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Peridynamic simulation of crack propagation in bulk superconductors with an electromagnetic-thermal model

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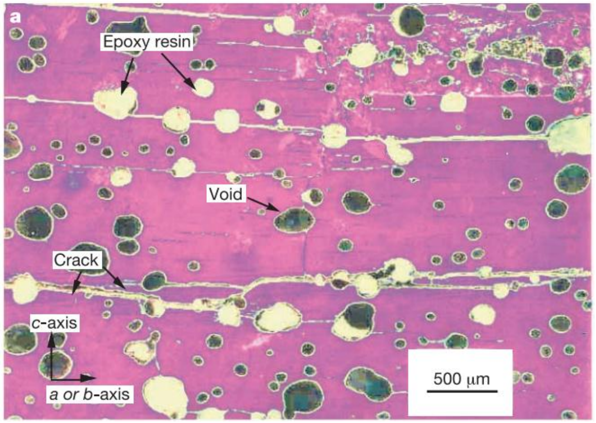
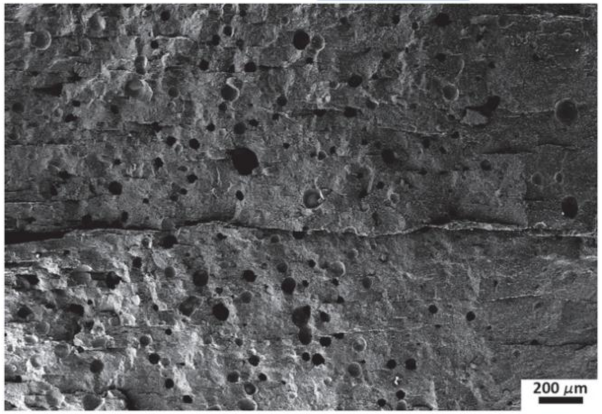
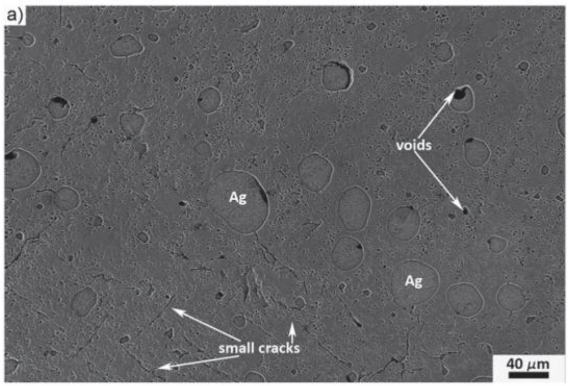
YBCO



GdBCO/Ag

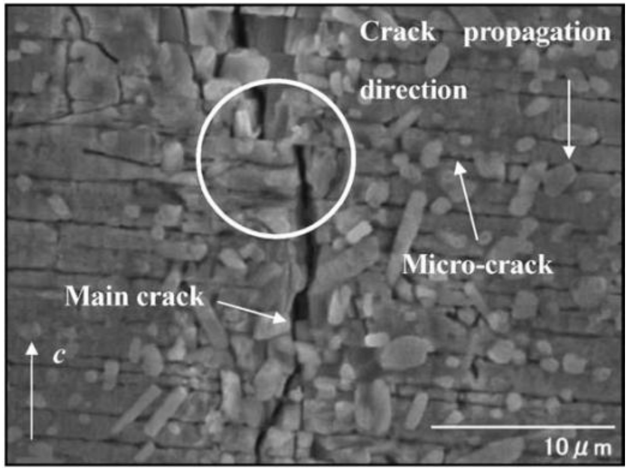


Cracks and voids



Masaru Tomita et al. / Nature, 421, (6922) 517, 2003

K Konstantopoulou et al. / Supercond. Sci. Technol.27(2014) 115011



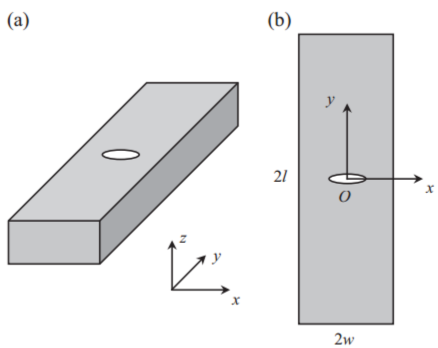
K. Katagiri et al. / Physica C 426–431 (2005) 709–713

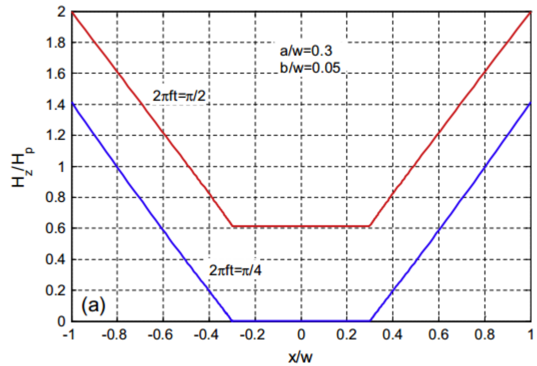
- Both **micro-cracks** and **voids** may appear in the superconducting materials.
- The **current density** and the **magnetic field** in the superconductor will be redistributed.
- **Damage** may occur in superconductors under high magnetic fields.

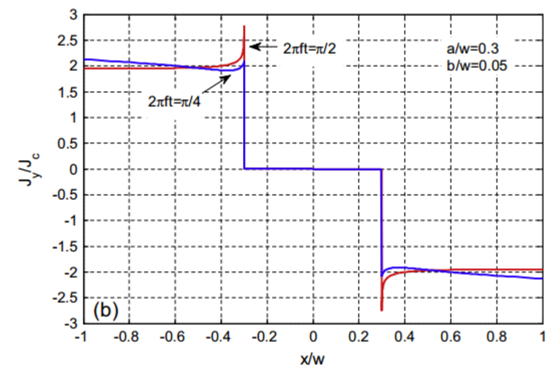
Introduction —previous work

The flaws can affect the AC losses of the superconductor materials.

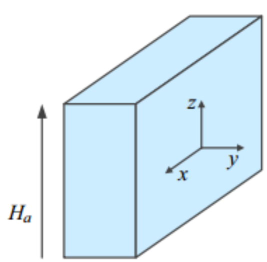
J. Xia, Y. Zhou / Cryogenics 69 (2015) 1–9

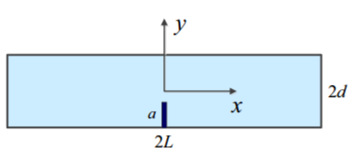


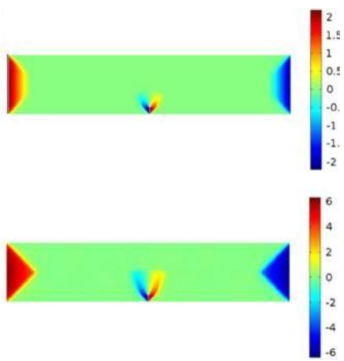


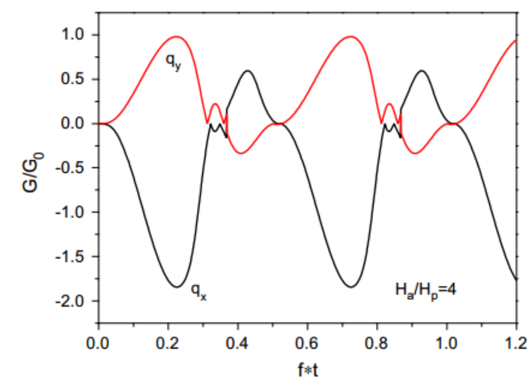


The dynamic fracture behavior of cracks in superconductors was also investigated.





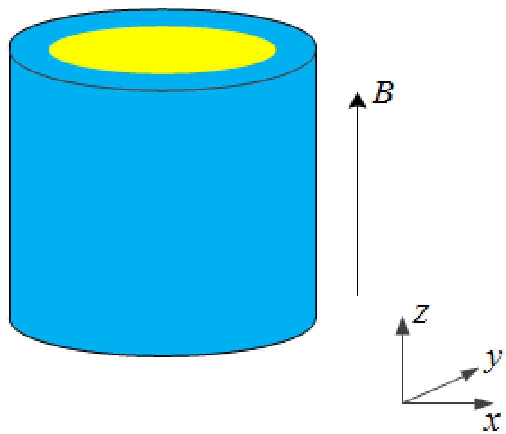


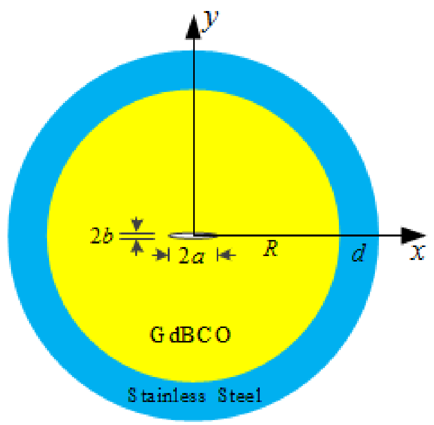


H.D. Yong et al. / Engineering Fracture Mechanics 158 (2016) 167.178

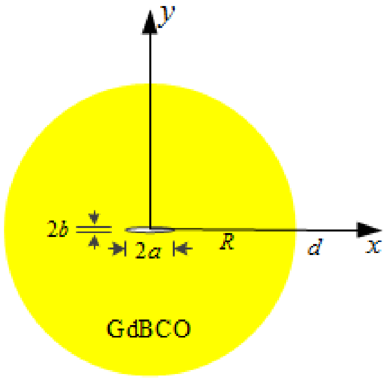
Model description —Electromagnetic-Thermal model

The schematic view of numerical simulation as follow:





bulk with a crack and stainless steel.



bulk with a crack without stainless steel.

Ru et al. / Engineering Fracture Mechanics 199 (2018) 257–273

The Maxwell equations and the electric field-current density (E-J) relationship

$$\mu_0 \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \qquad J_x = \frac{\partial H_z}{\partial y}, \qquad J_y = -\frac{\partial H_z}{\partial x} \qquad E_x = \rho J_x, \qquad E_y = \rho J_y$$

Model description —Electromagnetic-Thermal model

The resistivity ρ follows a modified power law relationship

$$\rho_{\mathrm{PL}} = \frac{E_0}{J_c}\left|\frac{\mathbf{J}}{J_c}\right|^{n-1}, \qquad \rho = \frac{\rho_{\mathrm{PL}} \cdot \rho_{\mathrm{normal}}}{\rho_{\mathrm{PL}} + \rho_{\mathrm{normal}}}$$

$$J_c(B) = \alpha\left(1-\left(\frac{T}{T_c}\right)^2\right)^{1.5}\frac{B_0}{|B|+B_0}$$

The law of heat transfer can be described by

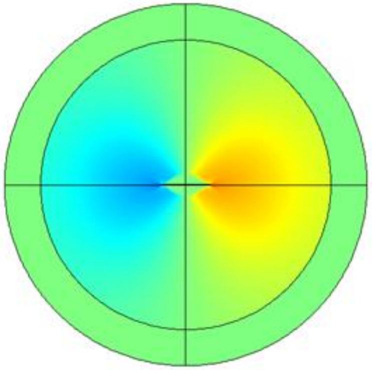
$$\tilde{\rho} c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q$$

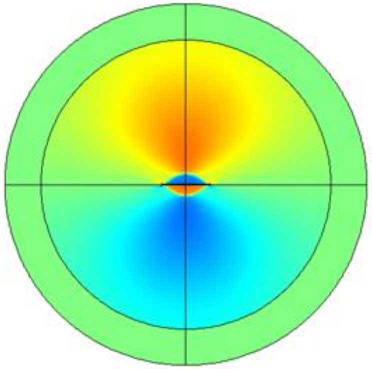
The applied pulsed field $B_{ex}(t)$ is expressed using

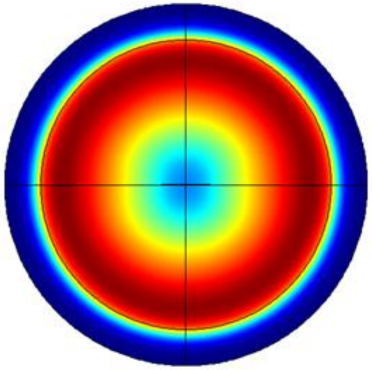
$$B_{\mathrm{ex}}(t) = B_{\max}\frac{t}{\tau}\exp\left(1-\frac{t}{\tau}\right)$$

Hiroyuki Fujishiro et al. / Supercond. Sci. Technol.23(2010)

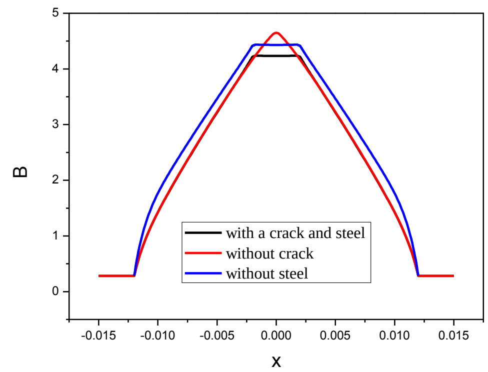
Model description —Electromagnetic-Thermal model



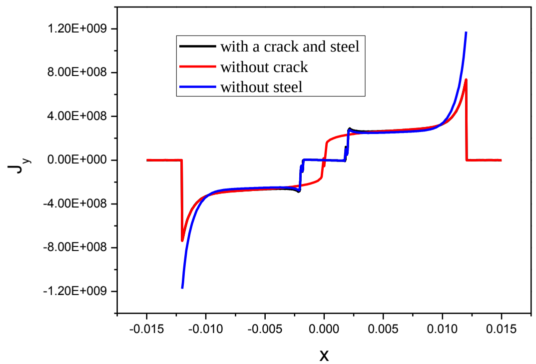




(a)



(b)

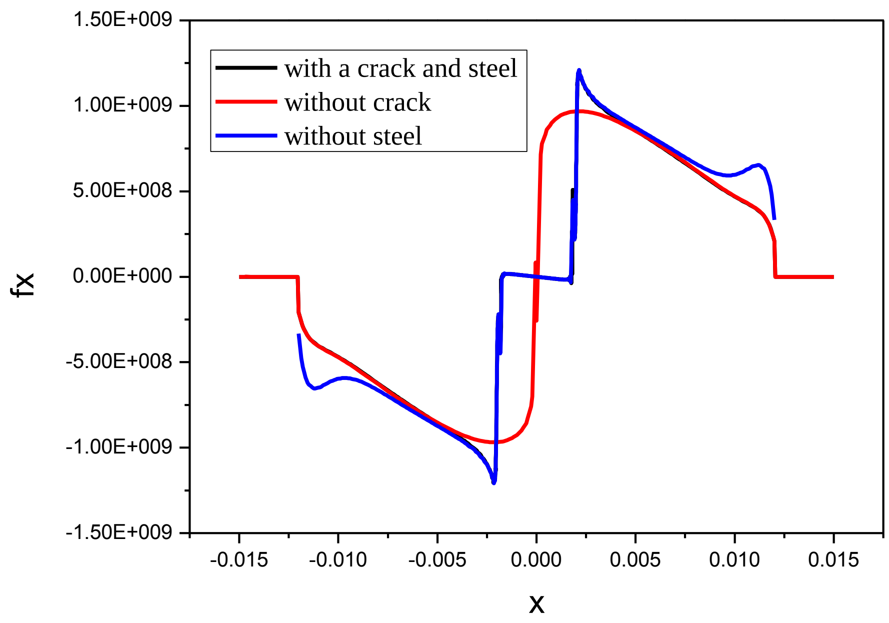


(a) Magnetic field and
(b) current profiles
along the x-axis

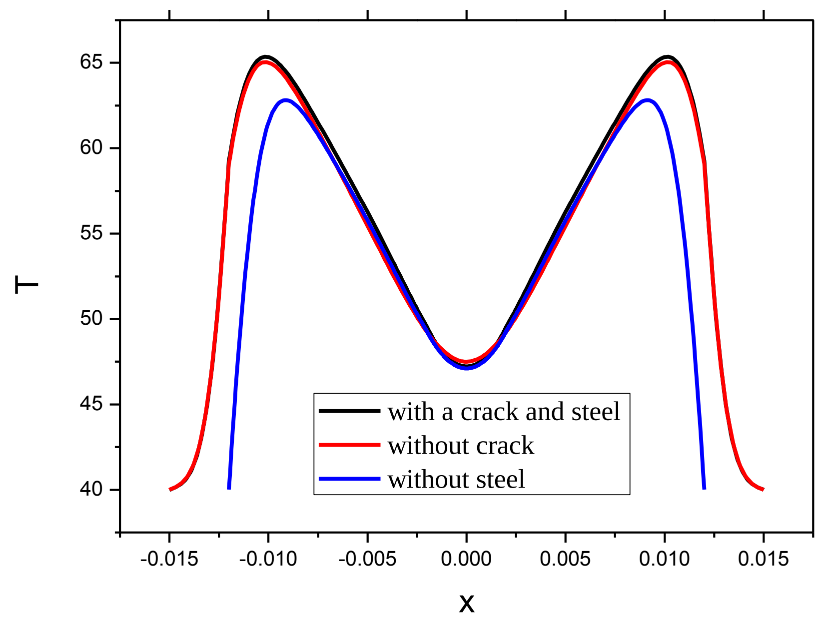
- The magnetic field and current density have difference near the crack between the bulk with crack and without crack.
- On the boundary, the difference of magnetic field is due to no heat exchange with stainless steel.

Model description —Electromagnetic-Thermal model

(a)

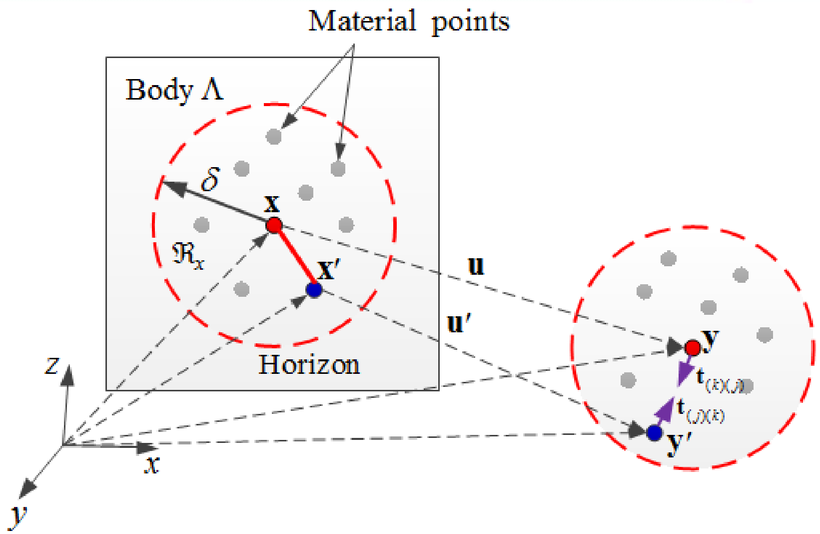


(b)



(a) Electromagnetic force and (b) Temperature profiles along the x-axis

Model description —Overview of peridynamic theory



In PD theory, each material particle **x** is assumed to interact with every other particle **x’** that is located within a finite distance.

The equation of motion is given as: Silling SA, Askari E. / Comput Struct (2000) 83:1526–1535

$$\tilde{\rho}(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t)=\int_{\Re_x}\left(\mathbf{t}(\mathbf{u}'-\mathbf{u},\mathbf{x}'-\mathbf{x},t)-\mathbf{t}'(\mathbf{u}-\mathbf{u}',\mathbf{x}-\mathbf{x}',t)\right)dV'+\mathbf{b}(\mathbf{x},t)$$

b(**x**,t) represents electromagnetic force.

Model description —Overview of peridynamic theory

The force density vectors is:

$$\mathbf{t}_{(k)(j)} = 2\delta \left\{ d \frac{\Lambda_{(k)(j)}}{\left|\mathbf{x}_{(j)} - \mathbf{x}_{(k)}\right|} \left(a\theta_{(k)} - \frac{1}{2} a_2 T_{(k)} \right) + b\left(s_{(k)(j)} - \alpha T_{(k)} \right) \right\} \times \frac{\mathbf{y}_{(j)} - \mathbf{y}_{(k)}}{\left|\mathbf{y}_{(j)} - \mathbf{y}_{(k)}\right|}$$

where the s is stretch

$$s_{(k)(j)} = \frac{\left|\mathbf{y}_{(j)} - \mathbf{y}_{(k)}\right| - \left|\mathbf{x}_{(j)} - \mathbf{x}_{(k)}\right|}{\left|\mathbf{x}_{(j)} - \mathbf{x}_{(k)}\right|}$$

The dilatation term θ can be calculated as follow:

$$\theta_{(k)} = d \sum_{j=1}^{N} w_{(k)(j)} \left(s_{(k)(j)} - \alpha T_{(k)} \right) \frac{\mathbf{y}_{(j)} - \mathbf{y}_{(k)}}{\left|\mathbf{y}_{(j)} - \mathbf{y}_{(k)}\right|} \cdot \left(\mathbf{x}_{(j)} - \mathbf{x}_{(k)} \right) V_{(j)} + 3\alpha T_{(k)}$$

Model description —Overview of peridynamic theory

The local damage at point **x** is defined as the ratio of the number of breakage interaction to the total number of initial interaction.

$$\varphi(\mathbf{x},t)=1-\frac{\int_H \mu(\mathbf{x}'-\mathbf{x},t)\,dV'}{\int_H dV'}$$

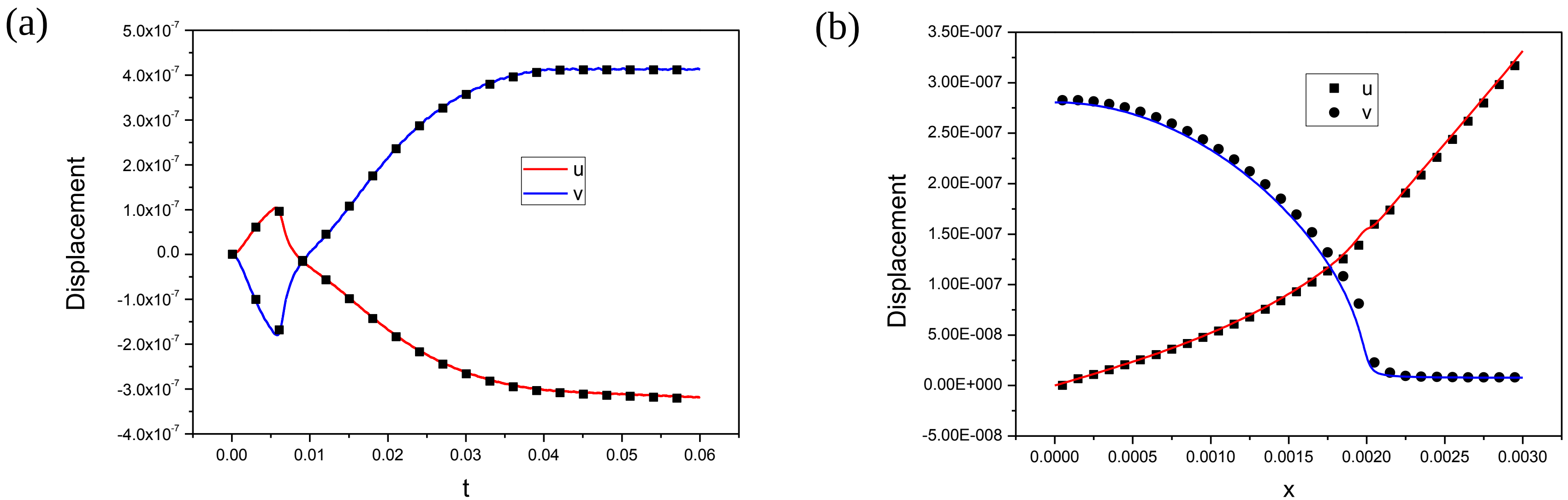
$$\mu(\mathbf{x}'-\mathbf{x},t)=\begin{cases}1 & s(\mathbf{x}'-\mathbf{x},t')-\left(T_{(j)}+T_{(k)}\right)/2<s_c, \quad 0<t' \\ 0 & \text{otherwise}\end{cases}$$

The relationship between the critical stretch s_c and the critical energy release rate G_c is:

$$G_c=\left(bh\delta^5+\frac{8}{9}ad^2h^2\delta^7\right)s_c^2$$

Model description —Verification

The mechanical response is analyzed using both the proposed PD model and the FEM:

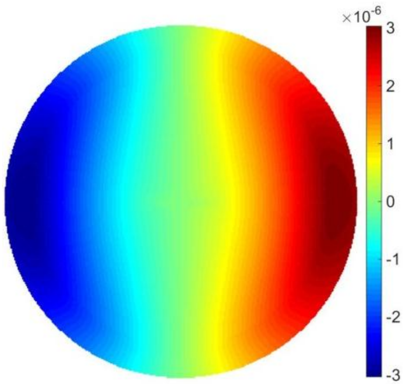


Displacements given by FEM and PD models (a)at different moments (b) profiles along the x-axis

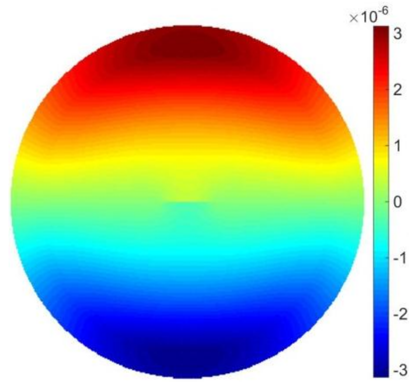
- Comparison shows that the results obtained from PD are well matched with those obtained from the FEM analysis.

Result —Bulk with stainless steel

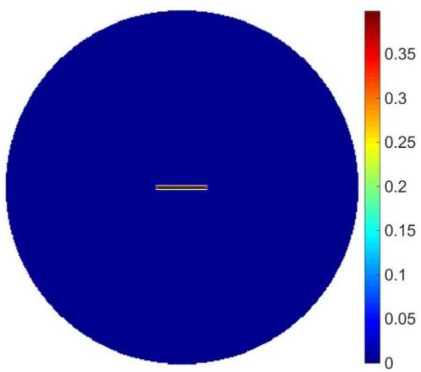
The crack will not propagate when the critical stress intensity factor K_{IC} is 0.53.



Horizontal displacement

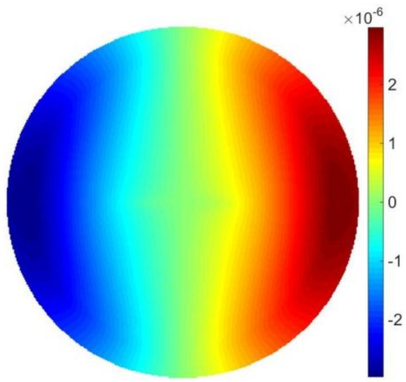


Vertical displacement

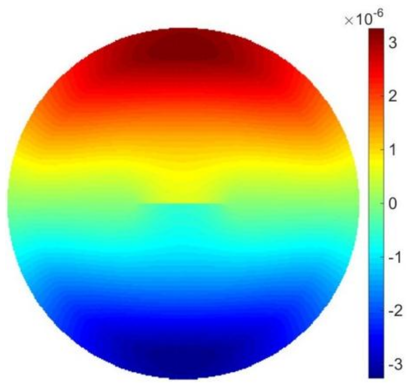


Local damage map

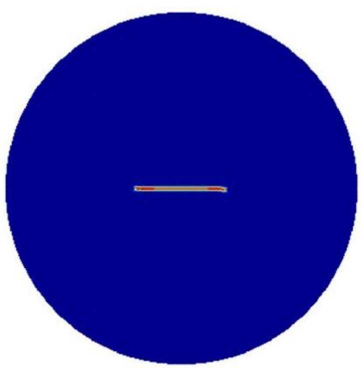
When the K_{IC} decrease to 0.52, damage appears at the crack tip and crack propagation occurs.



Horizontal displacement



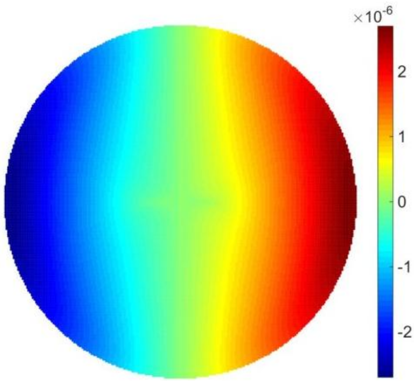
Vertical displacement



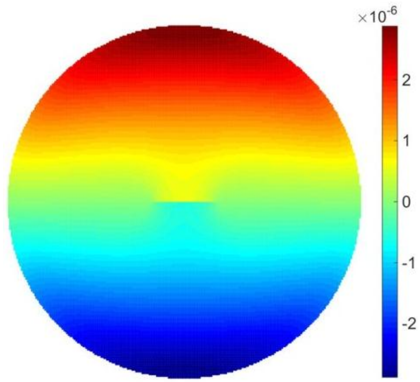
Crack propagation

Result —Bulk without stainless steel

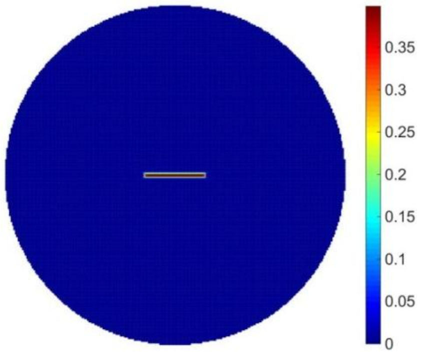
Similarly, The crack will not propagate when the K_{IC} is 0.96.



Horizontal displacement

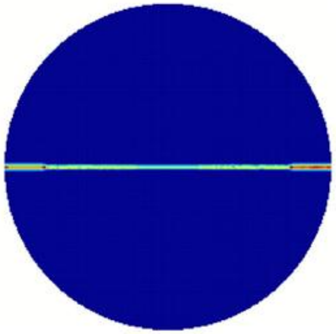


Vertical displacement



Local damage map

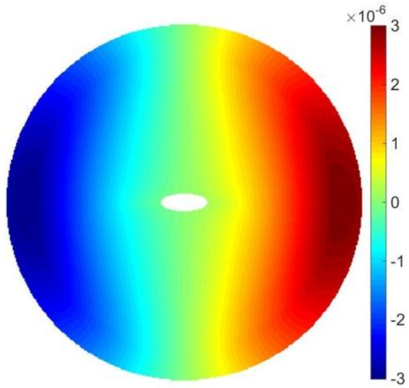
When the K_{IC} decrease to 0.95, the crack will propagate rapidly in the horizontal direction from the crack tip to either side of the bulk.


Crack propagation

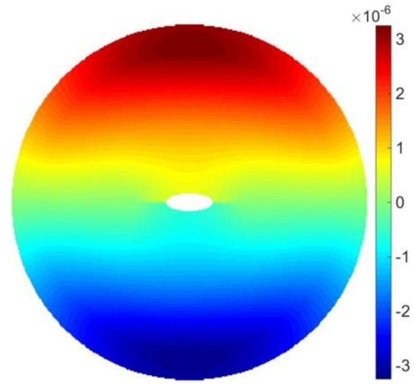
- The presence of stainless steel increases the stability of bulk effectively.

Result —Bulk containing a void with different a/b

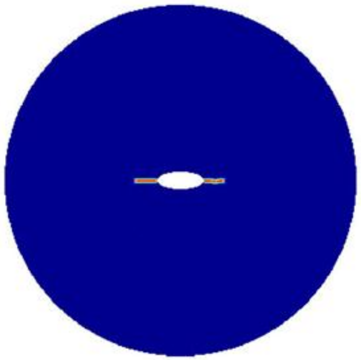
The crack length grows to 4.15mm when the *a/b* is 2.5. $K_{IC} = 0.5$



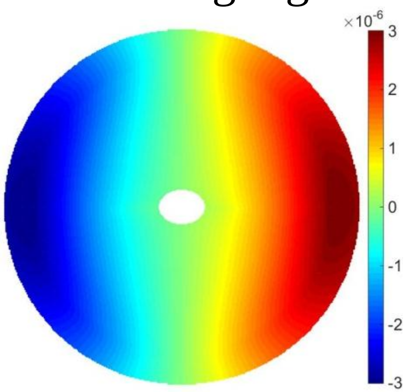
Horizontal displacement

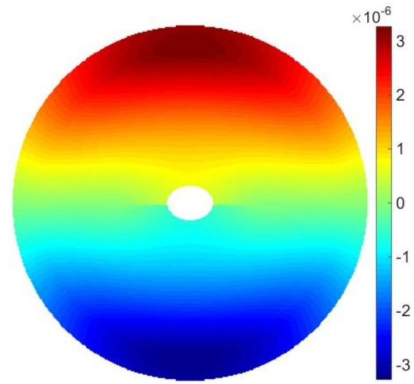


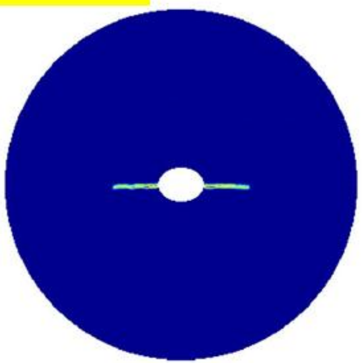
Vertical displacement


Crack propagation

The crack length grows to 5.85mm when the *a/b* is 4/3. $K_{IC} = 0.32$



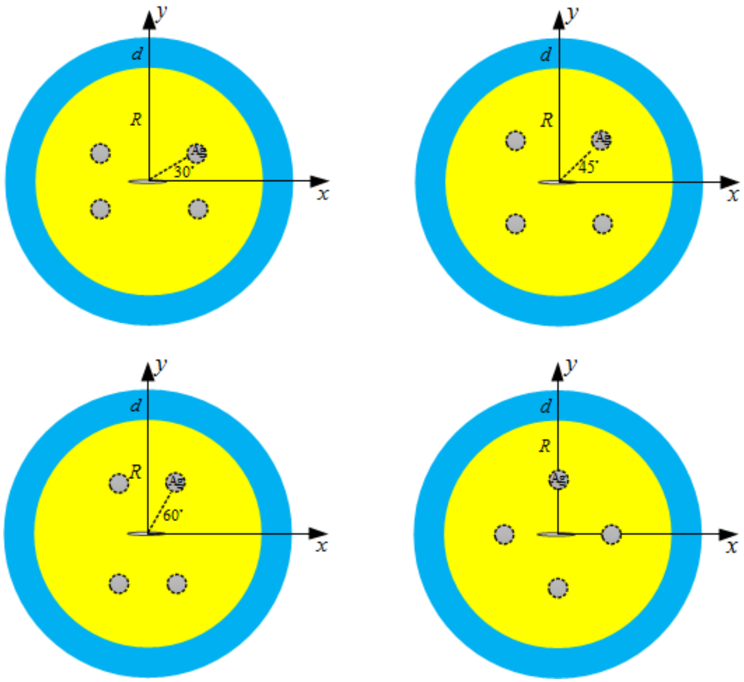


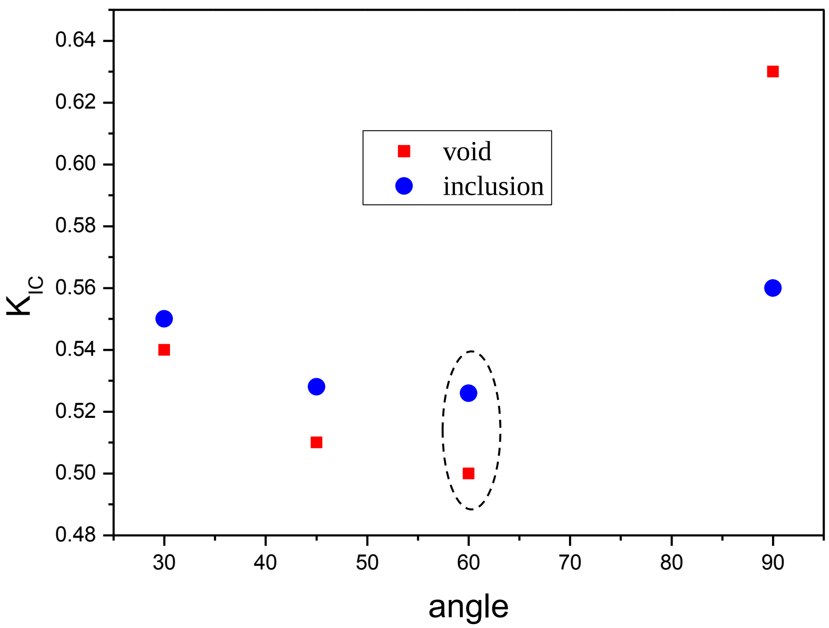


- The results show that the narrow void can lead to damage easier.

Result —Bulk with a crack and void or inclusion

Now, we consider the effects of angle between the crack and defect (void or inclusion) on the mechanical stability of the bulk.

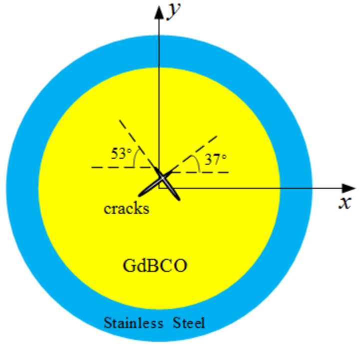


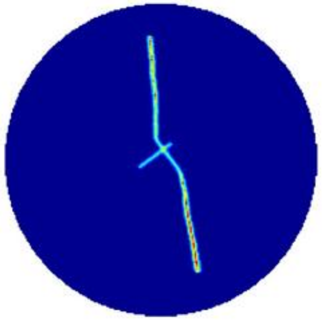


- The minimum value of K_{IC} is obtained at the angle of about 60° both the void and inclusion. For this case, the bulk sample has better mechanical stability.

Result —Bulk with cross cracks

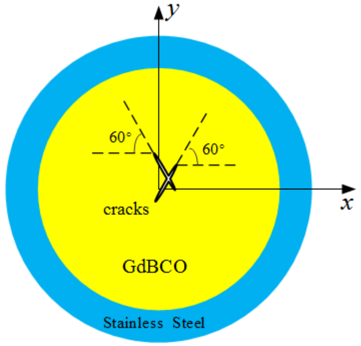
1. The crack propagation is investigated only under electromagnetic force, where the cross angle between the cracks is 90 degree.

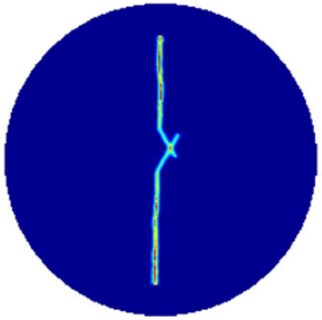




Only one crack begins to propagate while the other does not grow.

2. The cross angle between the cracks is 60 degree.





The crack propagation starts at one tip of the two cracks

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Conclusion

- The electromagnetic forces and temperature change in GdBCO bulk superconducting sample were solved using FEM models for different types of defects under pulsed fields.
- The crack can lead to strong local enhancement of the electromagnetic load at the crack tip, which may then cause crack propagation.
- A state-based PD theory was developed to study the dynamic fracture problems of materials and structures when subjected to an external electromagnetic force and thermal load.
- The stainless steel can protect the bulk effectively.
- When the angle between crack and defect is 60 degree, the stress intensity factor is smaller .

Thanks for your attention!