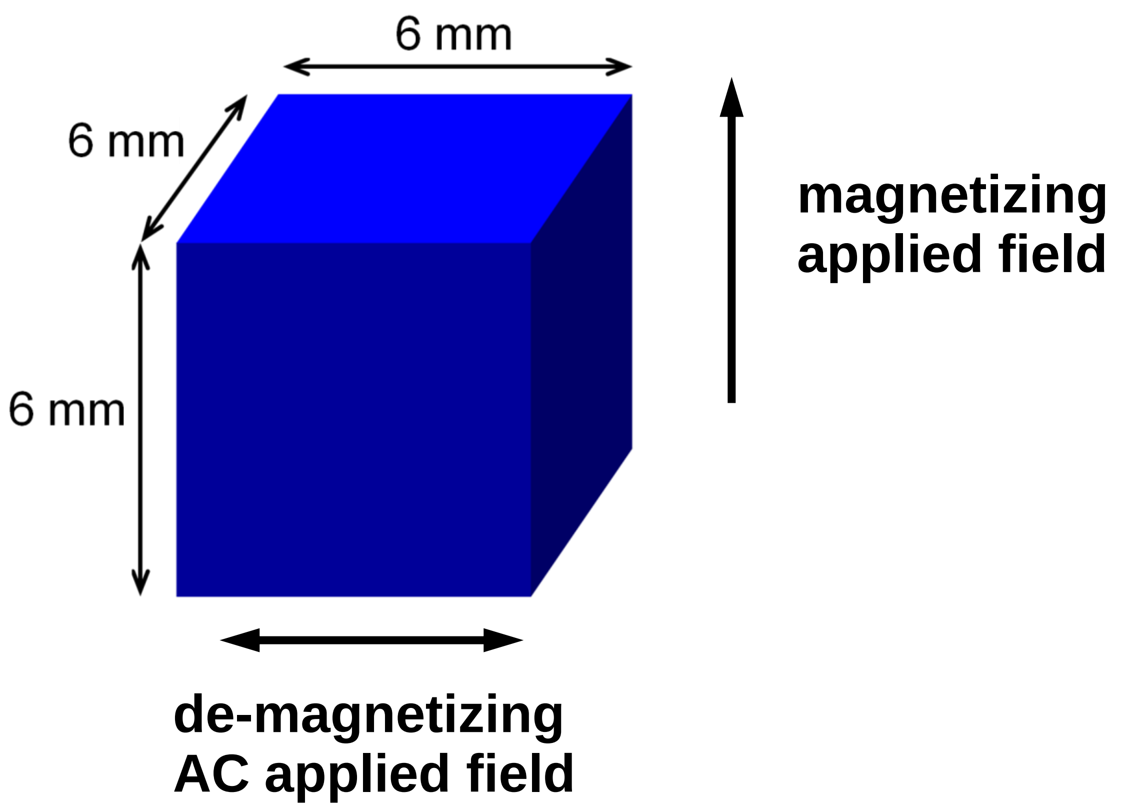
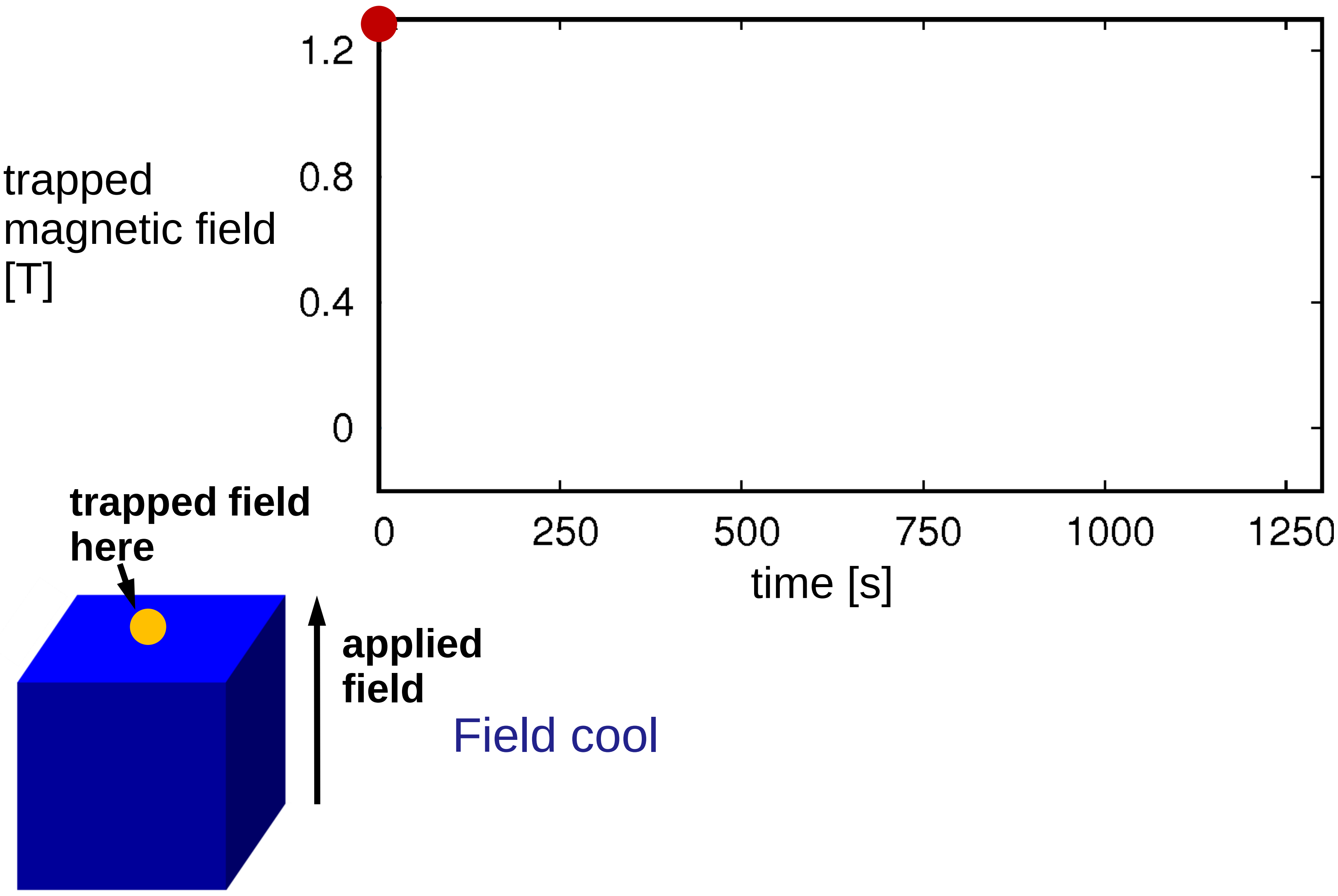
Cross-field demagnetization



Assumed constant J_c

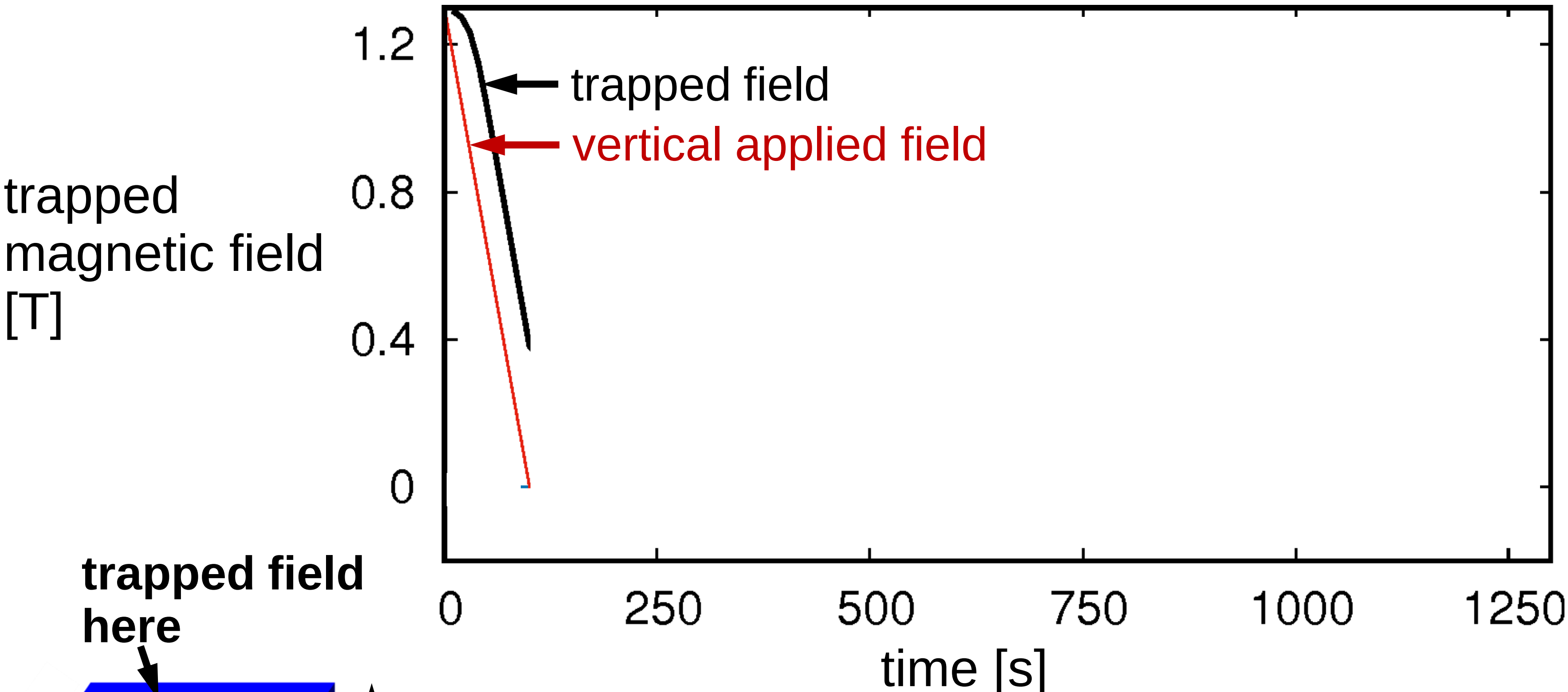
GdBCO sample prepared in Cambridge
in bulk superconductivity group

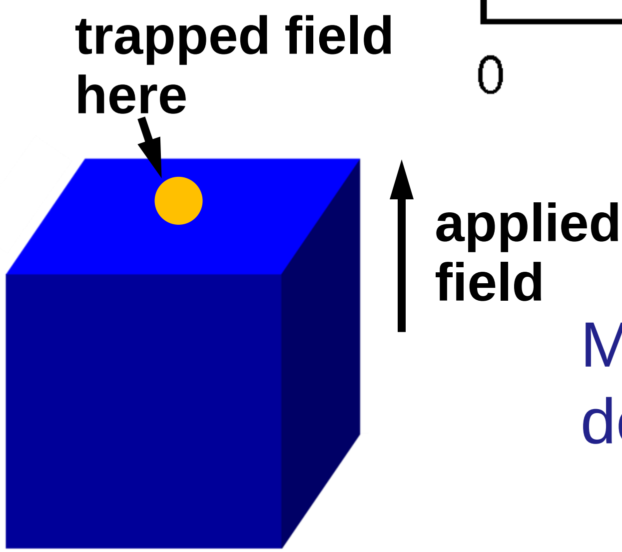
Cross-field demagnetizes sample



Field cool

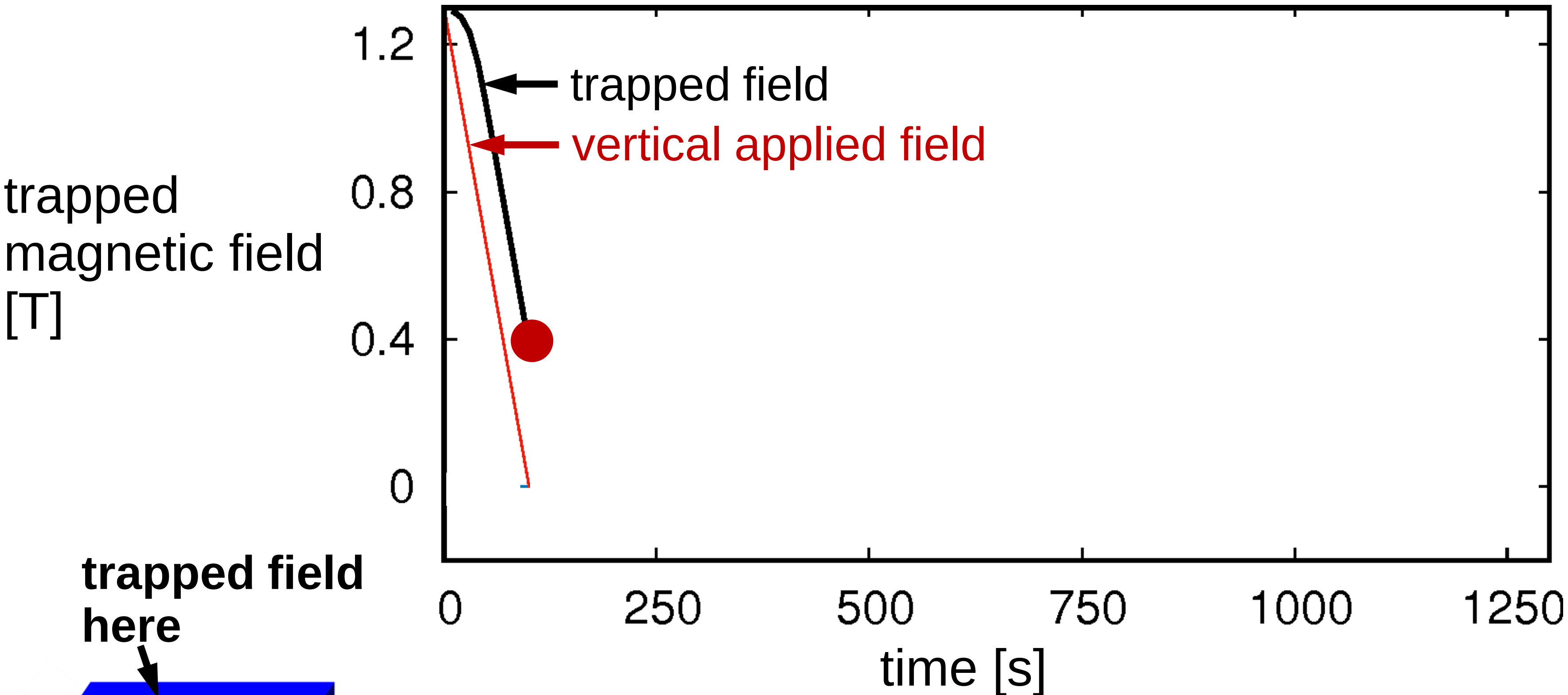
Cross-field demagnetizes sample

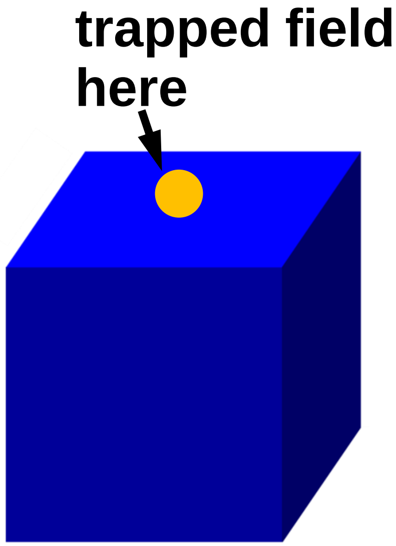




Magnetic field decrease

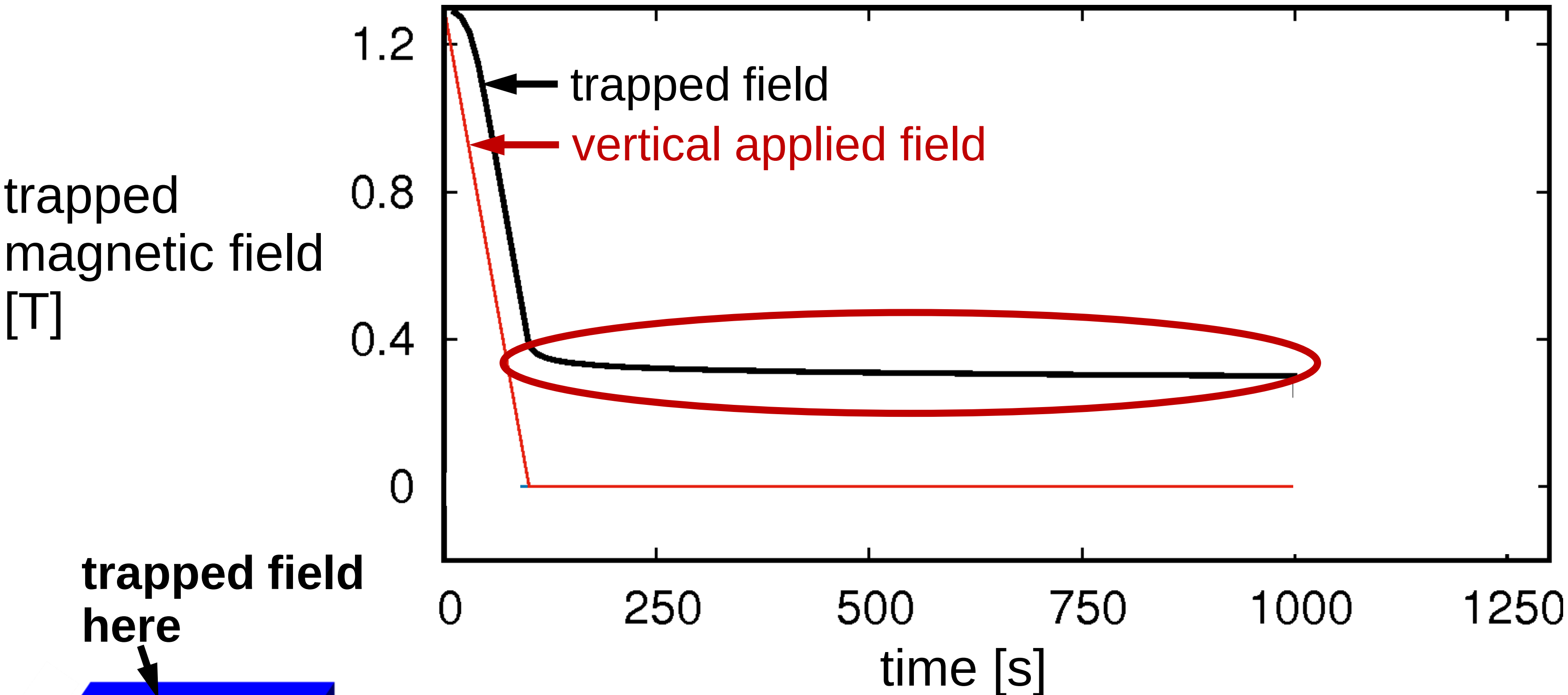
Cross-field demagnetizes sample

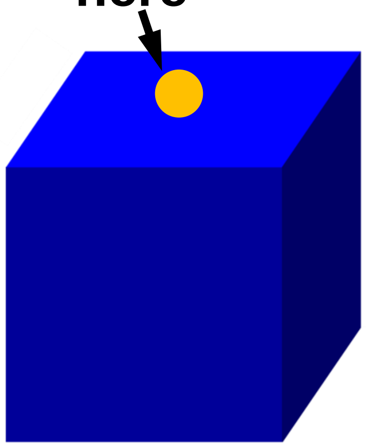




Remanence

Cross-field demagnetizes sample

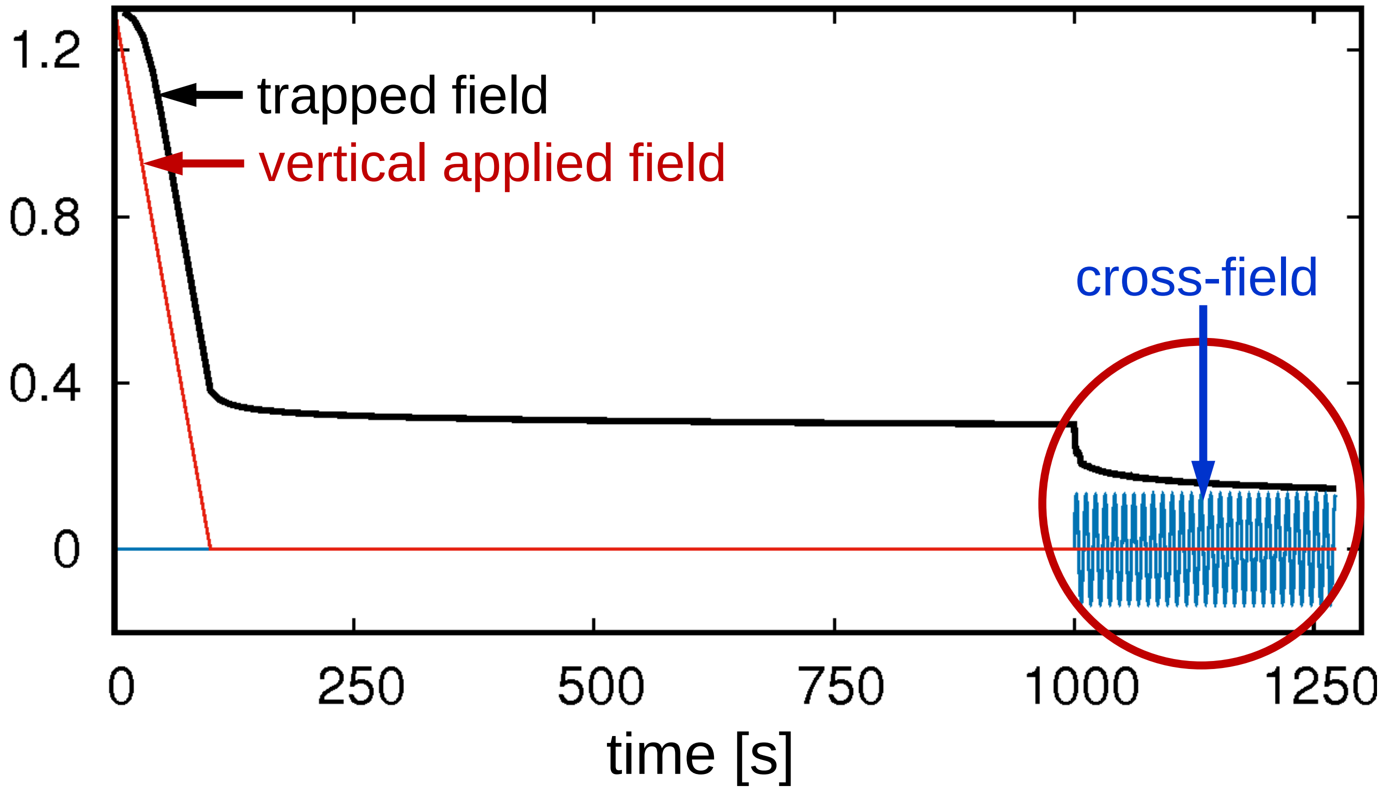




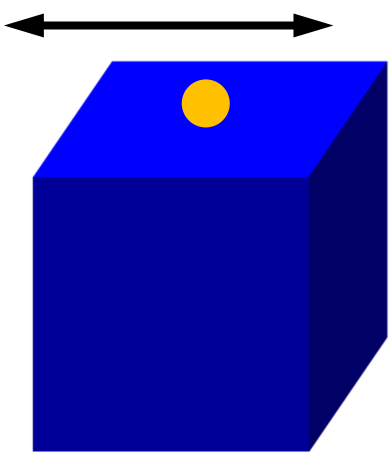
Relaxation

Cross-field demagnetizes sample

trapped
magnetic field
[T]



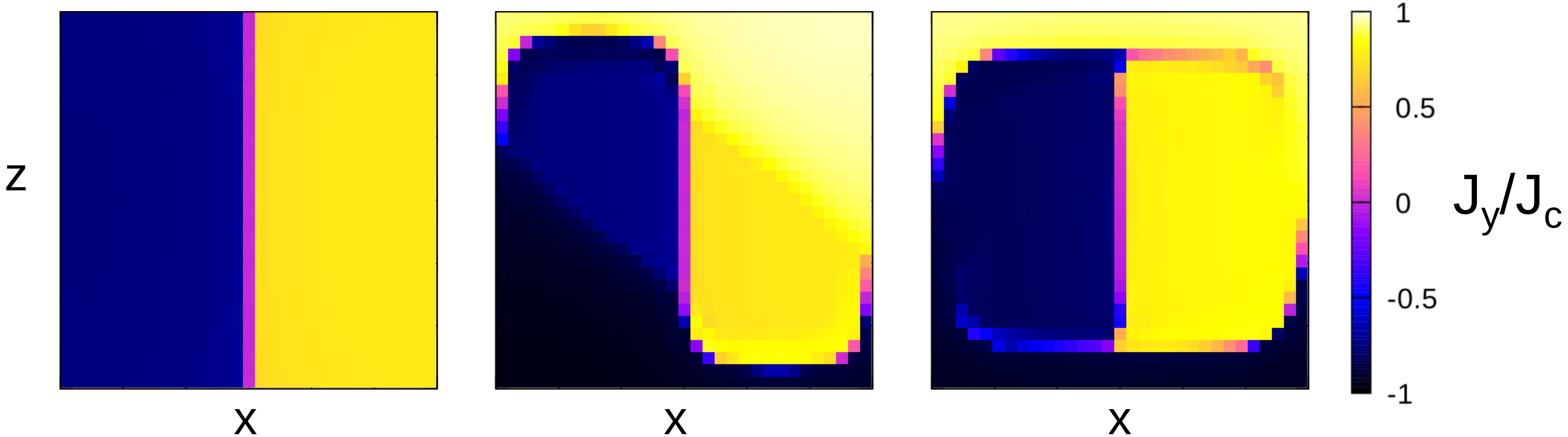
AC
applied field

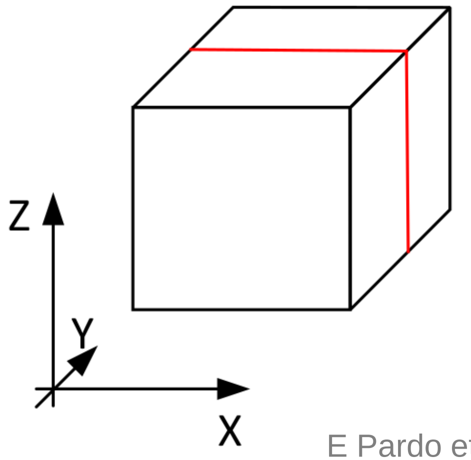


Cross-field
demagnetization

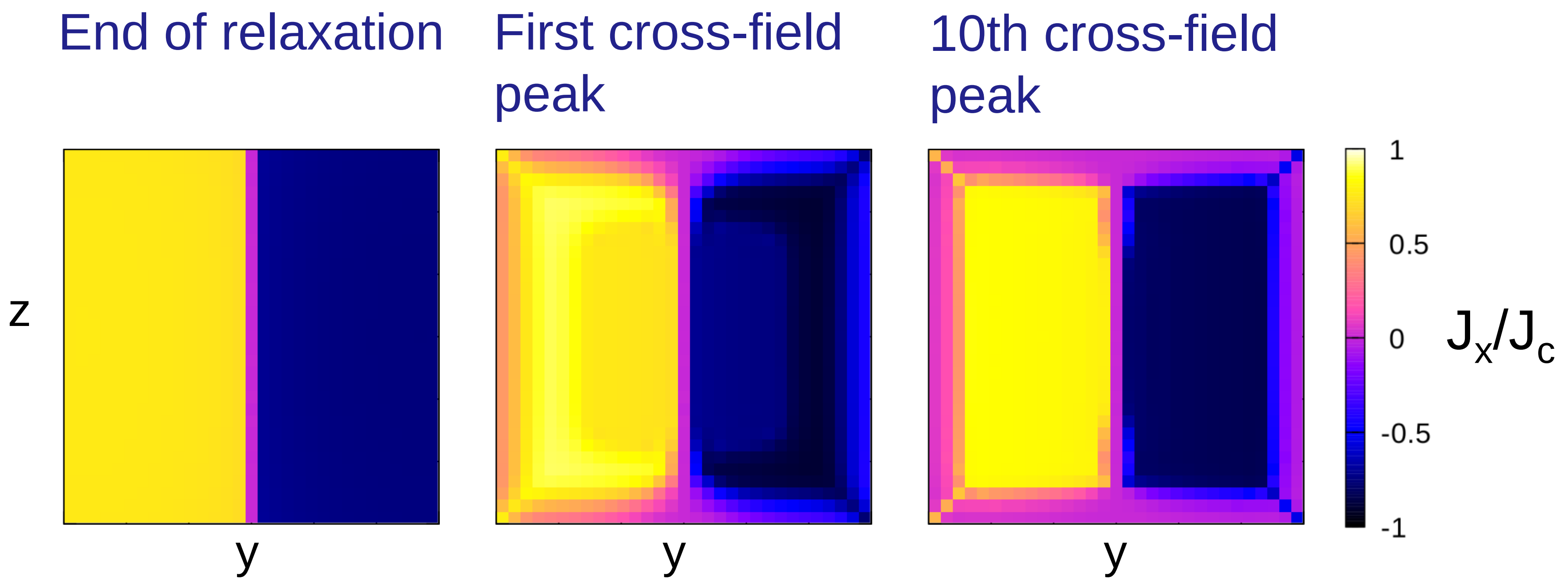
Transverse field reduces space for magnetization currents

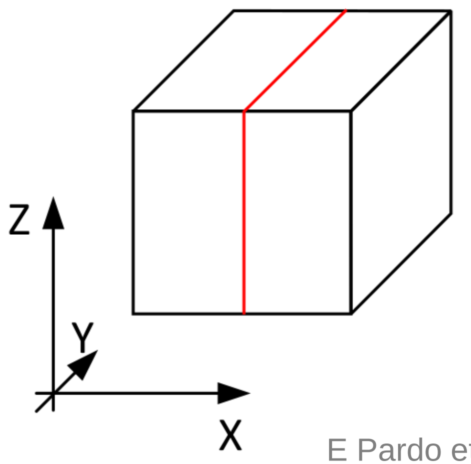
End of relaxation | First cross-field peak | 10th cross-field peak





Demagnetizing currents erase x-component of magnetization currents





Full 3D picture of demagnetizing currents

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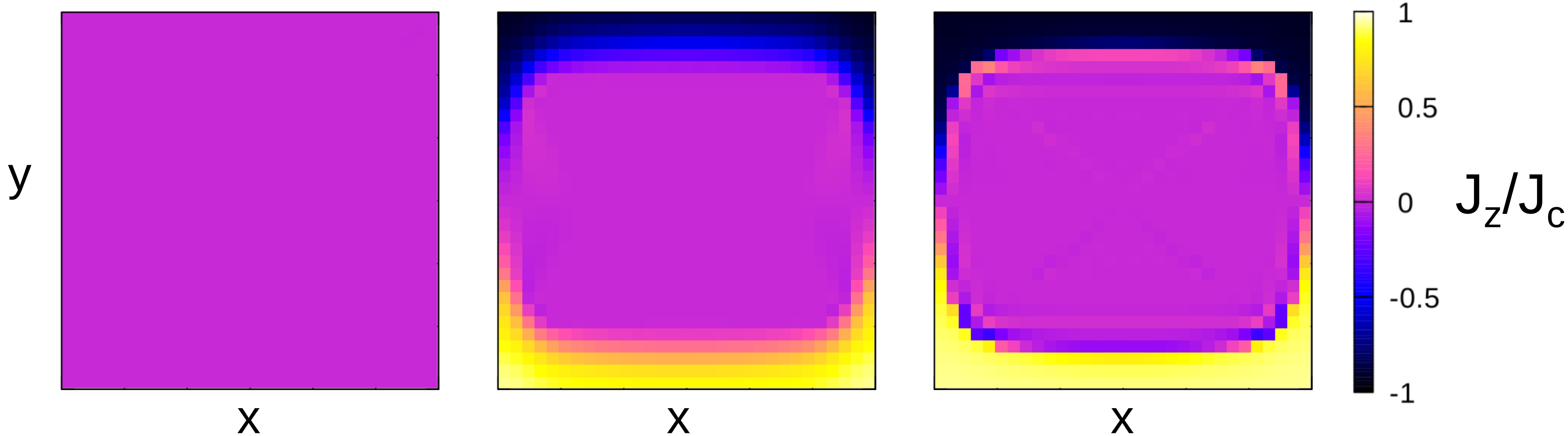
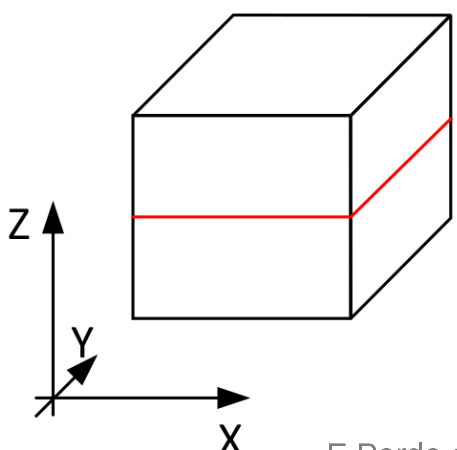
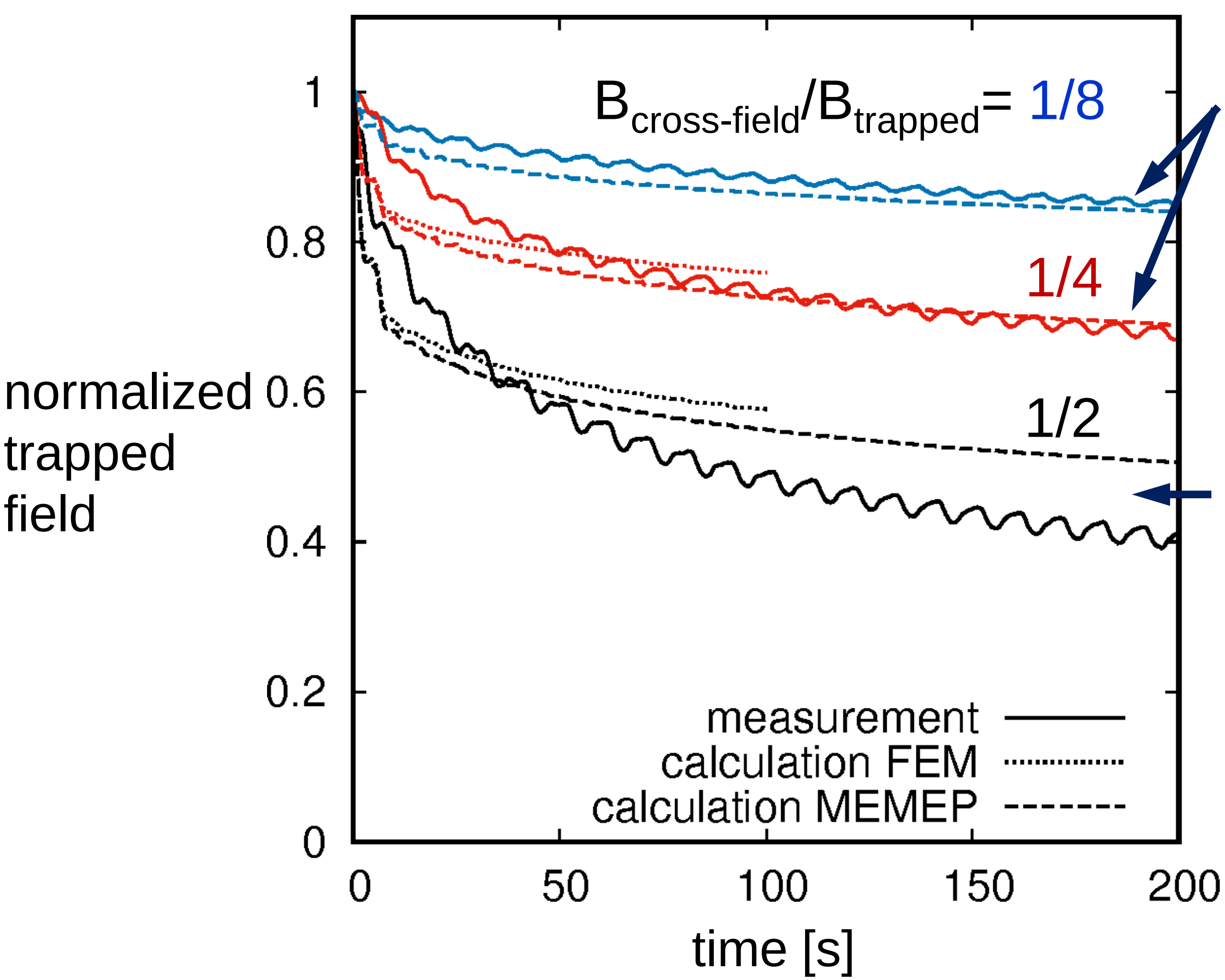
z component closes cross-field current loops

End of relaxation | First cross-field peak | 10th cross-field peak





Calculations by MEMEP agree with FEM and measurements



Good agreement with measurements at low applied fields

Disagreement at high applied fields due to $J_c(B)$

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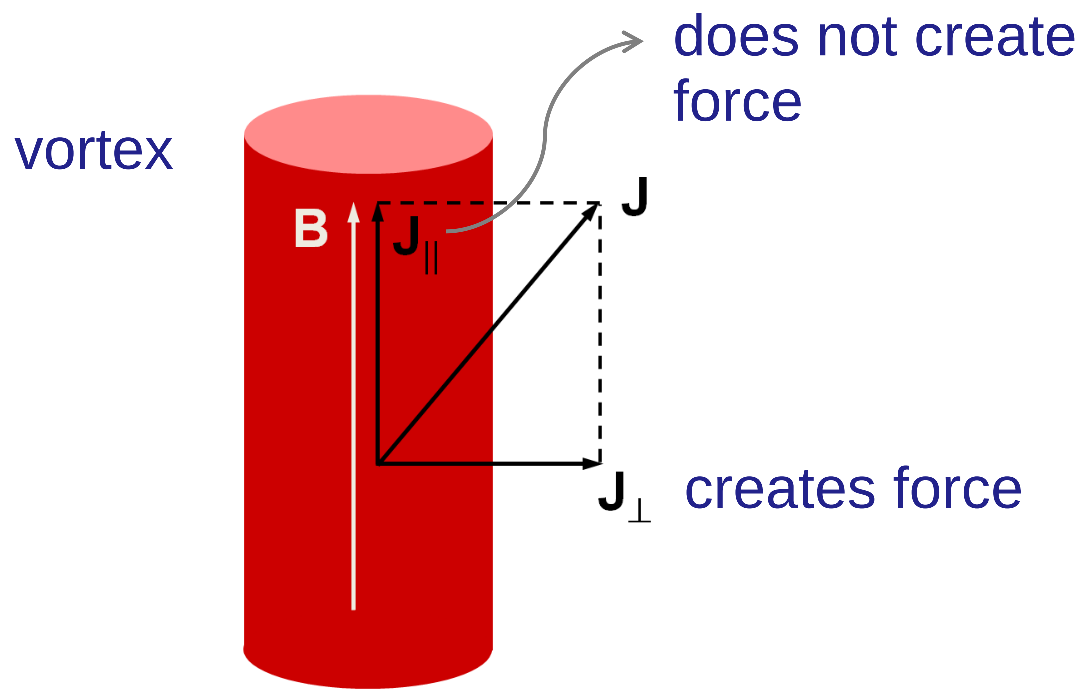
Flux-free effects cause anisotropic E(J)



Two critical currents:

$\mathbf{J}_{c\perp}$

$\mathbf{J}_{c||}$

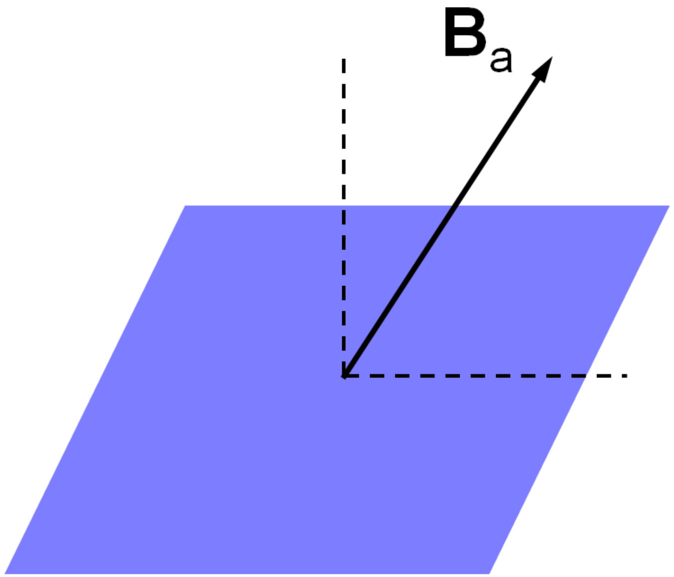
Anisotropic power law:

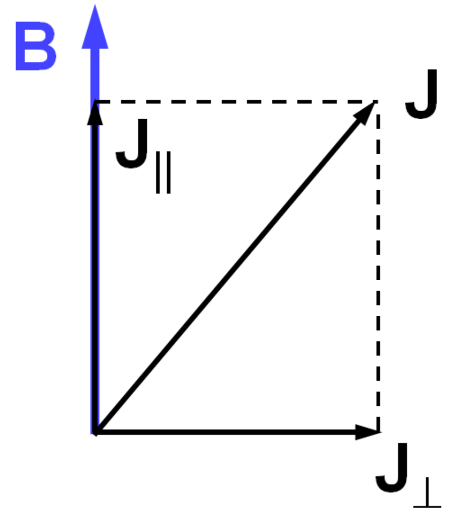
$$\mathbf{E}(\mathbf{J}) = E_c \left[\frac{J_{||}^2}{J_{c||}^2} + \frac{J_{\perp}^2}{J_{c\perp}^2} \right]^{\frac{n-1}{2}} \cdot \left(\frac{J_{||}}{J_{c||}} \frac{J_{\perp}}{J_{c||}} \mathbf{e}_{||} + \frac{J_{\perp}}{J_{c\perp}} \mathbf{e}_{\perp} \right)$$

A Badia, C Lopez DOI: 10.1088/0953-2048/28/2/024003

Flux-free effects in thin films

There is a force-free component





$$J_{c||}=3J_{c\perp}$$

Force-free J_c

Usual depining J_c

$J_{c\perp}=3\cdot10^{10}$ A/m^2

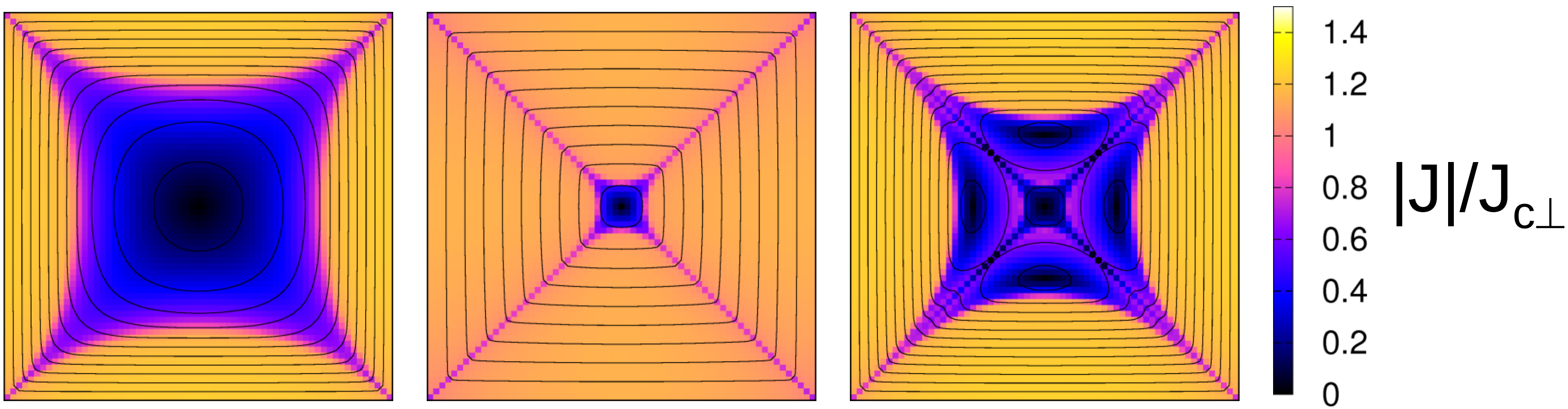
Mishev et al.

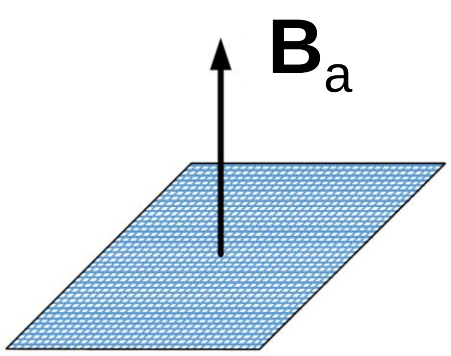
DOI: 10.1088/0953-2048/28/10/102001

Usual current penetration at perpendicular field

Perpendicular applied field:

23 mT **50 mT** **remanece**





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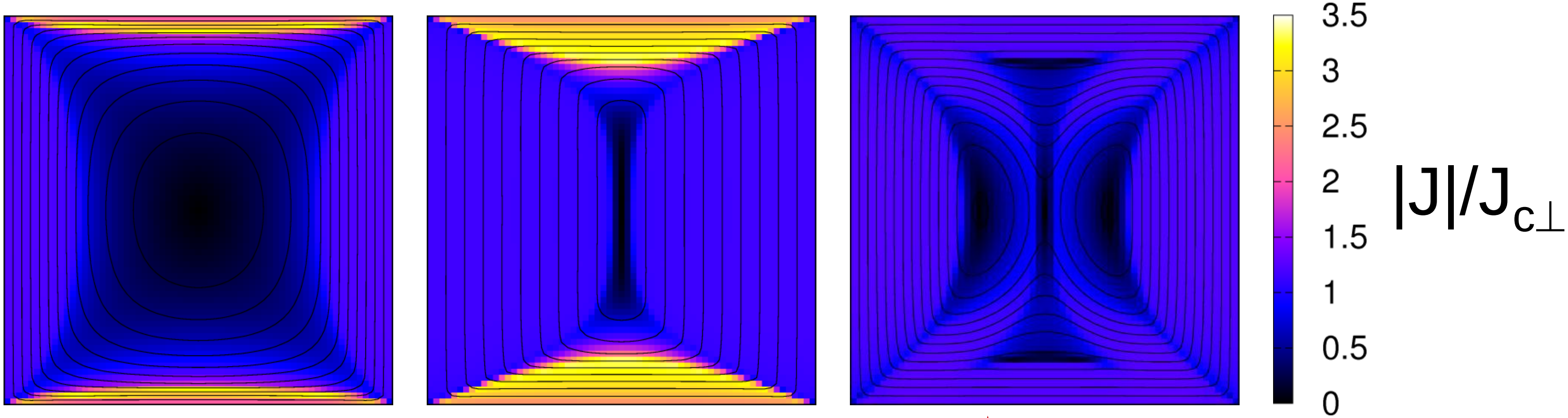
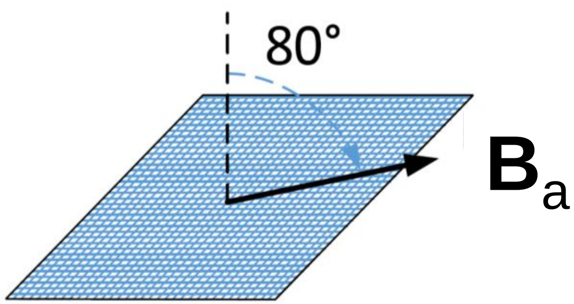
With angle, anisotropic penetration

Perpendicular applied field component:

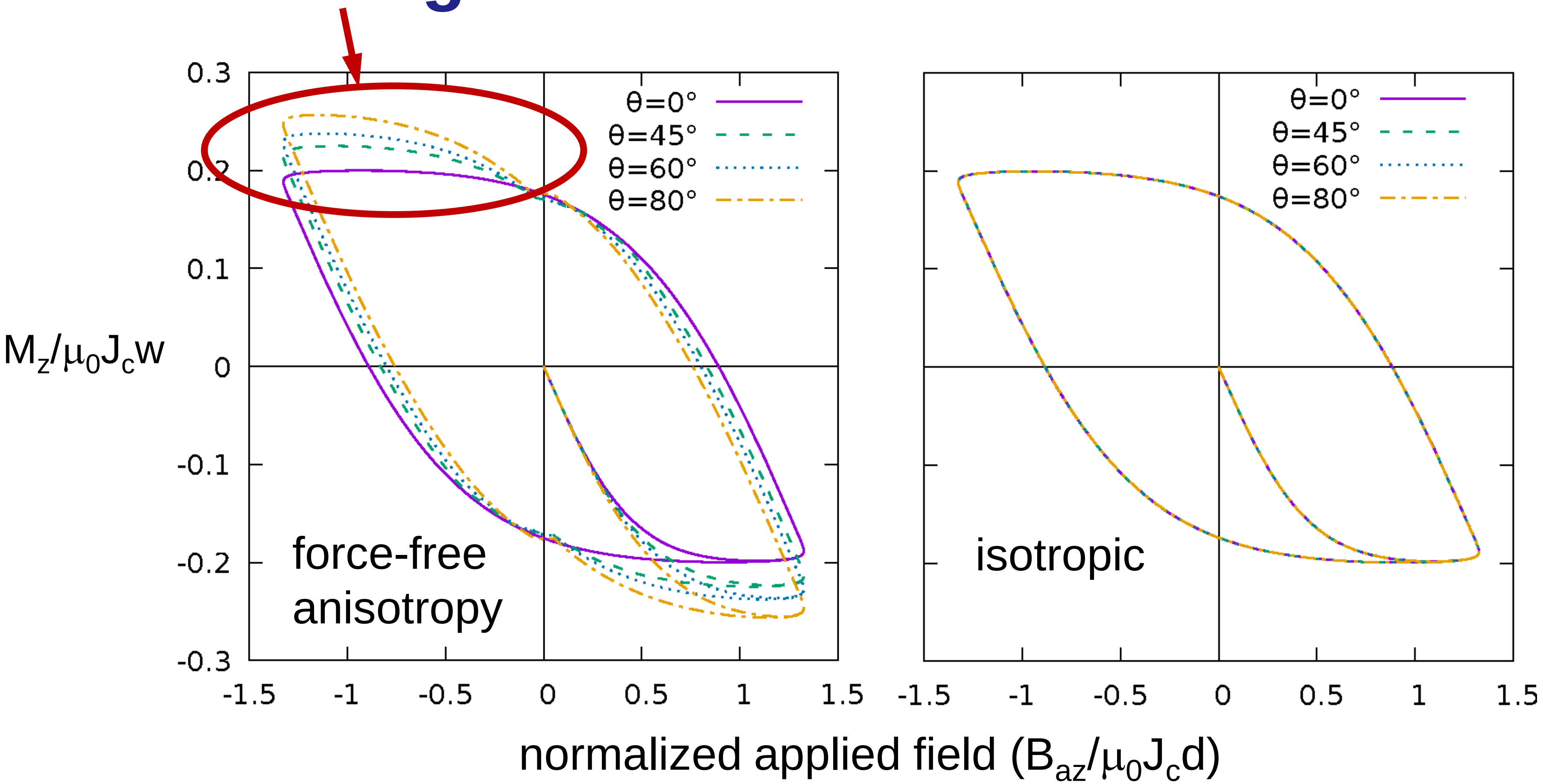
23 mT **50 mT** **remanece**



J_c limited to $J_{c\perp}$



Force-free effects increase magnetization



50 Hz sinusoidal applied field

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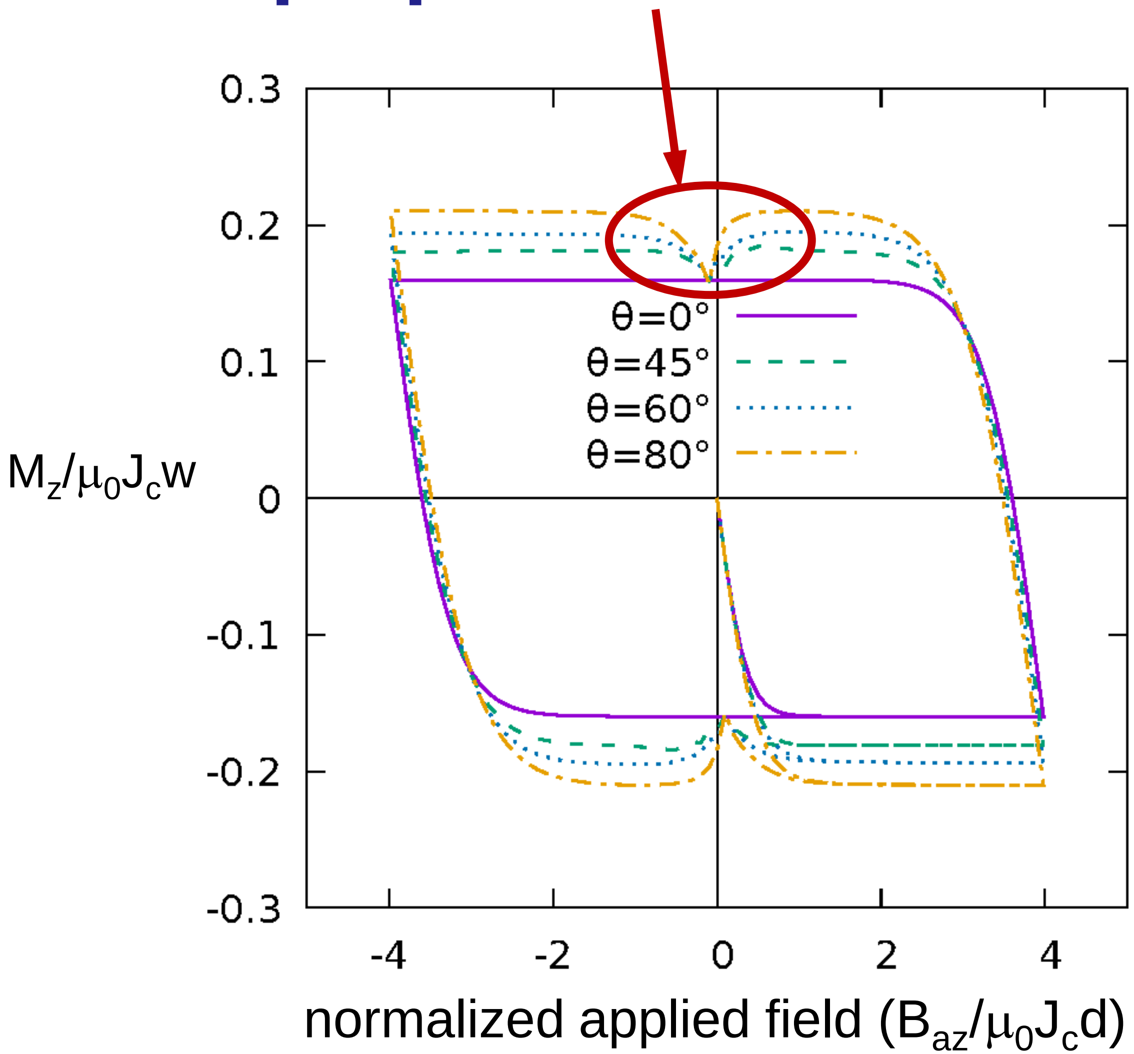
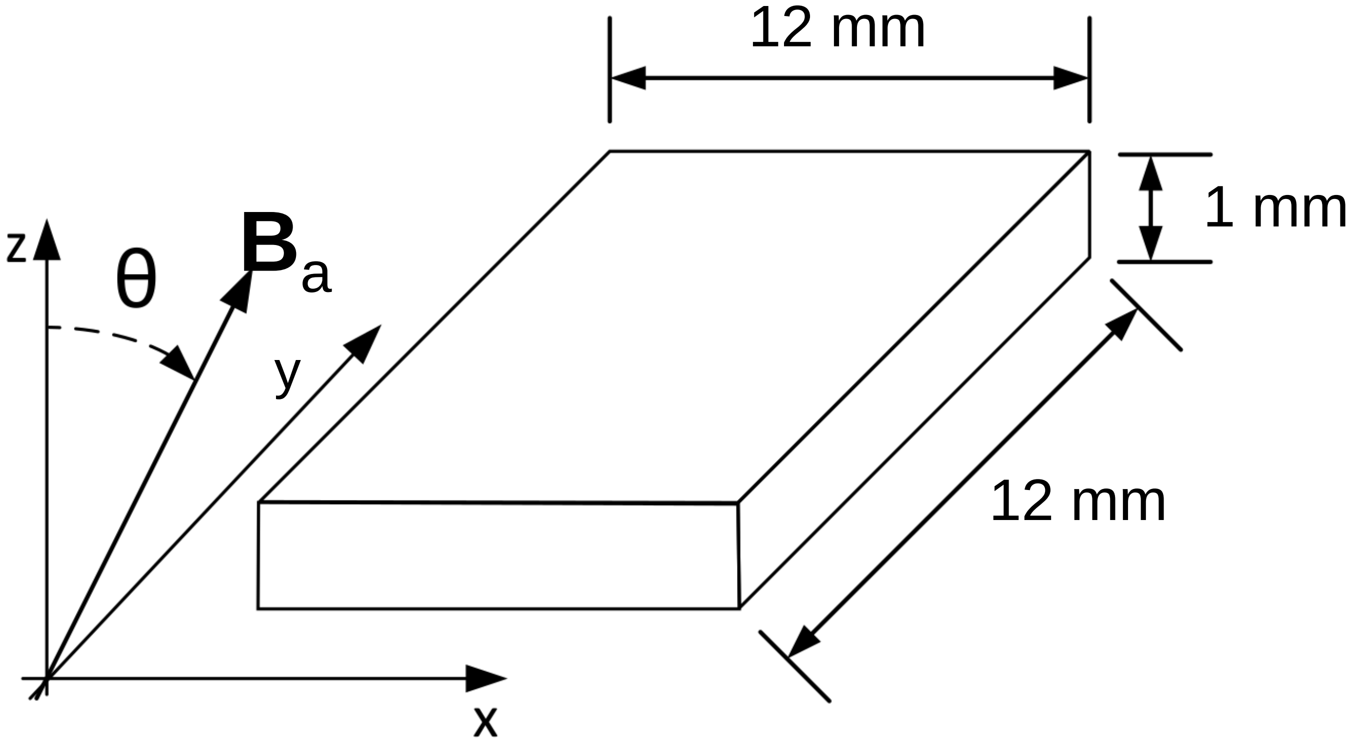
Minimum at remanence due to perpendicular self-field



1 mHz

constant ramp field

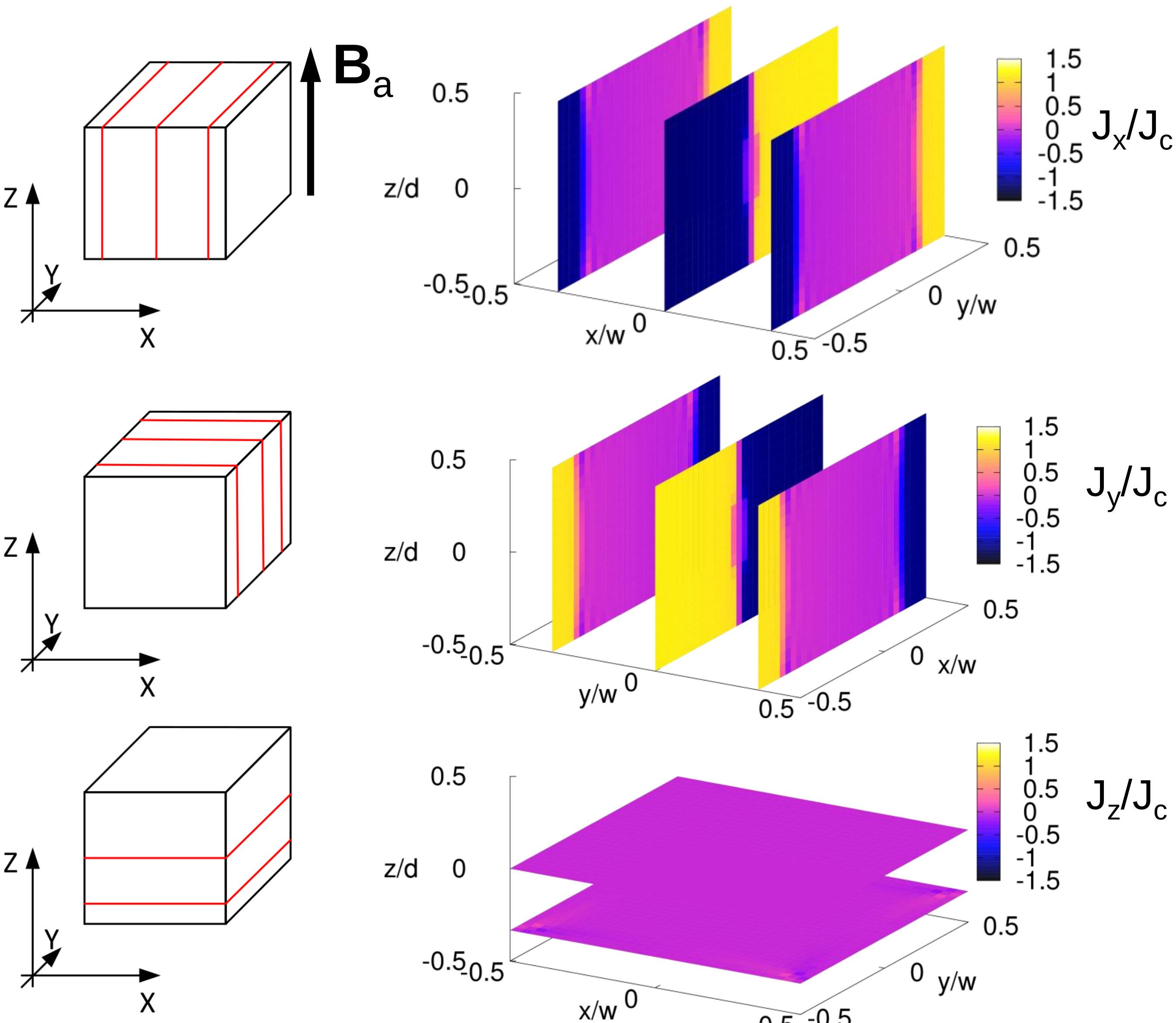
3D modeling with finite thickness

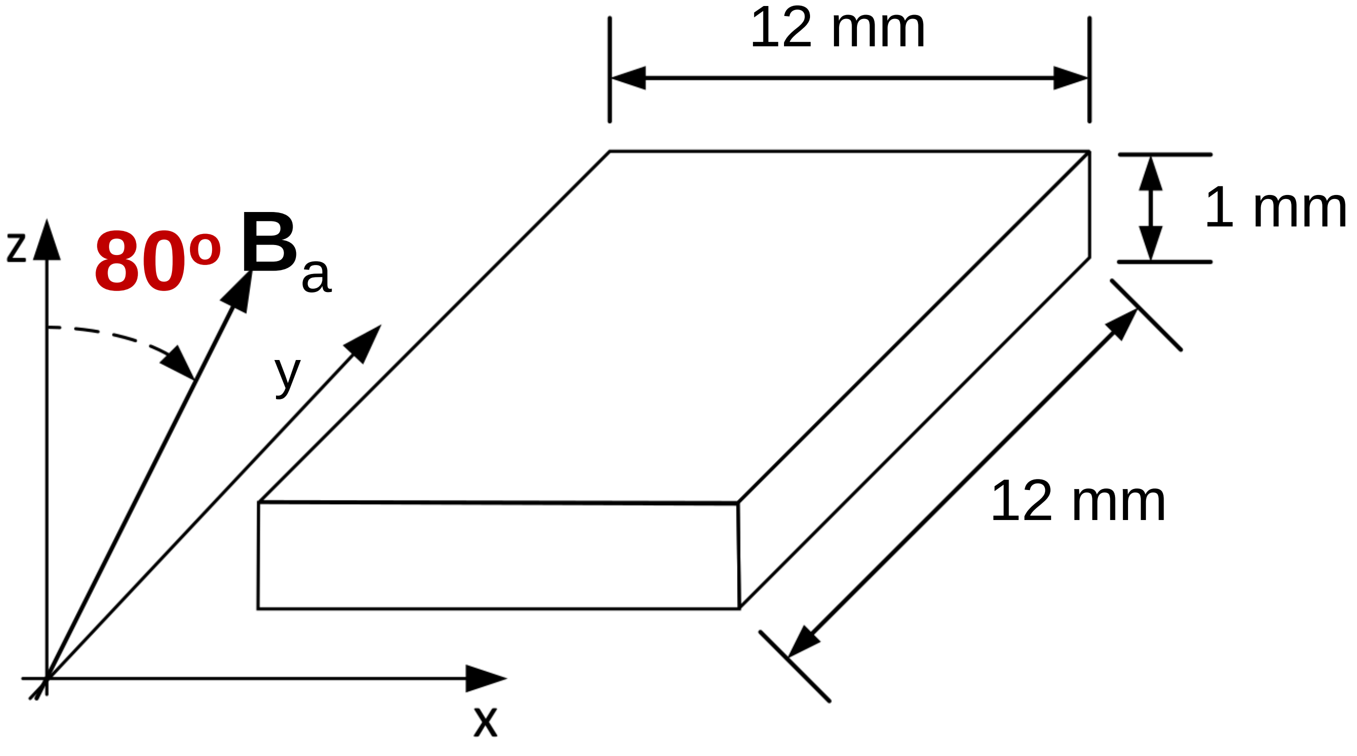


J_c perpendicular: $3 \cdot 10^7$ A/m^2

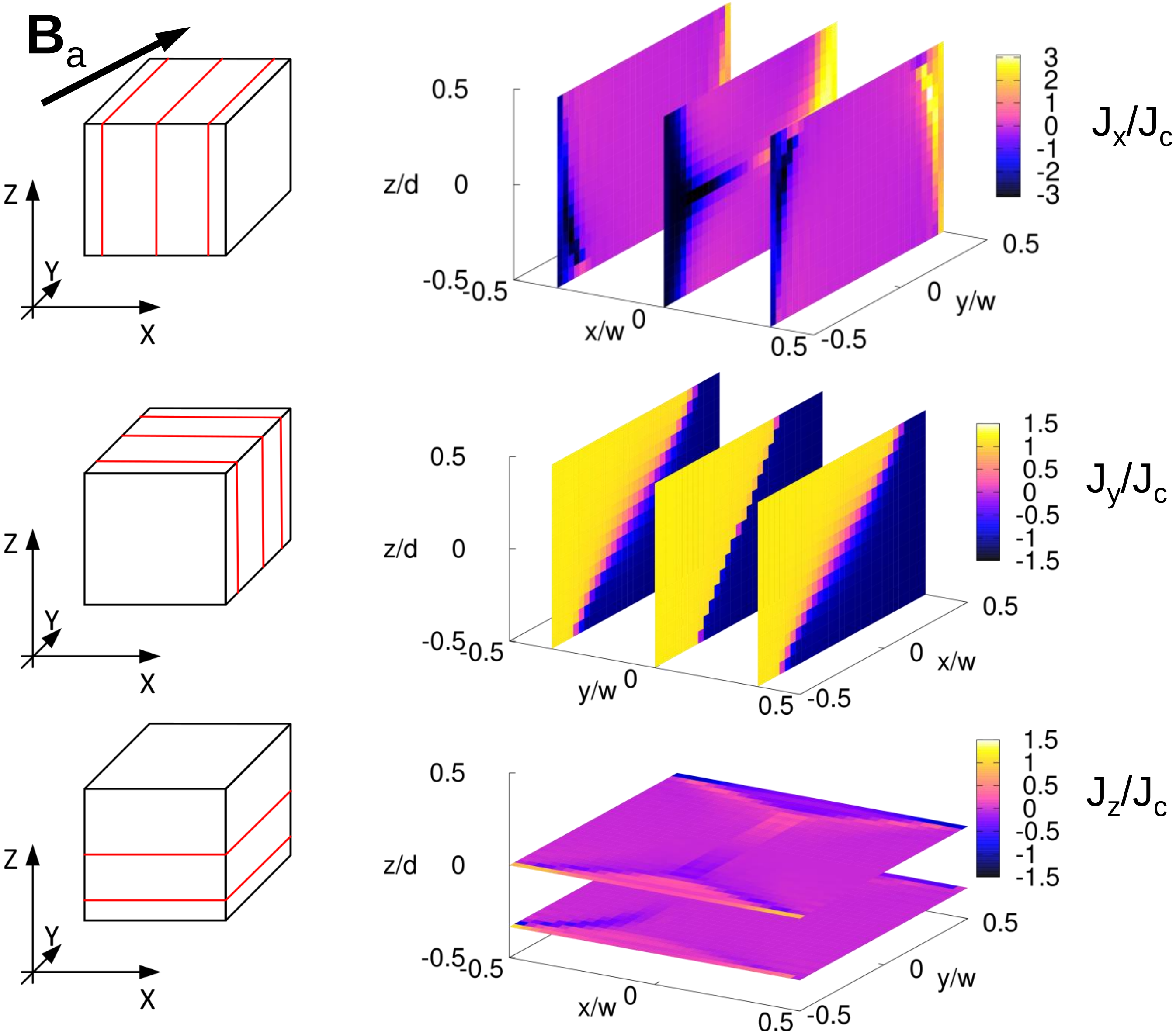
J_c parallel: $9 \cdot 10^7$ A/m^2

Parallel applied field: similar to isotropic




12 mm
1 mm
12 mm
z
80º
B_a
y
x

With angle: many new effects

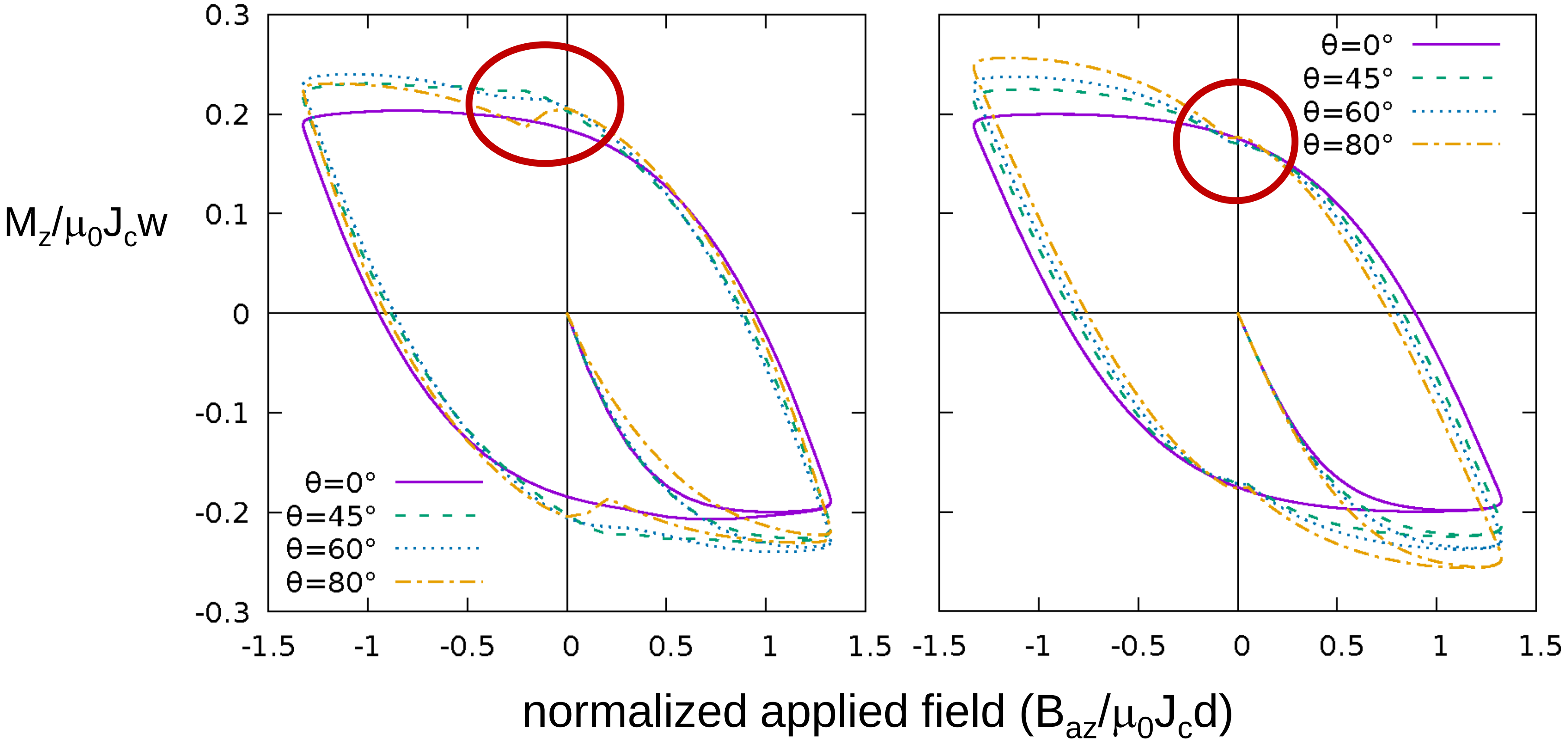


enhanced J_c

tilted J_c boundaries

large J_z

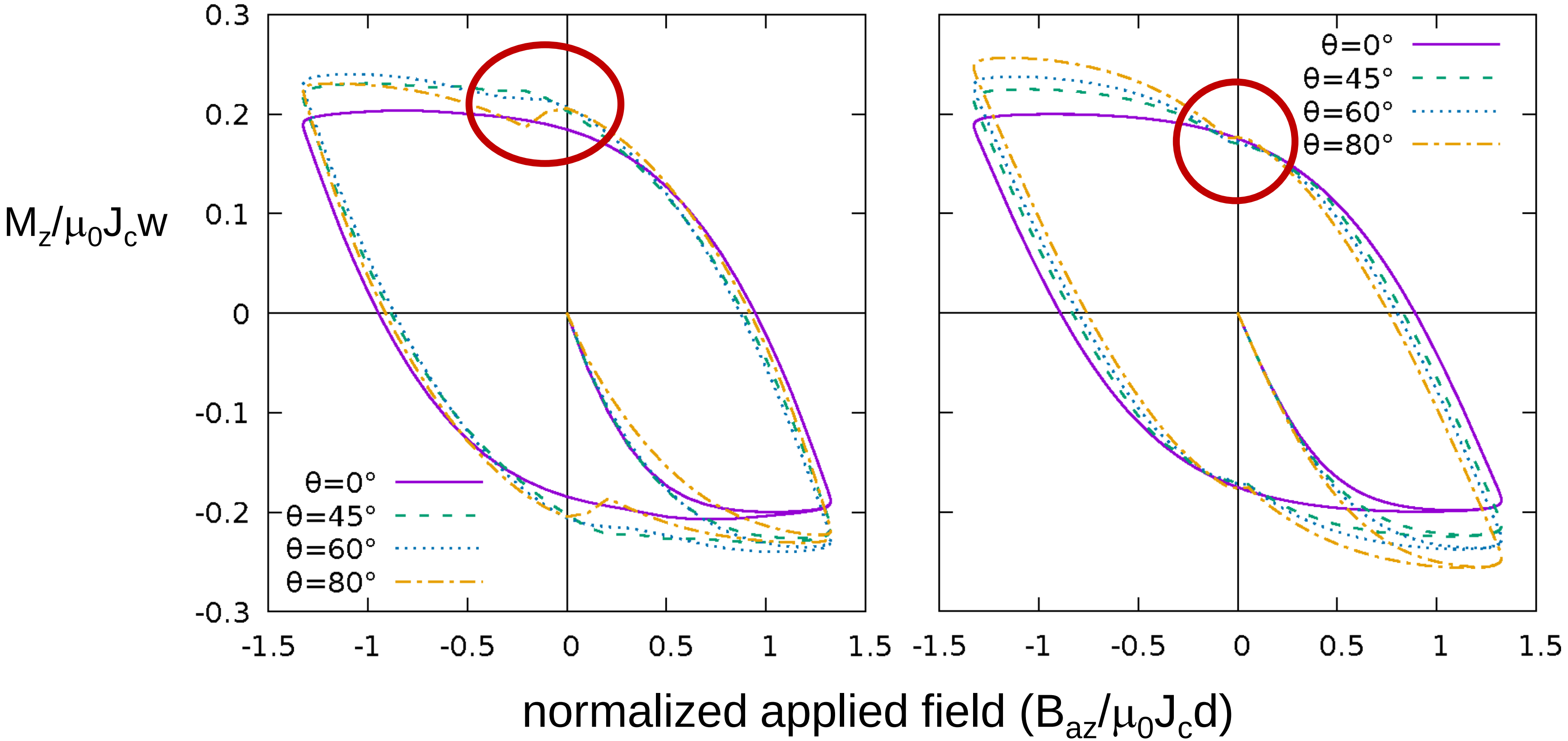
3D is necessary to see details close to self-field



3D is necessary to see details close to self-field

Self-field in 3D has a parallel component

Self-field in 2D is purely perpendicular



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Non-linear eddy currents

Interaction with ferromagnetic material

Electro-thermal modelling

Non-linear eddy currents

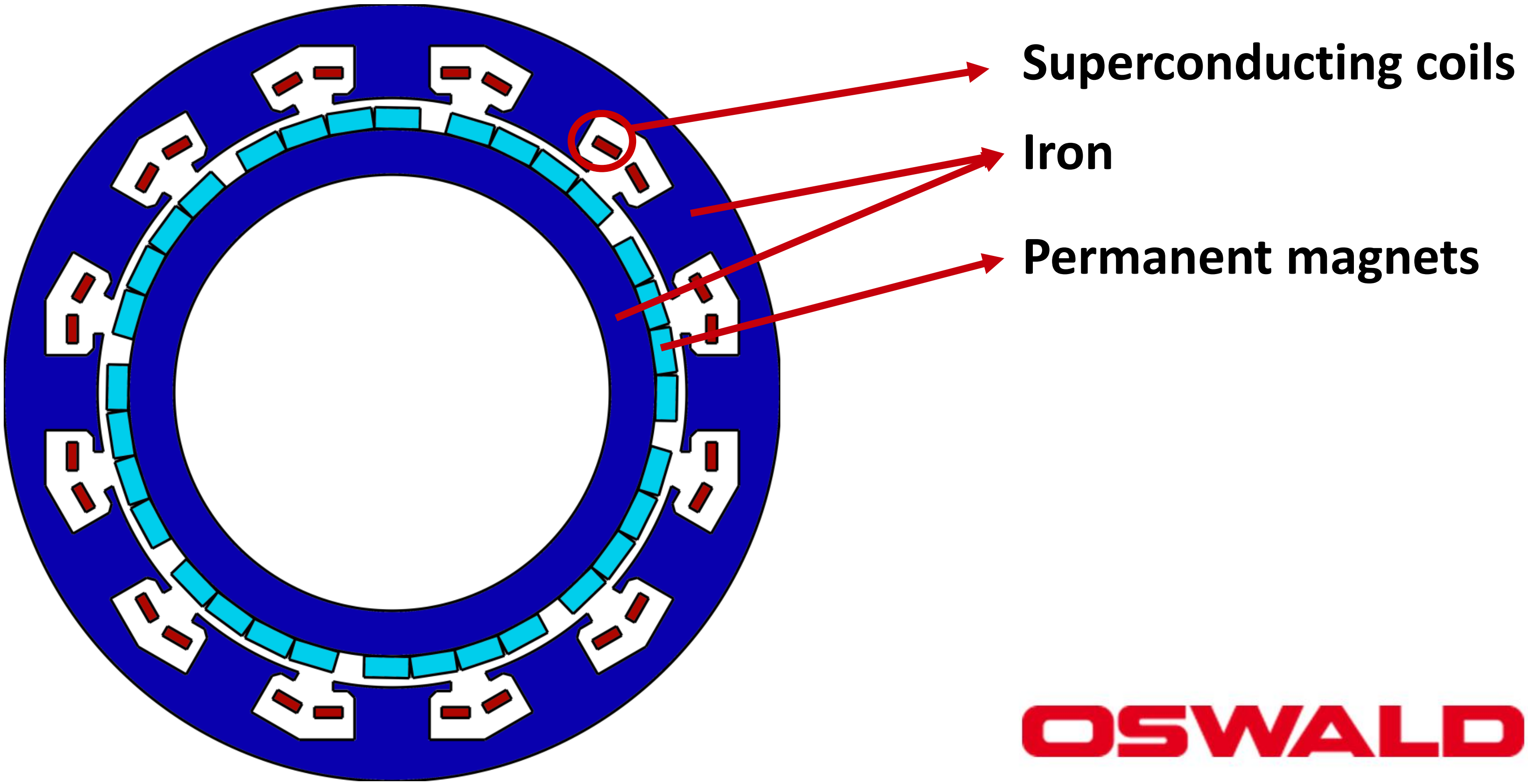
Interaction with ferromagnetic material

Coupling with FEM: motors

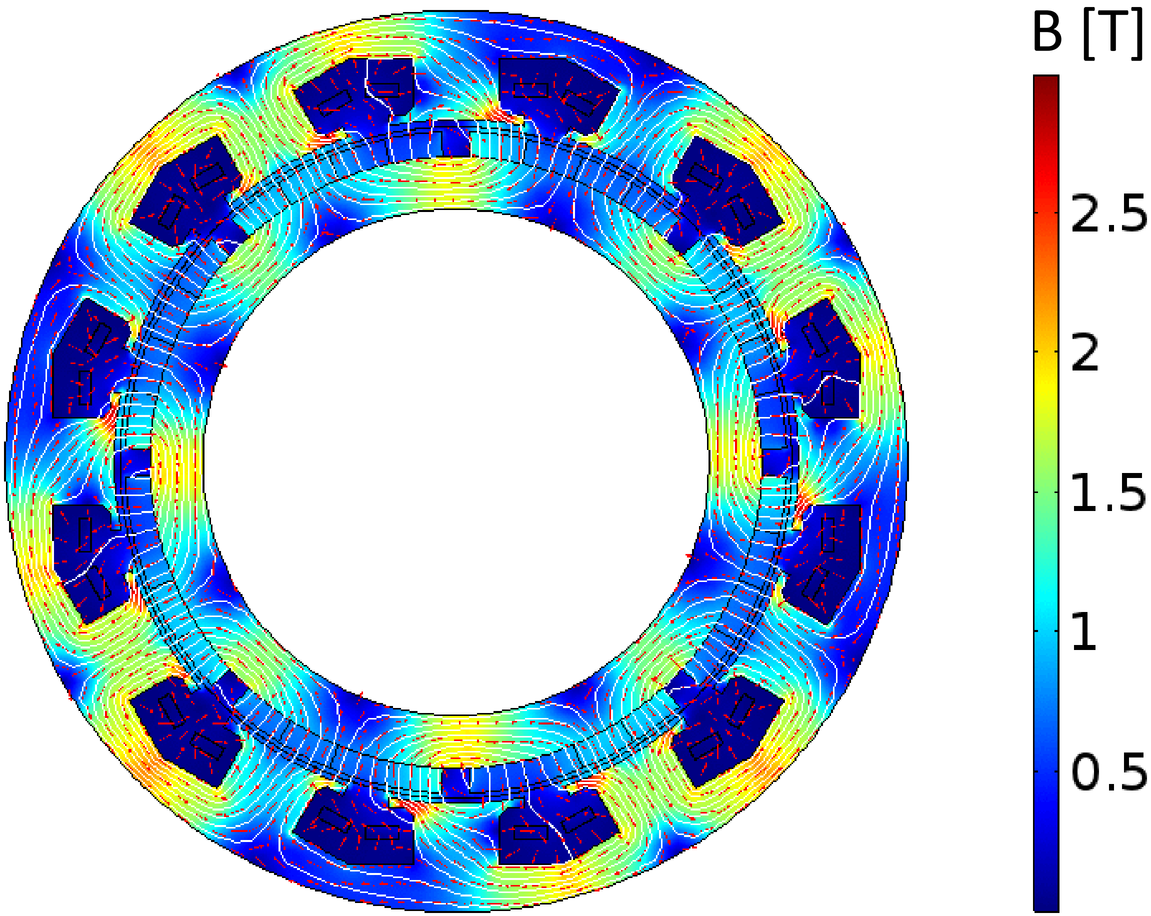
Fully variational method

Electro-thermal modelling

High-torque motor
SUTOR project



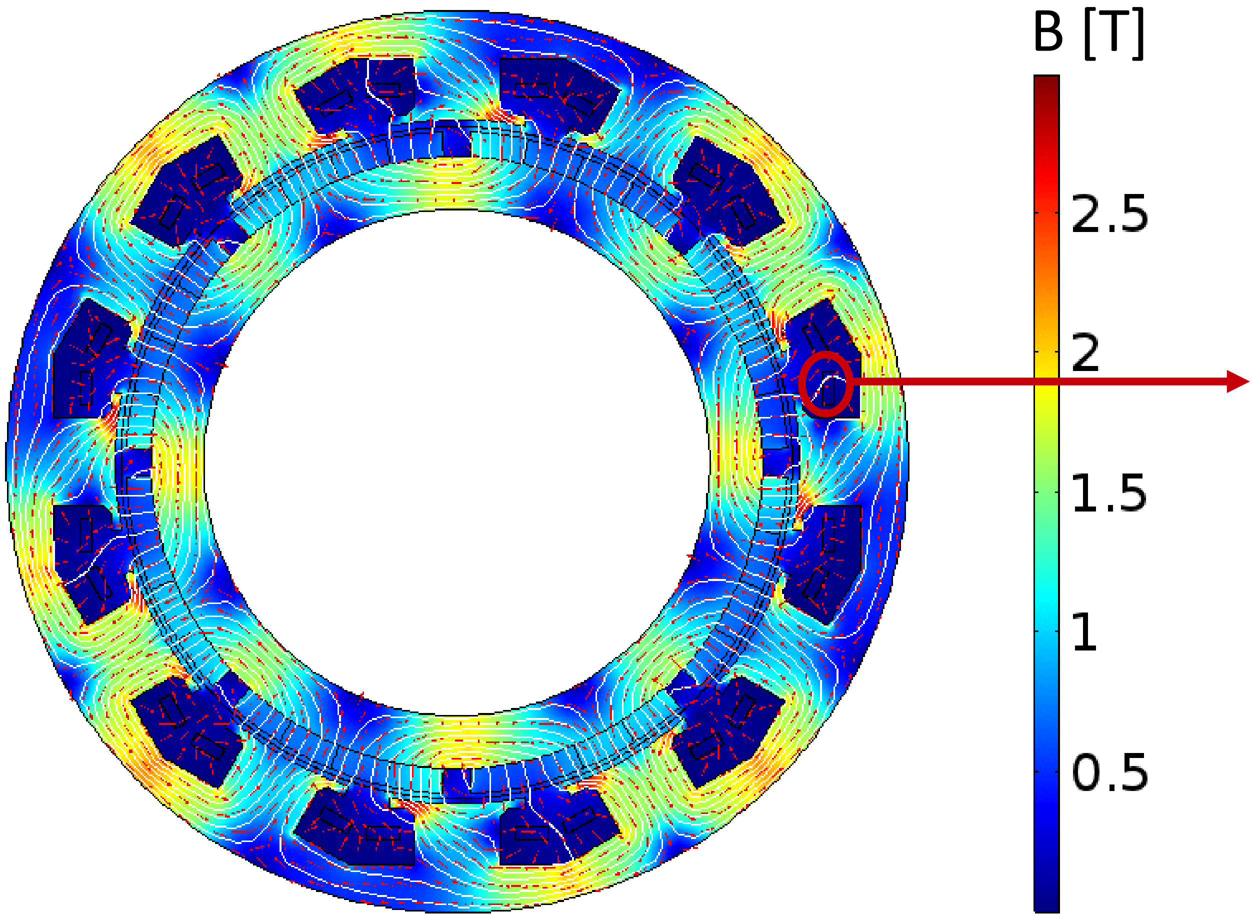
Commercial Finite-Element Method



COMSOL calculates magnetic flux density or vector potential

Map from KIT

Commercial Finite-Element Method

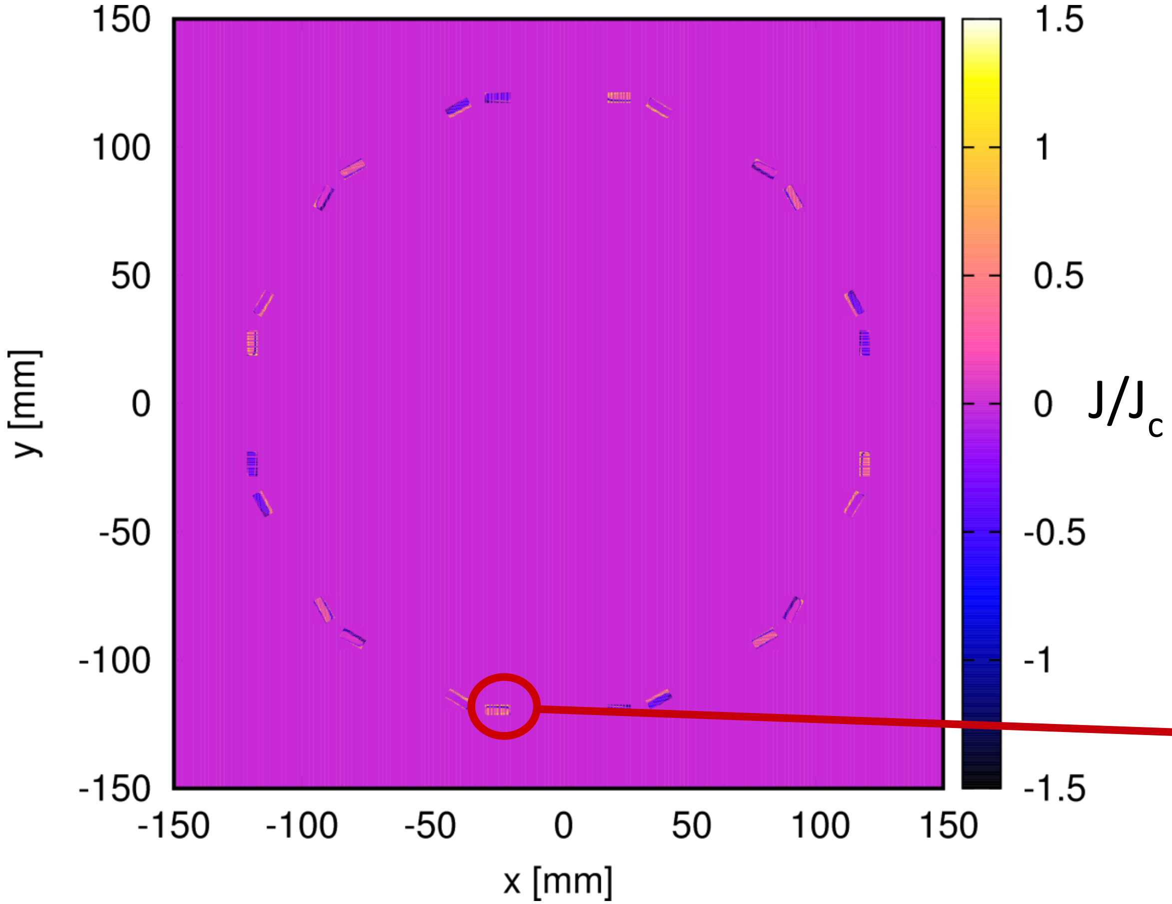


COMSOL calculates magnetic flux density or vector potential

Current density assumed uniform in coils

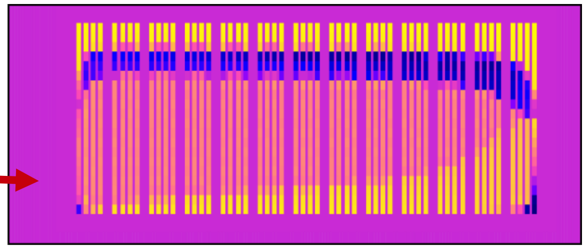
Map from KIT

Current density and AC loss in windings



MEMEP calculates real J and AC loss

Uses results from FEM as input „applied field“



Map from IEE

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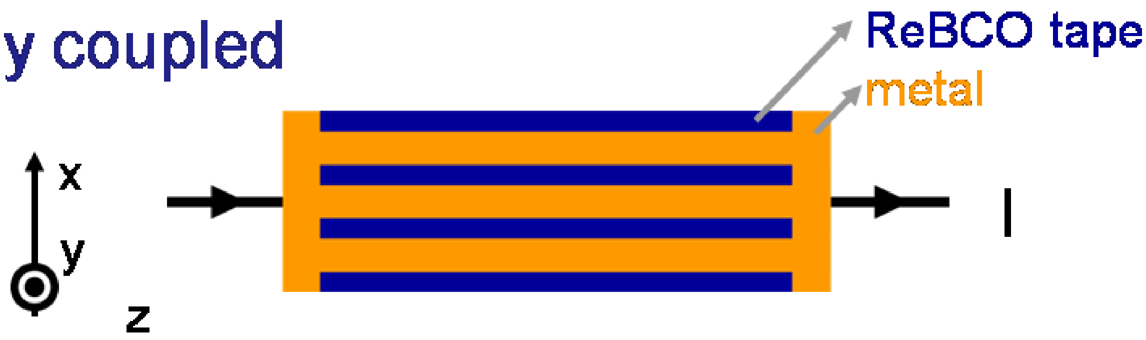
Only one assumption

No interaction between magnetization currents and iron

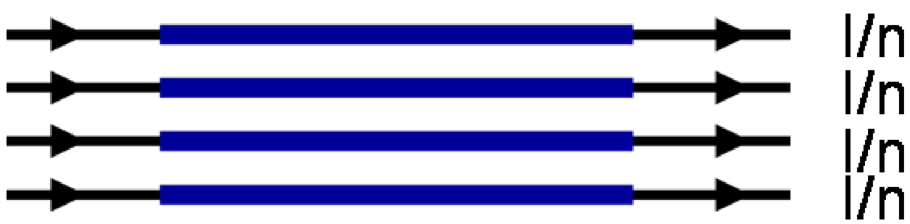
Distance from iron
of the order of tape width

Two coupling situations

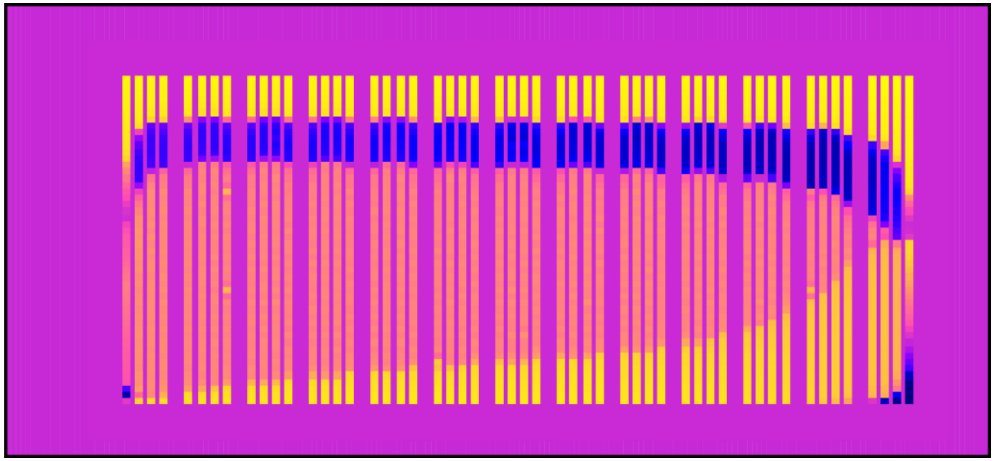
Fully coupled



Uncoupled

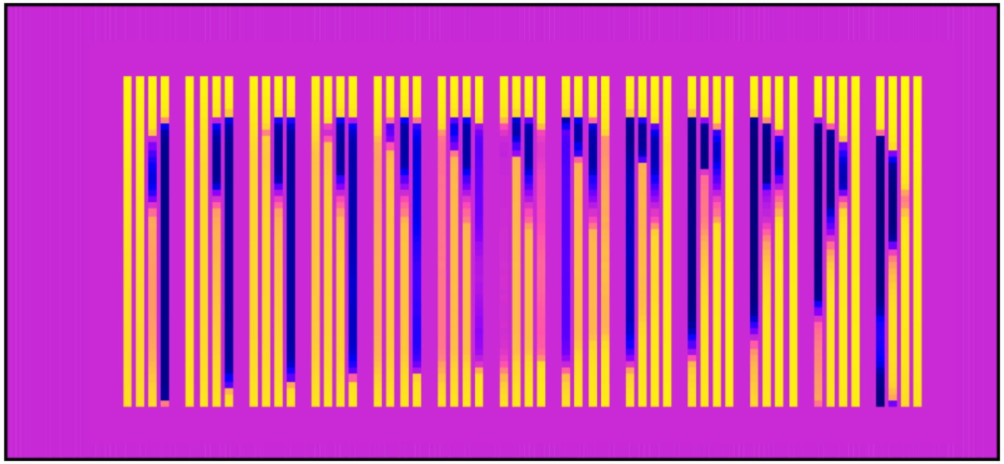


Homogeneous approximation is not suitable



Uncoupled

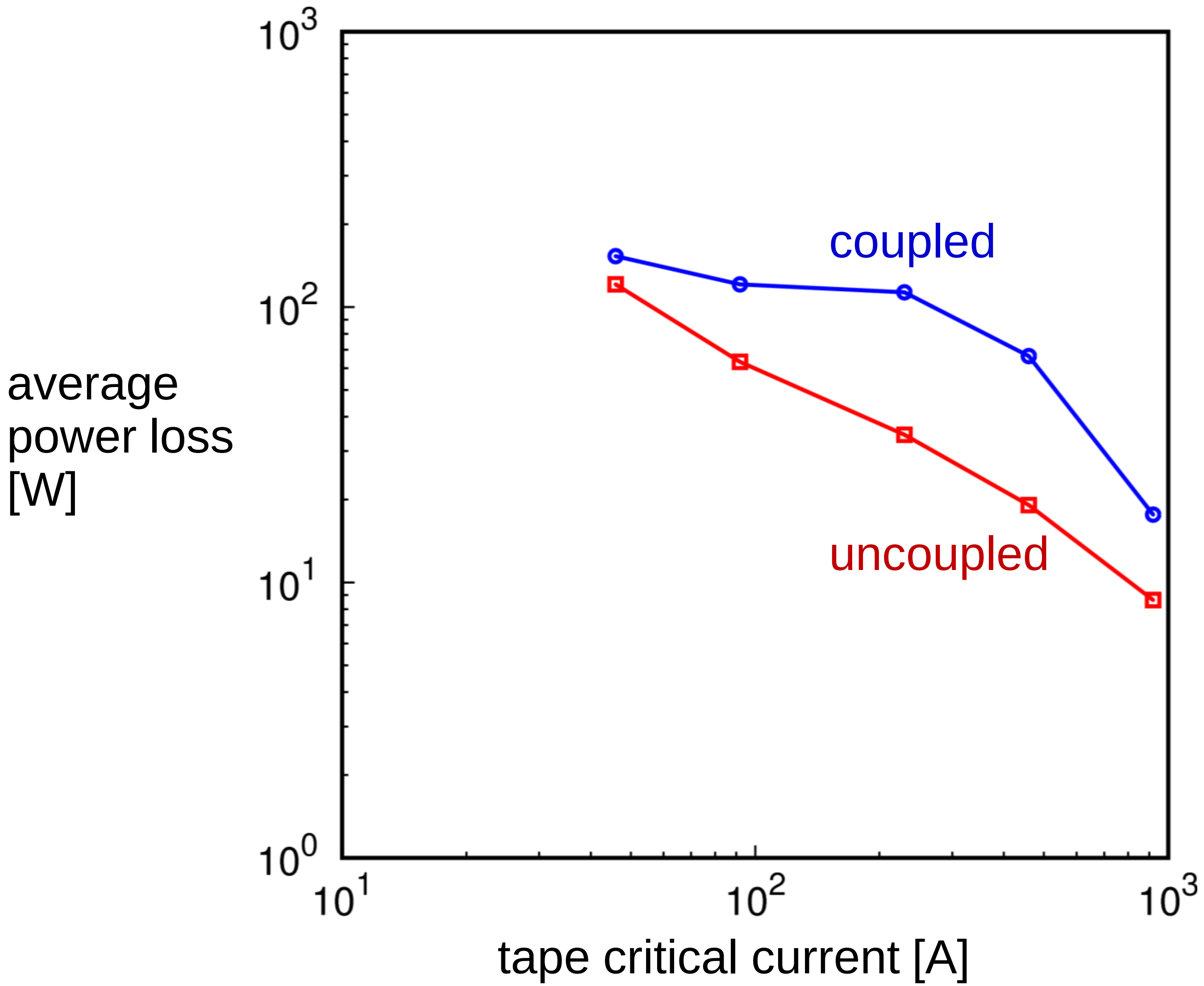
Homogeneous approximation still possible



Fully coupled

Homogeneous approximation not possible!

Coupling situation matters



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3D variational principle for the magnetic material

Reversible non-linear materials

Equation

$$\mathbf{B}(\mathbf{M}) = \mathbf{B}_M + \mathbf{B}_a + \mathbf{B}_J$$

- $\mathbf{B}(\mathbf{M})$: non-linear relation
- $\mathbf{B}_M$: **B** created by ∇×**M**
- $\mathbf{B}_a$: applied **B**
- $\mathbf{B}_J$: **B** from currents

is the Euler equation of

$$L_M = \int_V \mathrm{d}V \left[U(\mathbf{M}) - \frac{1}{2}\mathbf{B}_M \cdot \mathbf{M} - \mathbf{B}_a \cdot \mathbf{M} - \mathbf{B}_J \cdot \mathbf{M} \right]$$

$$U(\mathbf{M}) = \int_0^{\mathbf{M}} \mathrm{d}\mathbf{M}' \cdot \mathbf{B}(\mathbf{M}')$$

3D variational principle for the magnetic material

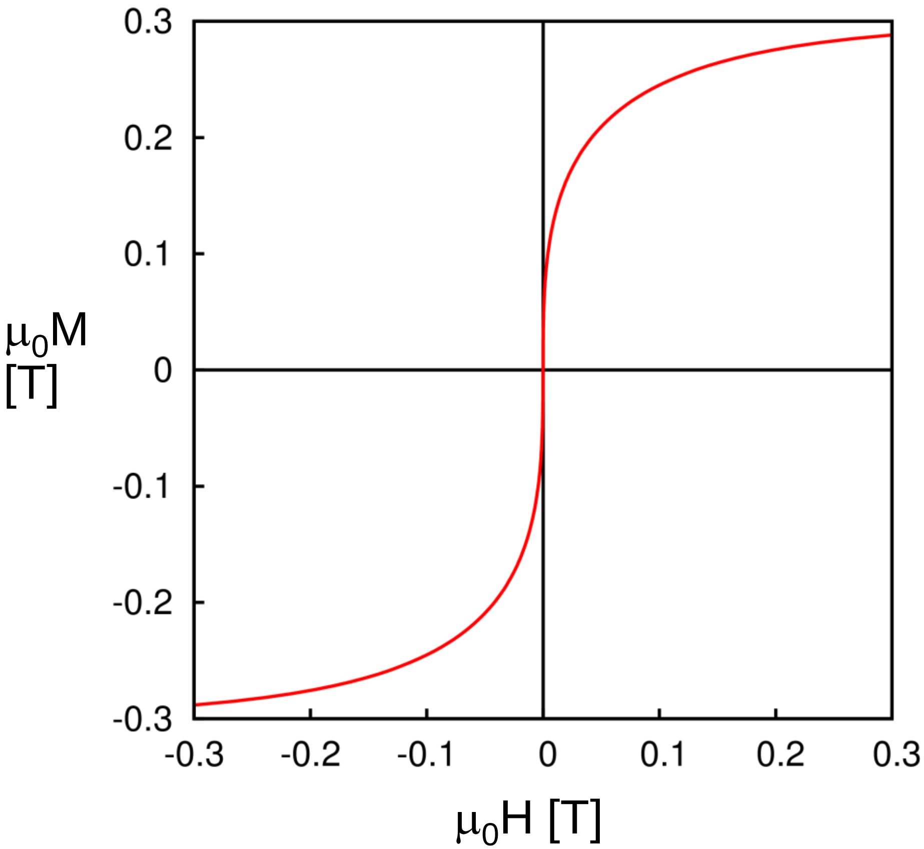
$$L_M = \int_V \mathrm{d}V \left[U(\mathbf{M}) - \frac{1}{2}\mathbf{B}_M \cdot \mathbf{M} - \mathbf{B}_a \cdot \mathbf{M} - \mathbf{B}_J \cdot \mathbf{M} \right]$$

$$U(\mathbf{M}) = \int_0^{\mathbf{M}} \mathrm{d}\mathbf{M}' \cdot \mathbf{B}(\mathbf{M}')$$

Problem restricted to the magnetic material volume

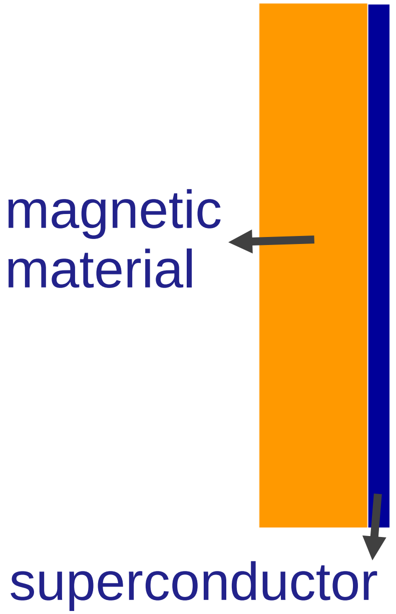
Functionals for magnetic material and superconductor **solved iteratively**

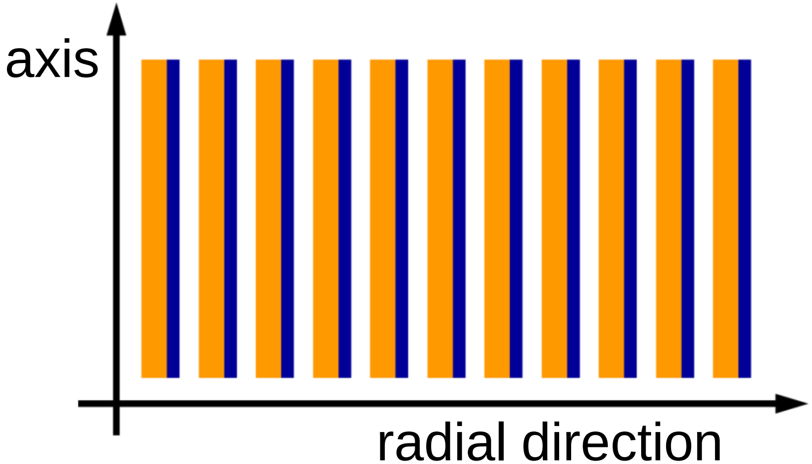
Superconductor with non-linear magnetic substrate



saturation $\mu_0 M$
300 mT

initial susceptibility
500





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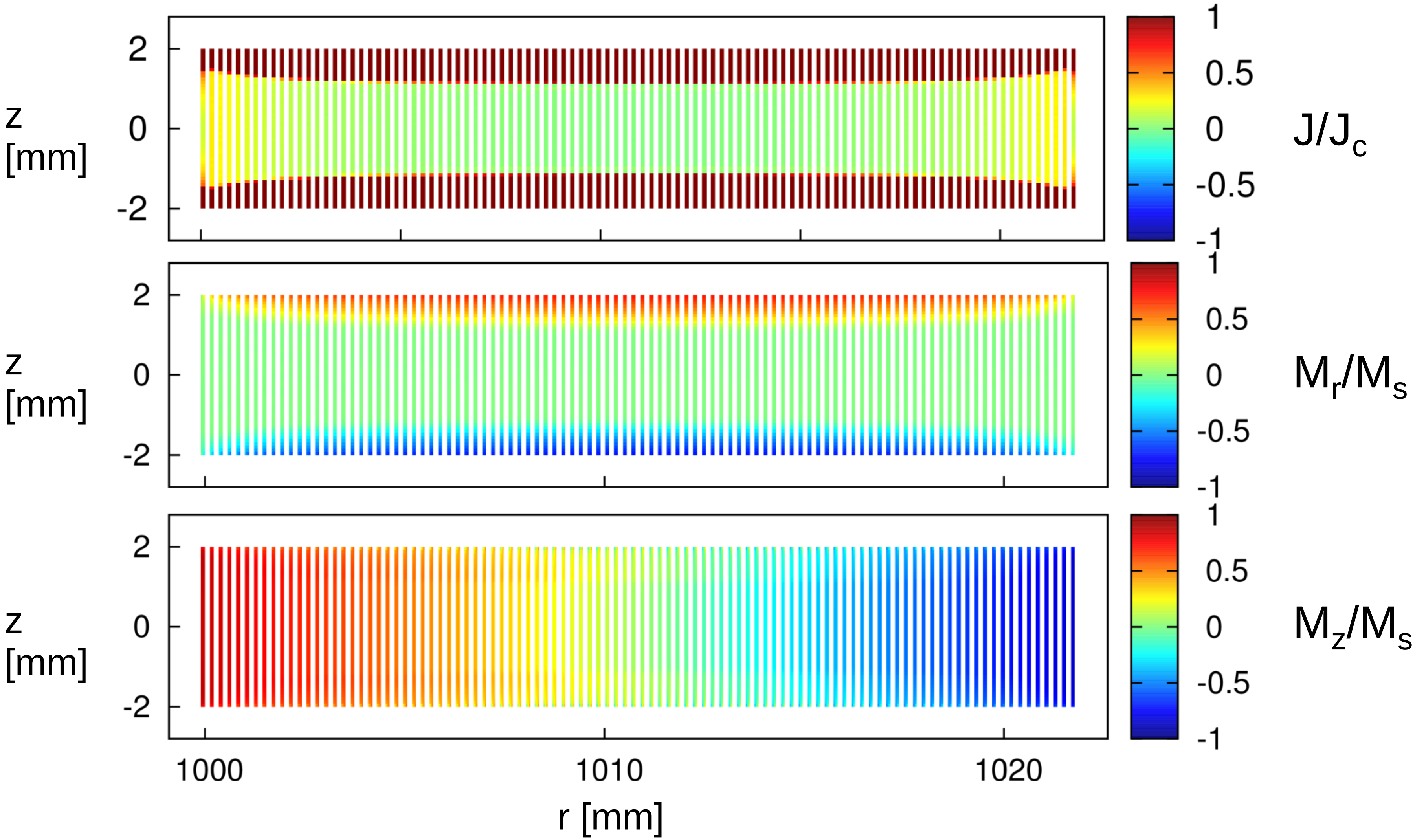
Magnetic substrate saturates in part of the coil

100 turns, 50% of critical current



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Non-linear eddy currents

Interaction with ferromagnetic material

Electro-thermal modelling

Non-linear eddy currents

Interaction with ferromagnetic material

Electro-thermal modelling

- Maths
- DC fault limitting

Thermal diffusion equation

Thermal energy

$$U_T(T) = \int_0^T \mathrm{d}T' C_v(T')$$

Heat capacity

$$\mathbf{E} \cdot \mathbf{J} = \frac{\partial U_T}{\partial T} - \nabla \cdot (\bar{\bar{k}} \nabla T)$$

Heat generation

Thermal conductivity tensor

Variational principle

Solving

$$\mathbf{E}\cdot\mathbf{J} = \frac{\partial U_T}{\partial T} - \nabla\cdot(\bar{\bar{k}}\nabla T)$$

is the same as minimizing

$$L_T = \int_V \mathrm{d}V \left\{ [h(T) - U_T(T_0)T]\frac{1}{\Delta t} + \frac{1}{2}\nabla T\bar{\bar{k}}\nabla T - T\,\mathbf{E}\cdot\mathbf{J} \right\}$$

$$h(T) = \int_0^T \mathrm{d}T' U_T(T')$$

Temperature
at previous time step

Coupled electro-thermal method at given time

Find **J**(**r**)
by MEMEP
at given T(**r**)

↓

Find T(**r**)
by thermal minimization
at given **J**(**r**)

↓

Update k(T)

Non-linear eddy currents

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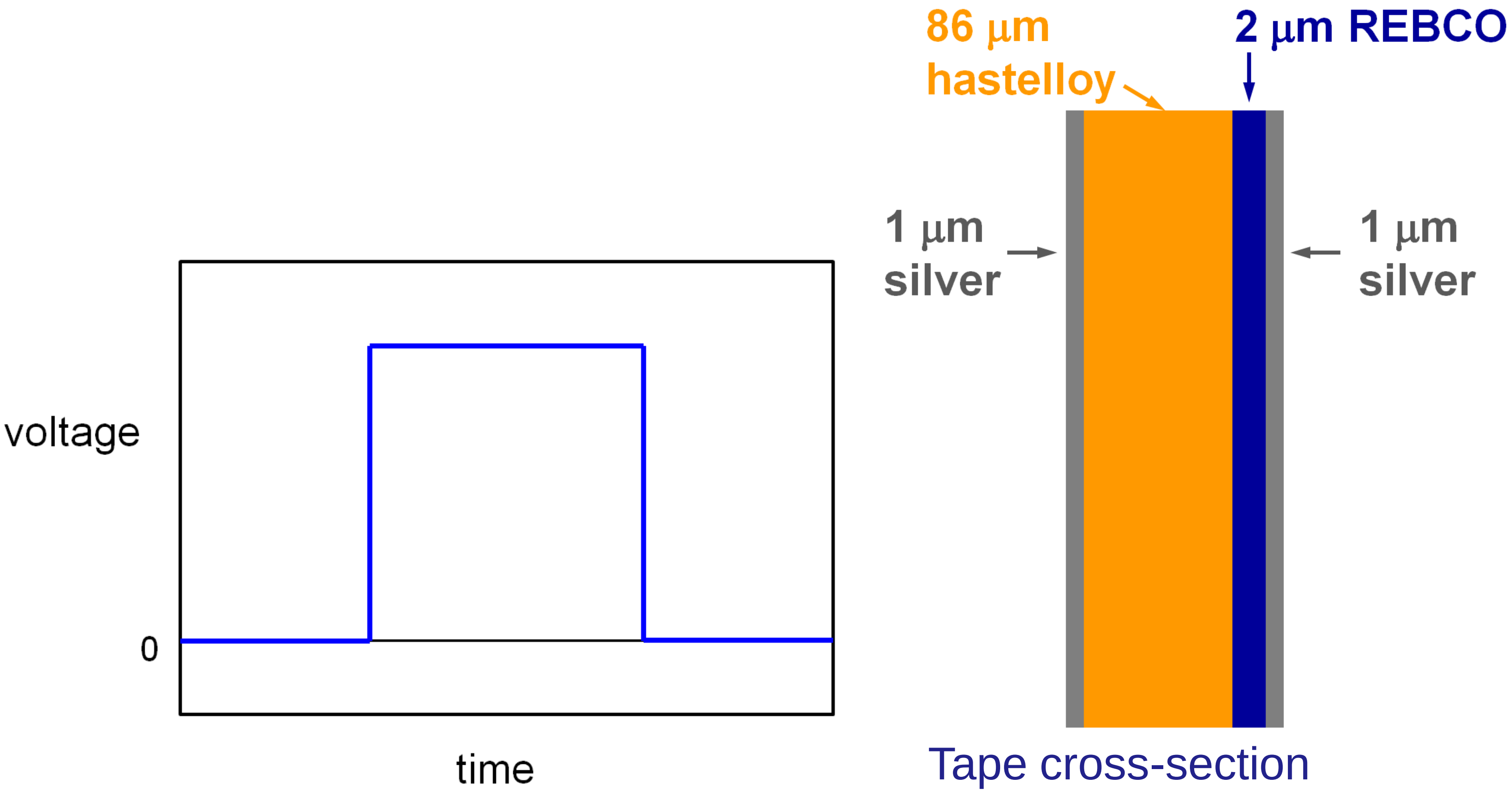
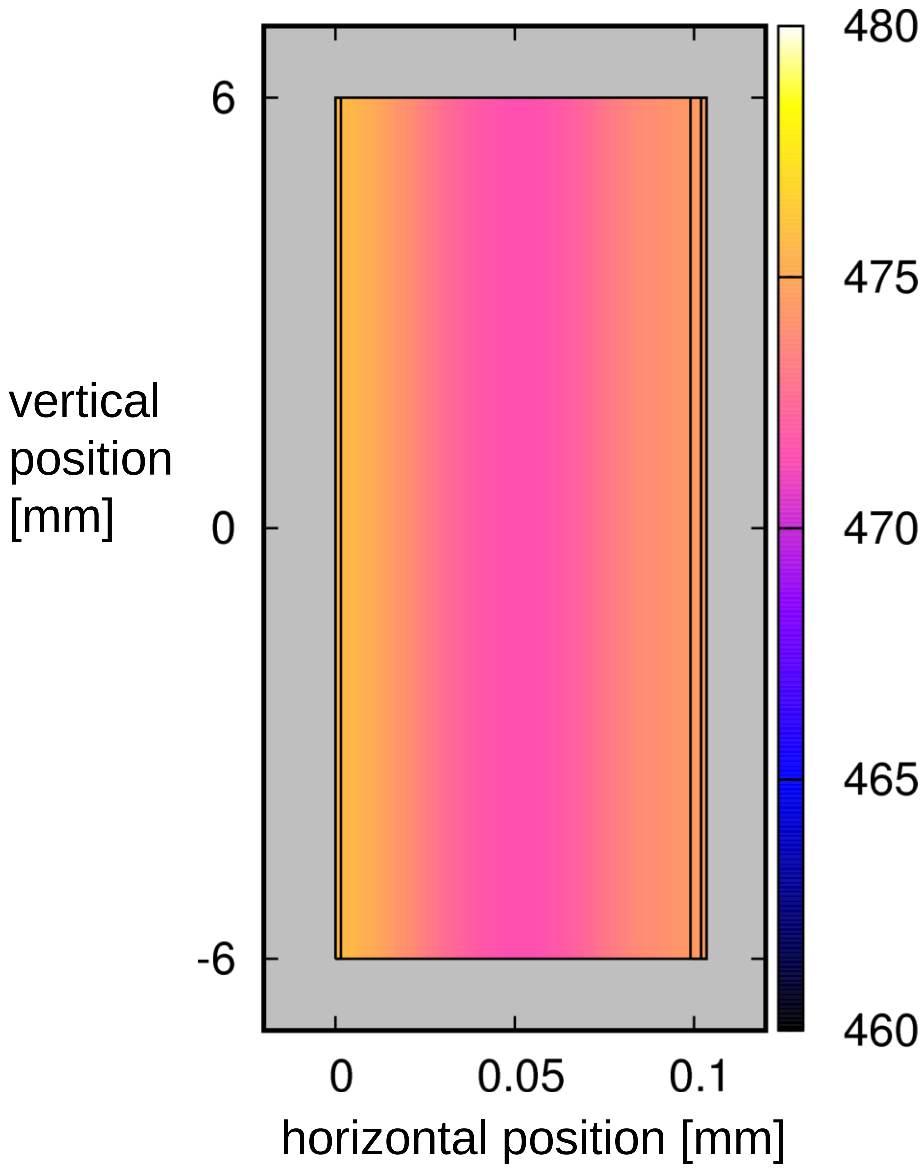
DC fault current limitting

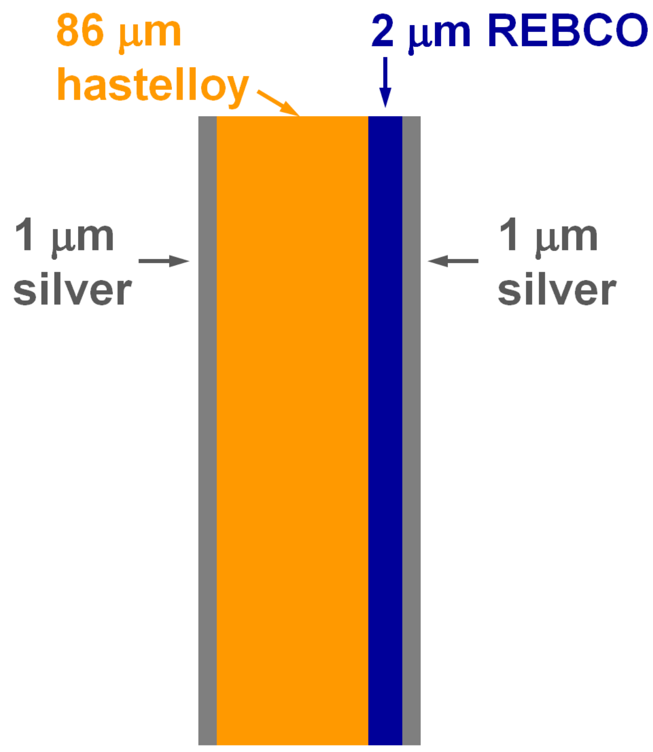


2D cross-sectional model

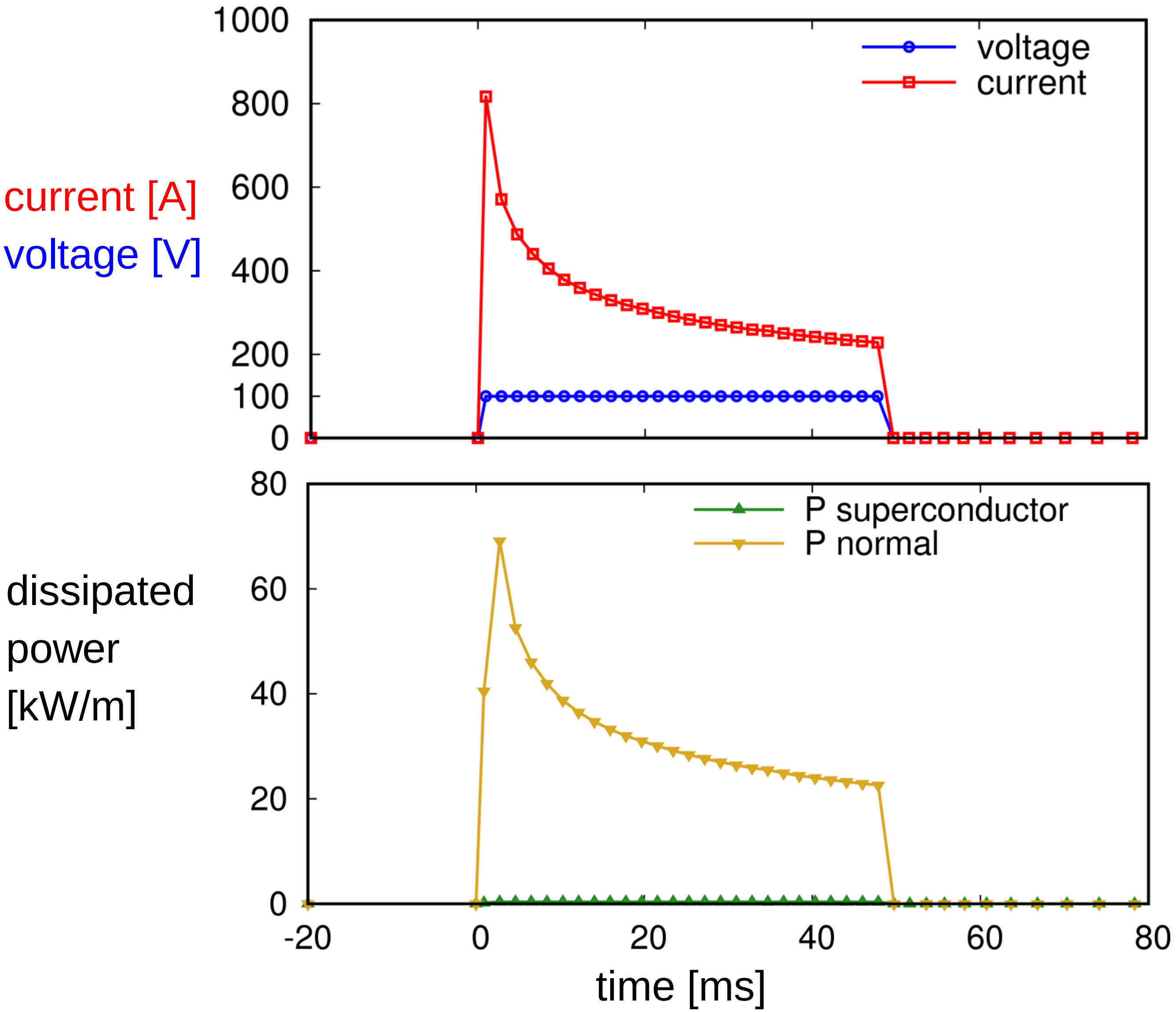


For this case:

Temperature almost uniform across width



Superconductor limits fault current



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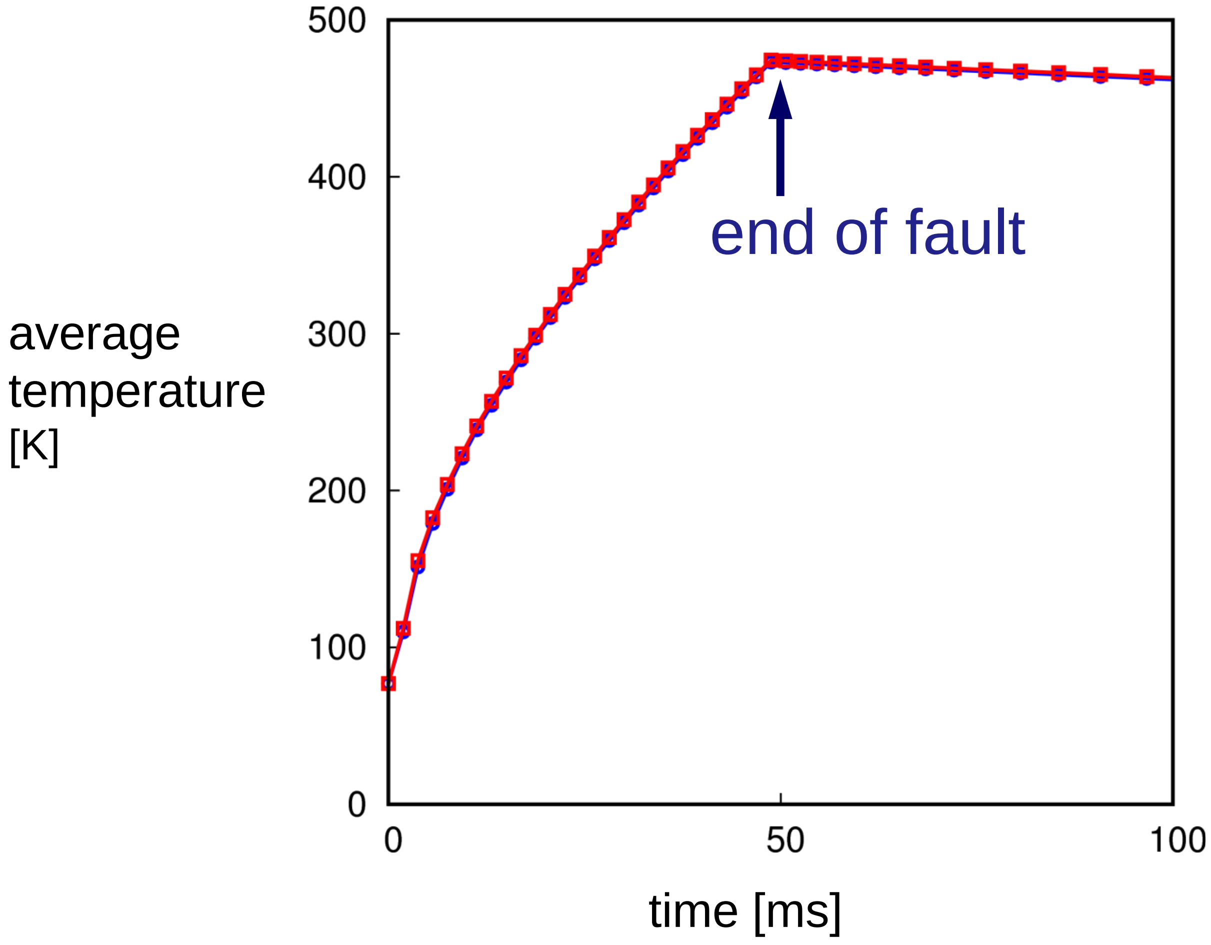
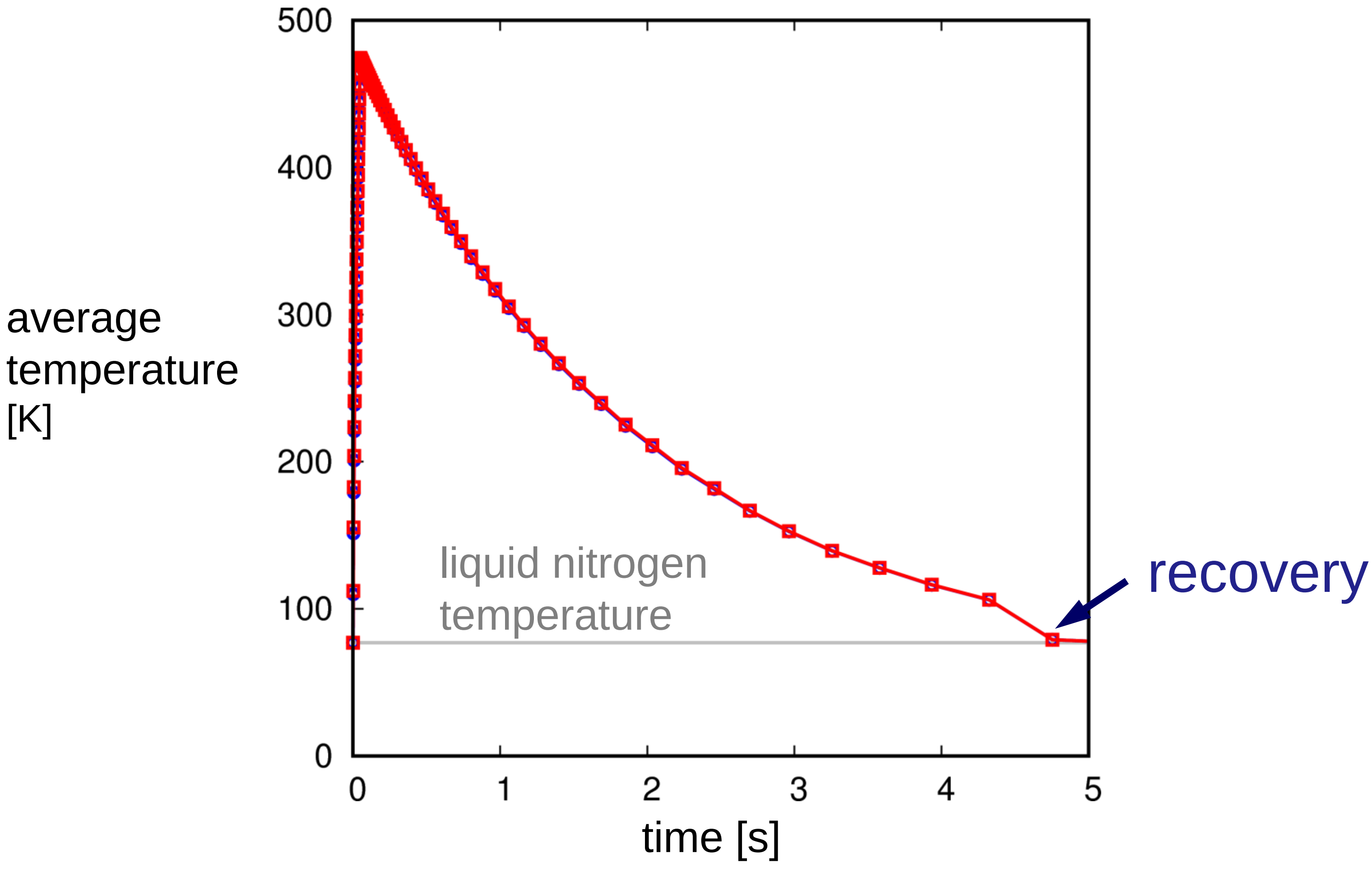
Initial temperature rise



Superconductivity recovers after relatively long time



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Conclusions

Multi-physics variational method

We developed the maths
for several physical systems:

Non-linear eddy current problem

Interaction with ferromagnetic material

Electro-thermal problem

Fast parallel numerical tool

Division into sectors enables:

Faster computations

Efficient parallelization

Mixed OpenMP/MPI implementation
runs in computer clusters

3D modelling shows novel effects

Cross-field demagnetization of bulks

Force-free anisotropy films and bulks

Thin-film approximation is not sufficient

Interaction with magnetic material

Coupling the variational method with FEM enables **modeling AC loss in motors**

Modeling entirely by variational methods is possible

Thank you for your attention!

Would you like to know more?

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