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A finite-element method framework for modeling rotating machines with superconducting windings

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A hybrid model

Source: Toyota

Motivation

- Build a "do-it-all" dynamical model for electrical machines with superconducting winding
  - Simulate the whole geometry
  - Calculate AC losses in superconducting winding
  - Implement the model in a widely used platform → Comsol Multiphysics
- Main idea: divide the (2D) geometry in two different parts:
  - One part for conventional material, the other for superconductors
  - Exploit the best formulation of Maxwell's equations for each part:
    - *A*-formulation for conventional materials
    - *H*-formulation for superconductors

Main issue: how to join the two parts?

Before continuing: why two formulations?

A couple of observations:

- The FEM simulation of flux dynamics in superconductors characterized a power-law seems to be more reliable with the *H*-formulation than with the *A*-formulation*
- The conditions of continuity of the physical quantities between the rotating and the fixed parts are more easily obtained with the continuity of a scalar quantity (the potential *A* in 2D) than with a vector one (the H field components).

*For the record: *A*-formulation models have been developed both with commercial[1] and home-made[2] codes.

[1] Nibbio et al 2001 IEEE TAS **11** 1 2631

[2] Lathinen et al 2012 SuST **25** 11 115001

The power-law and the formulations

- Equivalent algebraic forms of the E-J power law

(1) $\rho(J) = \rho_0 |J/J_c|^{n-1}$
(2) $\sigma(J) = \sigma_0 |J/J_c|^{1-n}$
(3) $\rho(E) = \rho_0 |E/E_c|^{1-1/n}$
(4) $\sigma(E) = \sigma_0 |E/E_c|^{1/n-1}$

- In the model implementation, circular definitions must be avoided
- *A*-formulation:

$$\sigma(\mathrm{E})\partial_t \boldsymbol{A} - \frac{1}{\mu}\nabla^2 \boldsymbol{A} = \boldsymbol{J}_\mathbf{e}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}, \qquad E = -\partial_t \boldsymbol{A},$$
$$\boldsymbol{J} = \sigma(E)\boldsymbol{E} + \boldsymbol{J}_\mathbf{e}$$

- *E* is a primary quantity → (4) can be used

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The power-law and the formulations

- Equivalent algebraic forms of the E-J power law

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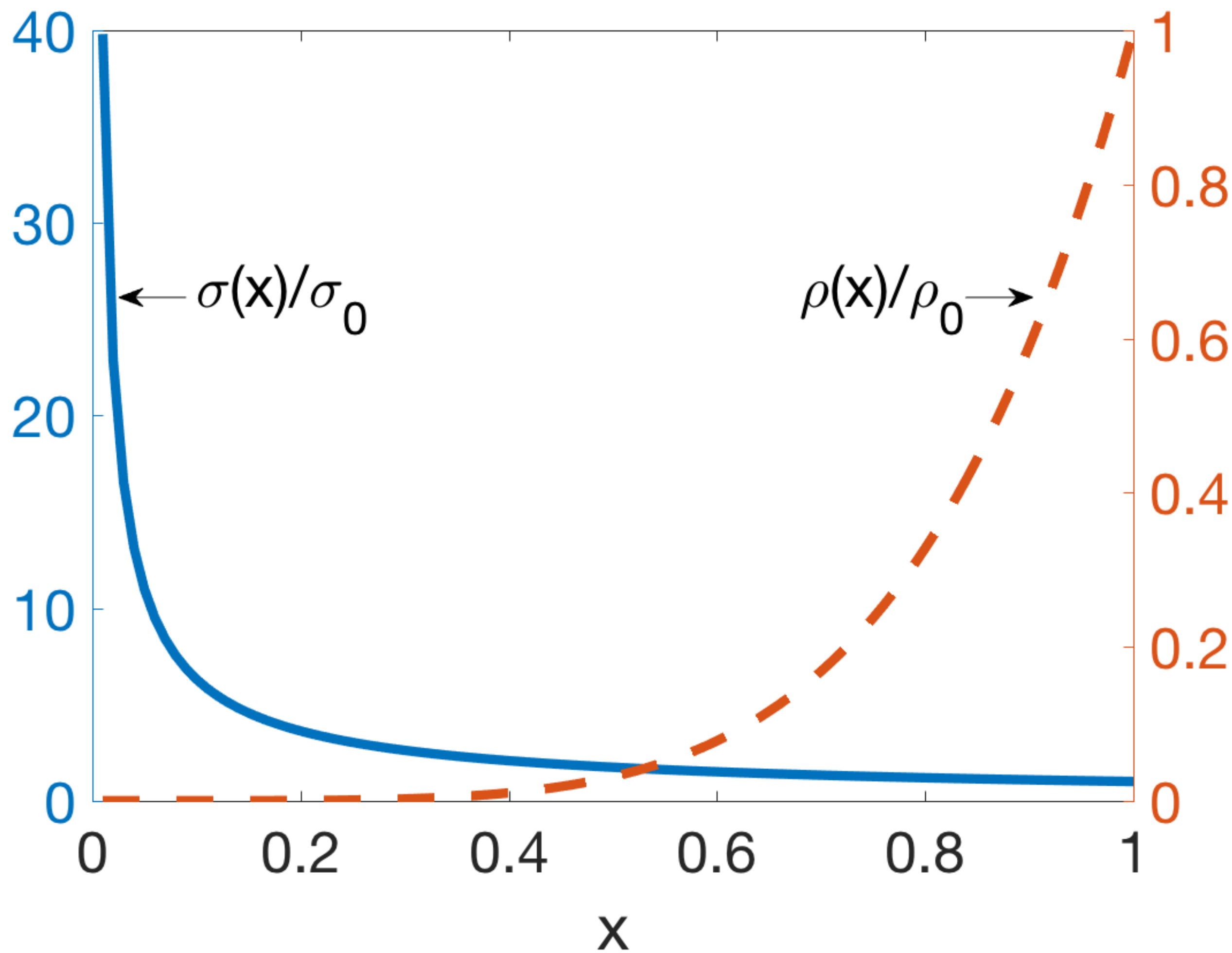
(4) $\sigma(E) = \sigma_0 |E/E_c|^{1/n-1}$

- In the model implementation, circular definitions must be avoided
- *H*-formulation:

$$\mu \partial_t \boldsymbol{H} + \nabla \times \boldsymbol{E} = 0 \qquad \boldsymbol{J} = \nabla \times \boldsymbol{H}, \qquad \boldsymbol{E} = \rho(J)\boldsymbol{J}$$

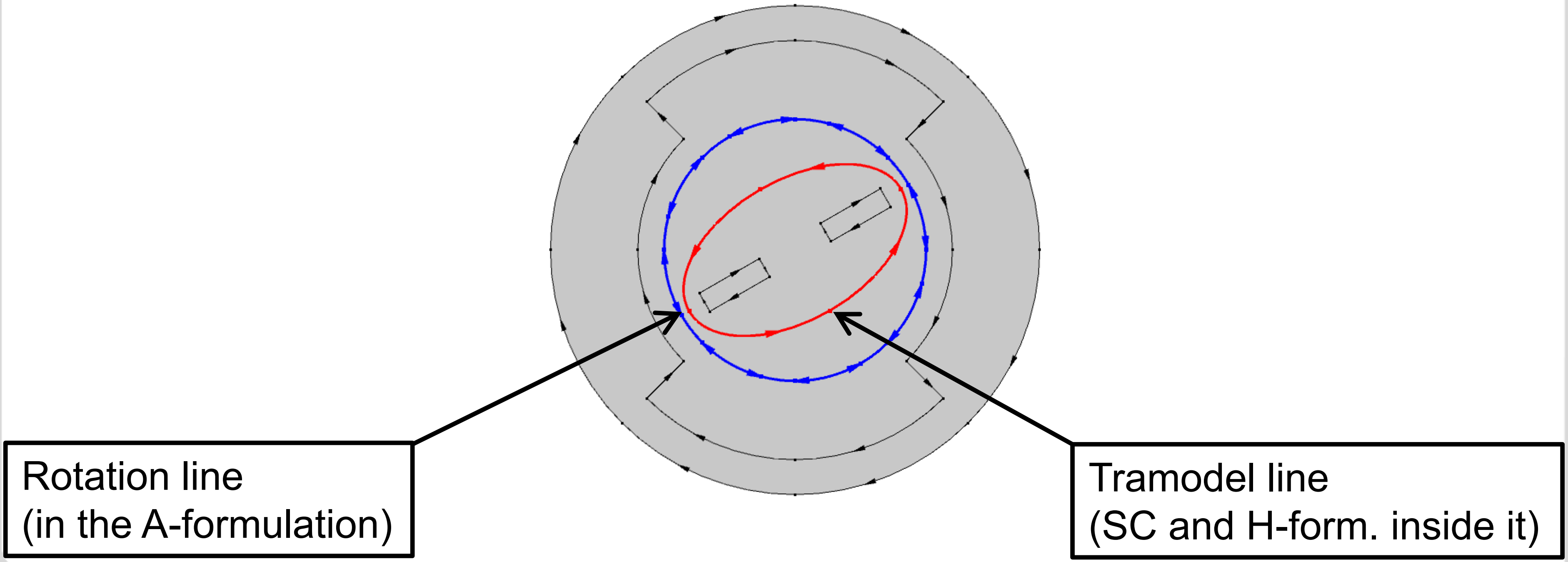
- *J* is a primary quantity → (1) can be used

The power-law and the formulations



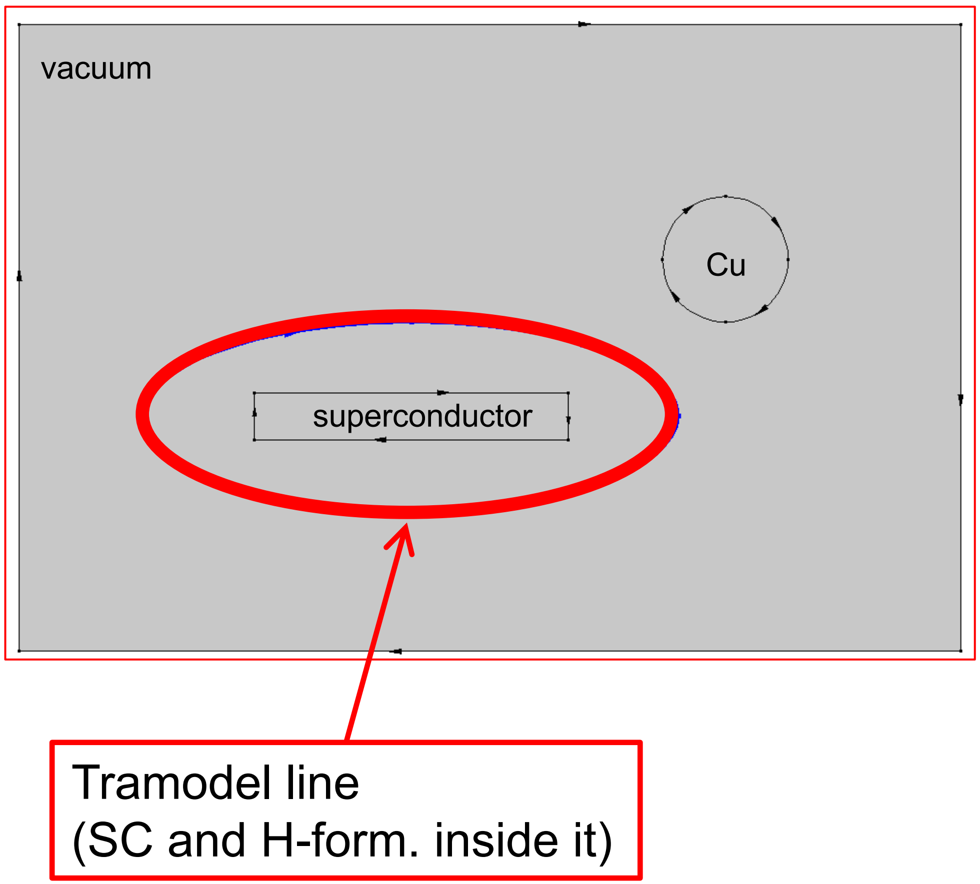
Two important lines

- Tramodel line: separates the *A*- and *H*-formulation parts
- Rotation line: separates the rotating and fixed parts
- In 2D, *A* is a scalar → Ideal solution:
  - Divide the geometry so that the rotation line is inside the *A*-formulation part
  - Scalar continuity between two (rotating/fixed) coordinate systems



Coupling the *A*- and *H*-formulation parts

- Rectangular superconductor and round Cu wire carrying opposite currents



Current definitions

$$I_{\mathrm{SC}} = \int_{\mathrm{SC}} J_z(t)\mathrm{d}s = I_{\mathrm{a}}(t)$$

$$J_{\mathrm{e}}(t) = \sigma V_{\mathrm{wire}}(t)/L$$

Setting tangential components of the magnetic field equal

$$H_{\mathrm{t}}^{(A)} = t_x \cdot H_x + t_y \cdot H_y$$

$$H_{\mathrm{t}}^{(H)} = t_x \cdot H + t_y \cdot K$$

$$H_{\mathrm{t}}^{(H)} = H_{\mathrm{t}}^{(A)}$$

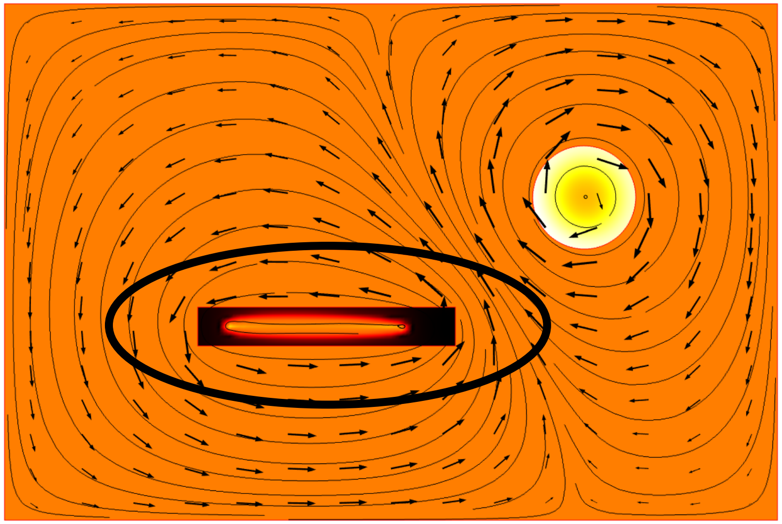
Ineffective!!!

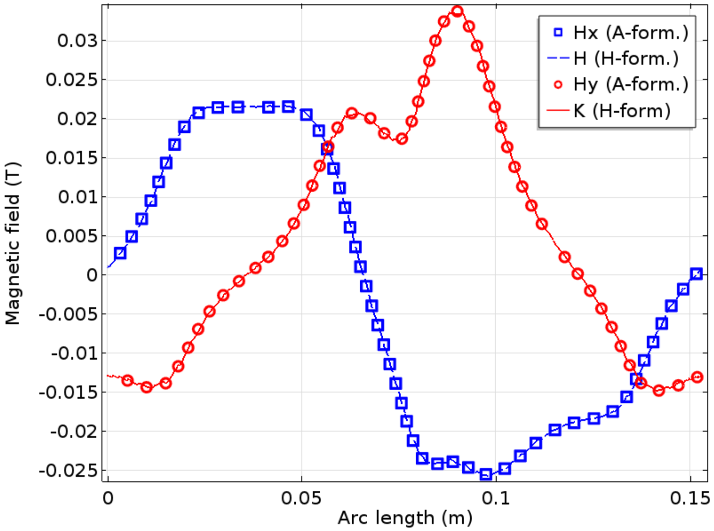
Coupling the *A*- and *H*-formulation parts

- In Comsol, coupling conditions are effective if we add two Weak Contribution instructions on the tramodel line:
  - *A*-formulation $H_t \cdot \mathrm{test}(A_z)$
  - *H*-formulation $E_z \cdot \mathrm{test}(H_t)$

Effective!!!

- This forces the common values as sources:
  - $H_\mathrm{t}^{(H)}$ acts as a source for the *A*-formulation part
  - $\partial_t A_z = E_z$ acts as a source for the *H*-formulation part





Application to rotating systems

- Two reference systems
  - Fixed part (spatial frame) → (x, y)
  - Rotating part (material frame) → (X, Y)
- Link between temporal coordinates

$$\begin{pmatrix} X(x,y,t) \\ Y(x,y,t) \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix} \qquad T = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix}$$

- The vectors undergo a similar transformation

$$\begin{pmatrix} B_X \\ B_Y \end{pmatrix} = T \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

Application to rotating systems

- Field equations solved in both reference systems
- What is the transformation of physical quantities from one reference system to the other one?

$$\boldsymbol{B}_{\text{spatial}}(x, y) = \boldsymbol{B}_{\text{material}}(X, Y)$$

$$\boldsymbol{E}_{\text{spatial}}(x, y) = \boldsymbol{E}_{\text{material}}(X, Y) - \boldsymbol{v}(x, y) \times \boldsymbol{B}_{\text{material}}(X, Y)$$

- $\boldsymbol{v}(x, y)$ velocity of a point of the rotor in the spatial frame
- $\boldsymbol{v} = (v_x, v_x, 0) = \boldsymbol{\omega} \times \boldsymbol{x} = (-\omega y, \omega x, 0)$
- In the 2D case, we will have:

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = T^{-1} \begin{pmatrix} B_X \\ B_Y \end{pmatrix} \qquad E_z = E_Z + \omega(X B_X + Y B_Y) = E_Z + \mathrm{d}E_Z$$

In passing from one reference system to the other:
- The magnetic field simply rotates
- The electric field rotates and changes its magnitude

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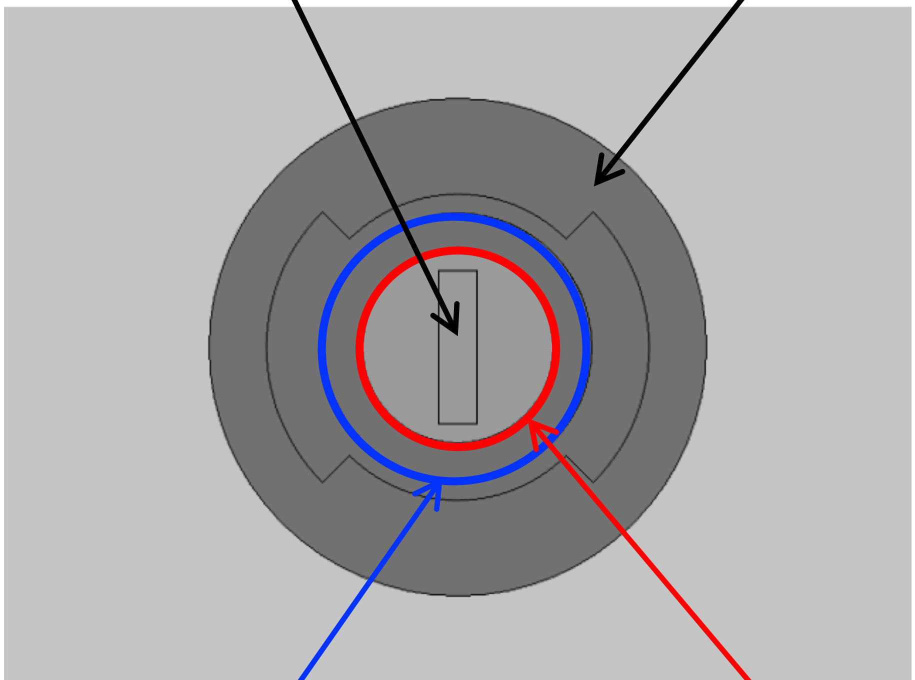
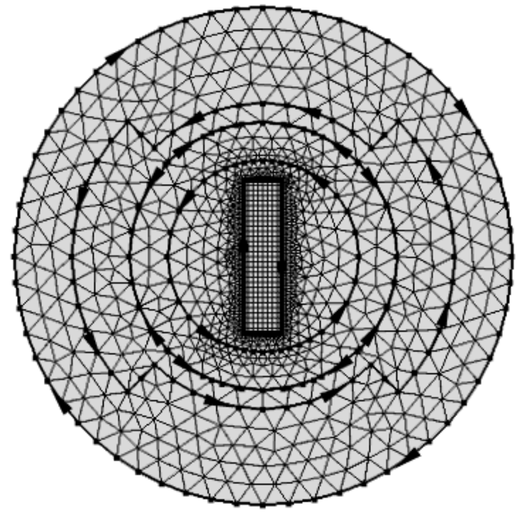
An example

Rotating superconductor and air (H-formulation)

Fixed magnetic material and air (A-formulation)

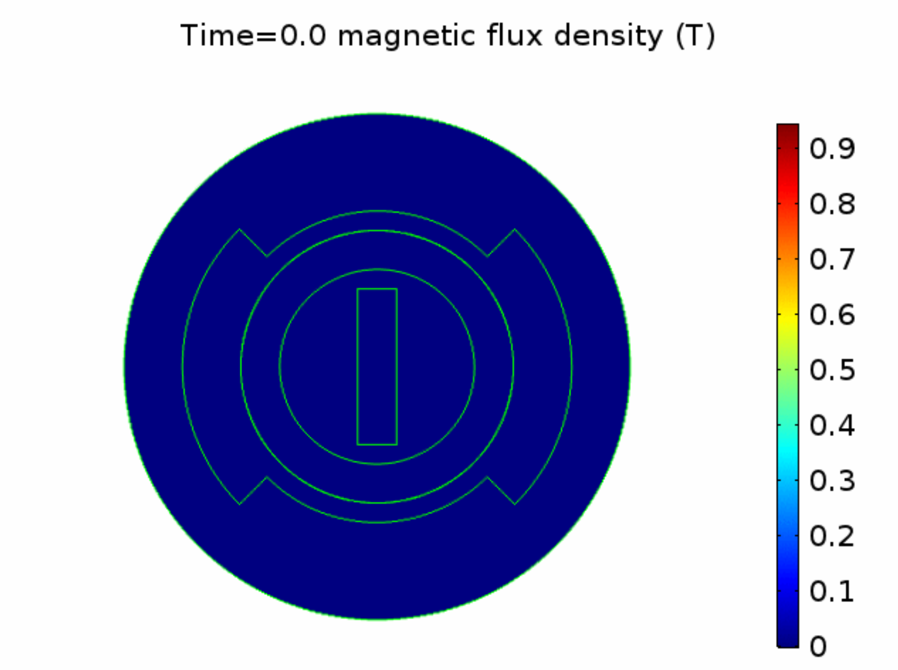


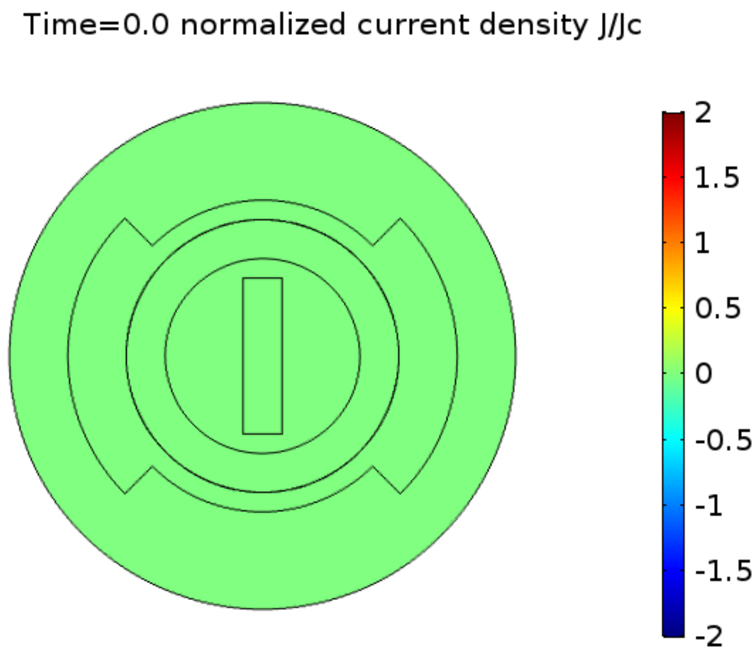


Rotation line (in the A-formulation)

Tramodel line (SC and H-form. inside it)

An example


Time=0.0 magnetic flux density (T)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


Time=0.0 normalized current density J/Jc
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

Conclusion

- New FEM framework for modeling electrical machines employing superconductors:
  - *H*-formulation for regions containing superconductors
  - *A*-formulation for the rest
- Electromagnetic quantities joined on the line separating the two formulations (“tramodel” line) by means of weak contribution boundary conditions
- Tramodel line can be in the rotating or fixed part → joining conditions different in the two cases

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One more thing…

The model will be freely available on

http://www.htsmodelling.com/

Source: Toyota

Just to be clear: **not** the car model!!!

Thank you very much for your attention!

Muito obrigado pela vossa atenção!