Peridynamic simulation of crack propagation in bulk superconductors with an electromagnetic-thermal model

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Contents

- Introduction
- Model description
- Result
- Conclusion
Introduction

- Both **micro-cracks** and **voids** may appear in the superconducting materials.
- The **current density** and the **magnetic field** in the superconductor will be redistributed.
- **Damage** may occur in superconductors under high magnetic fields.

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**YBCO**


**GdBCO/Ag**


**Cracks and voids**

The flaws can affect the AC losses of the superconductor materials.

The dynamic fracture behavior of cracks in superconductors was also investigated.
Model description — Electromagnetic-Thermal model

The schematic view of numerical simulation as follow:

The Maxwell equations and the electric field-current density (E-J) relationship

\[
\mu_0 \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \\
J_x = \frac{\partial H_z}{\partial y}, \quad J_y = -\frac{\partial H_z}{\partial x} \\
E_x = \rho J_x, \quad E_y = \rho J_y
\]
The resistivity $\rho$ follows a modified power law relationship

$$\rho_{\text{PL}} = \frac{E_0}{J_c} \left| \frac{J}{J_c} \right|^{n-1}, \quad \rho = \frac{\rho_{\text{PL}} \cdot \rho_{\text{normal}}}{\rho_{\text{PL}} + \rho_{\text{normal}}}$$

$$J_c(B) = \alpha \left( 1 - \left(\frac{T}{T_c}\right)^2 \right)^{1.5} \frac{B_0}{|B| + B_0}$$

The law of heat transfer can be described by

$$\tilde{\rho} c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

The applied pulsed field $B_{\text{ex}}(t)$ is expressed using

$$B_{\text{ex}}(t) = B_{\text{max}} \frac{t}{\tau} \exp \left( 1 - \frac{t}{\tau} \right)$$

• The magnetic field and current density have difference near the crack between the bulk with crack and without crack.
• On the boundary, the difference of magnetic field is due to no heat exchange with stainless steel.
Model description — Electromagnetic-Thermal model

(a) Electromagnetic force and (b) Temperature profiles along the x-axis
In PD theory, each material particle $x$ is assumed to interact with every other particle $x'$ that is located within a finite distance.

The equation of motion is given as:

$$
\tilde{\rho}(x) \ddot{u}(x, t) = \int_{\mathcal{R}_x} \left( t(u' - u, x' - x, t) - t'(u - u', x - x', t) \right) dV' + b(x, t)
$$

$b(x, t)$ represents electromagnetic force.
Model description — Overview of peridynamic theory

The force density vectors is:

\[ t_{(k)(j)} = 2\delta \left\{ d \frac{\Lambda_{(k)(j)}}{|x_{(j)} - x_{(k)}|} \left( a\theta_{(k)} - \frac{1}{2} a_2 T_{(k)} \right) + b \left( s_{(k)(j)} - \alpha T_{(k)} \right) \right\} \times \frac{y_{(j)} - y_{(k)}}{|y_{(j)} - y_{(k)}|} \]

where the \( s \) is stretch

\[ s_{(k)(j)} = \frac{|y_{(j)} - y_{(k)}| - |x_{(j)} - x_{(k)}|}{|x_{(j)} - x_{(k)}|} \]

The dilatation term \( \theta \) can be calculated as follow:

\[ \theta_{(k)} = d \sum_{j=1}^{N} w_{(k)(j)} \left( s_{(k)(j)} - \alpha T_{(k)} \right) \frac{y_{(j)} - y_{(k)}}{|y_{(j)} - y_{(k)}|} \cdot \left( x_{(j)} - x_{(k)} \right) V_{(j)} + 3\alpha T_{(k)} \]
Model description — Overview of peridynamic theory

The local damage at point $x$ is defined as the ratio of the number of breakage interaction to the total number of initial interaction.

$$\varphi(x, t) = 1 - \frac{\int_{H} \mu(x' - x, t) \, dV'}{\int_{H} \, dV'}$$

$$\mu(x' - x, t) = \begin{cases} 
1 & s(x' - x, t') - \left( T_j + T_k \right)/2 < s_c, \\ 0 & \text{otherwise} 
\end{cases}$$

The relationship between the critical stretch $s_c$ and the critical energy release rate $G_c$ is:

$$G_c = \left( bh\delta^{5} + \frac{8}{9} ad^2 h^2 \delta^7 \right)s_c^2$$
The mechanical response is analyzed using both the proposed PD model and the FEM:

Displacements given by FEM and PD models (a) at different moments (b) profiles along the x-axis

- Comparison shows that the results obtained from PD are well matched with those obtained from the FEM analysis.
The crack will not propagate when the critical stress intensity factor $K_{IC}$ is 0.53.

When the $K_{IC}$ decrease to 0.52, damage appears at the crack tip and crack propagation occurs.
Similarly, The crack will not propagate when the $K_{IC}$ is 0.96.

When the $K_{IC}$ decrease to 0.95, the crack will propagate rapidly in the horizontal direction from the crack tip to either side of the bulk.

- The presence of stainless steel increases the stability of bulk effectively.
**Result** — Bulk containing a void with different a/b

The crack length grows to 4.15mm when the \( a/b \) is 2.5. \( K_{IC} = 0.5 \)

The crack length grows to 5.85mm when the \( a/b \) is 4/3. \( K_{IC} = 0.32 \)

- The results show that the narrow void can lead to damage easier.
Now, we consider the effects of angle between the crack and defect (void or inclusion) on the mechanical stability of the bulk.

- The minimum value of $K_{IC}$ is obtained at the angle of about $60^\circ$ both the void and inclusion. For this case, the bulk sample has better mechanical stability.
1. The crack propagation is investigated only under electromagnetic force, where the cross angle between the cracks is 90 degree.

Only one crack begins to propagate while the other does not grow.

2. The cross angle between the cracks is 60 degree.

The crack propagation starts at one tip of the two cracks.
• The electromagnetic forces and temperature change in GdBCO bulk superconducting sample were solved using FEM models for different types of defects under pulsed fields.
• The crack can lead to strong local enhancement of the electromagnetic load at the crack tip, which may then cause crack propagation.
• A state-based PD theory was developed to study the dynamic fracture problems of materials and structures when subjected to an external electromagnetic force and thermal load.
• The stainless steel can protect the bulk effectively.
• When the angle between crack and defect is 60 degree, the stress intensity factor is smaller.

Thanks for your attention!