Comparison of constitutive laws for modeling high-temperature superconductors

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Motivation

- Numerical models routinely used to model electromagnetic response of superconductors
- Many numerical methods, codes and software available
- Inputs: geometry, boundary conditions, constraints,..., **material properties**
- Superconductors usually simulated with critical state or power-law models

Question

Are these simple choices OK for all practical cases?

We compare three models

1. Critical state model – CSM (1 parameter: J_c)

$$J = 0 \text{ or } J_\mathrm{c} \quad \text{(thus } E \text{ is multi-valued)} \tag{1}$$

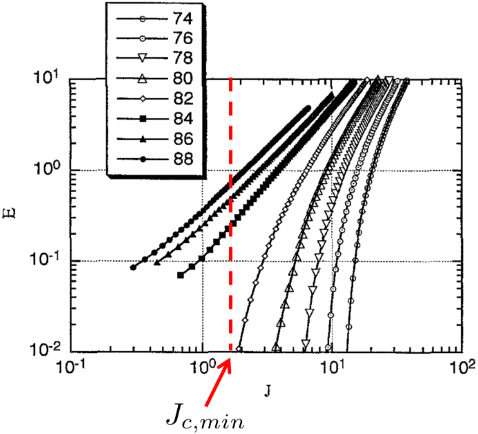
2. Power law model (2 parameters: J_c and n)

$$E = E_\mathrm{c} \left(\frac{J}{J_\mathrm{c}} \right)^n \tag{2}$$

3. Percolation model (3 parameters: J_c, $J_\mathrm{c,min}$ and n)

$$E = \begin{cases} 0 & \text{if } J \leq J_\mathrm{c,min} \\ E_\mathrm{c} \left(\dfrac{J - J_\mathrm{c,min}}{J_\mathrm{c} - J_\mathrm{c,min}} \right)^n = k \left(J_\mathrm{c} - J_\mathrm{c,min} \right)^n & \text{if } J > J_\mathrm{c,min} \end{cases} \tag{3}$$

Percolation model



Percolation model

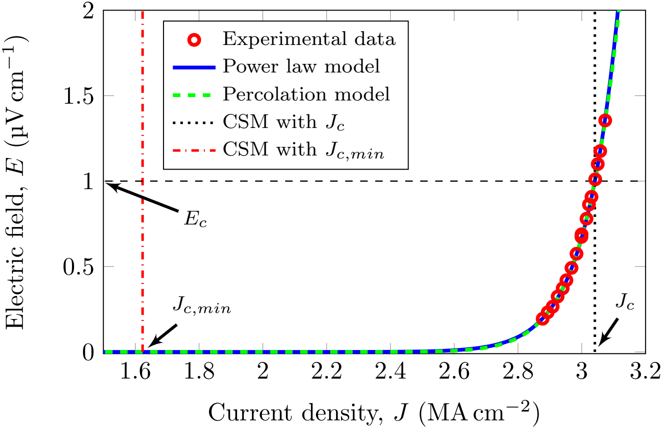
$$E = k\left(J_{\mathrm{c}} - J_{\mathrm{c,min}}\right)^{n}$$

$$E = 0 \text{ if } J < J_{\mathrm{c,min}}$$

True lossless behavior

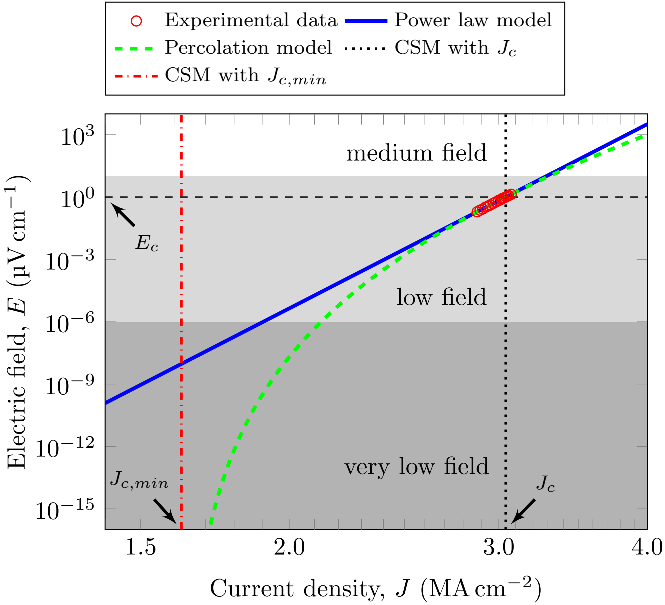
Source: Yamafuji and Kiss 1996 Physica C **258** 197

One material, four models



(RE)BaCuO tape, 4 mm wide, characterized in self-field at 77 K

One material, four models



What determines the operating field range?

- The magnitude of the transport current (higher $J =$ higher E)
- The rate of variation of the magnetic field $\mathrm{d}B/\mathrm{d}t$ (current ramp in a magnet, frequency of AC excitations)

Here we consider three scenarios

1. Magnetic relaxation in an HTS slab (DC operation)
2. AC transport current in a HTS strip (AC operation)
3. AC ripples in HTS strip carrying DC current (DC+AC operation)

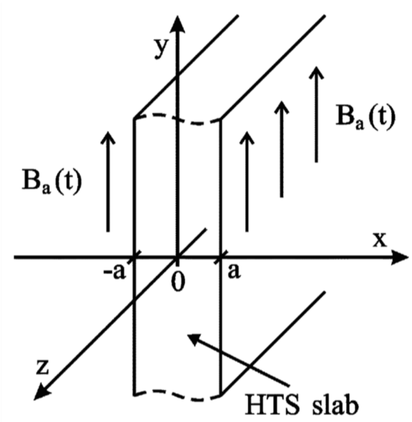
Material parameters and numerical methods

Model name	Parameter values and units
Power law model	$J_\mathrm{c} = 3.041$ MA cm^{-2}, $n = 29.44$, $E_\mathrm{c} = 1\ \mu$V cm^{-1} (not a fitting parameter)
Percolation model	$J_\mathrm{c,min} = 1.623$ MA cm^{-2}, $n = 13.37$, $k = 0.0093775$
CSM with J_c	$J_\mathrm{c} = 3.041$ MA cm^{-2}
CSM with $J_\mathrm{c,min}$	$J_\mathrm{c} = J_\mathrm{c,min} = 1.623$ MA cm^{-2}

Employed numerical methods

- 1-D finite element model for slabs (COMSOL)
  Sirois/Grilli, 2008 IEEE TAS **18** (3) 1733-1742
- 1-D integral method for thin strips (COMSOL)
  Brambilla *et al* 2008 SuST **21** (10) 105008
- 2-D self-programmed code
  Morandi/Fabbri 2015 SuST **28** (2) 024004

Case 1: slab in DC field



$$2a = 1\,\mu\text{m}$$

$$B_p = \mu_0 J_c a = 19.1\,\text{mT}$$

$$B_a(t) = \begin{cases} 0 & \text{if } t < 0 \\ B_p/2 & \text{if } t \geq 0 \end{cases}$$

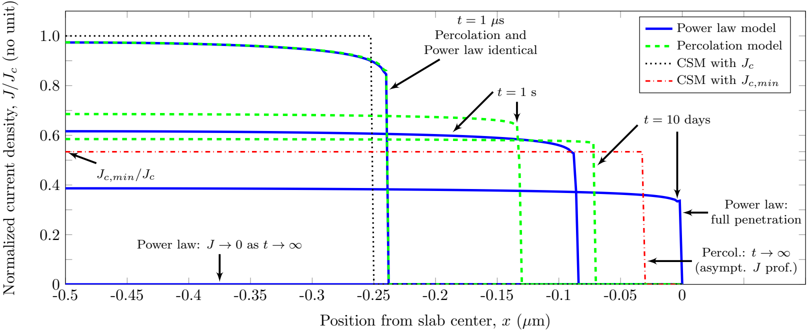
B_a turned on
“instantaneously” at $t = 0$

Numerically, the step function is approximated by a 100 ns ramp

Case 1: slab in DC field

Current density profiles between 1 µs and more than 10 days

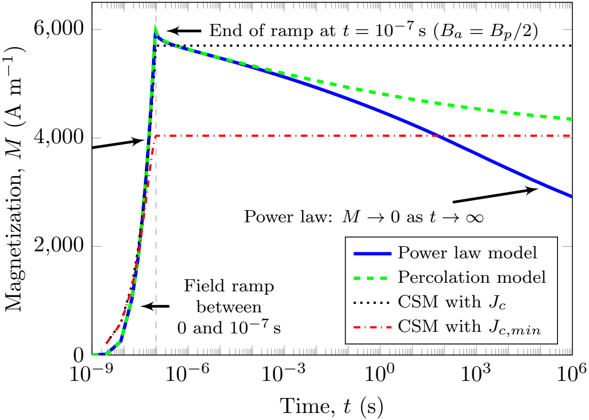
- PM relaxes more slowly than PL and converges to CSM($J_{\mathrm{c,min}}$) as $t \to \infty$
- PL fully penetrates the samples after 303 s, and converges to 0 as as $t \to \infty$



Case 1: slab in DC field

Magnetic moment vs. time

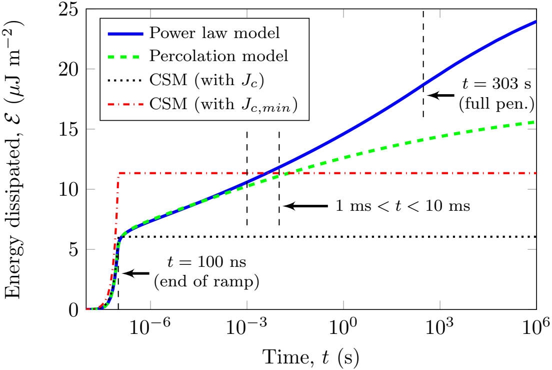
- PM converges asymptotically to CSM($J_{\mathrm{c,min}}$) as $t \to \infty$
- PL results in complete loss of magnetization as $t \to \infty$
- CSM(J_{c}) has no special significance in this case



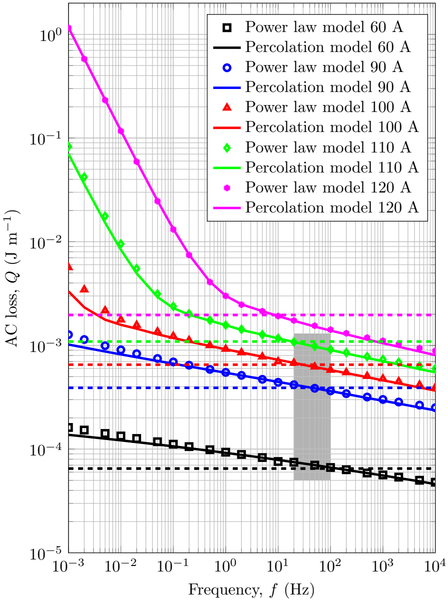
Case 1: slab in DC field

Accumulated energy loss vs. time

- CSM models do not account for relaxation losses
- Loss trends for PL and PM split after 10-20 ms (approx. time scale of AC power signals)



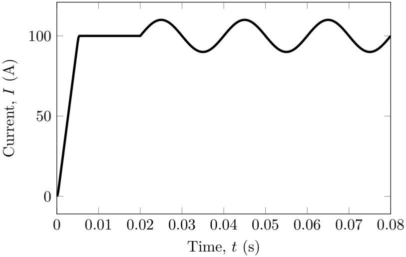
Case 2: thin strip with AC transport current

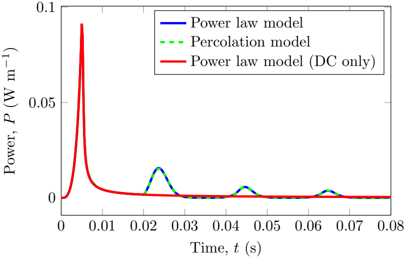


Shaded area:
frequency range where Norris's formula (dashed lines) agrees with percolation and power-law models

Case 3: thin strip with DC transport current + AC ripples

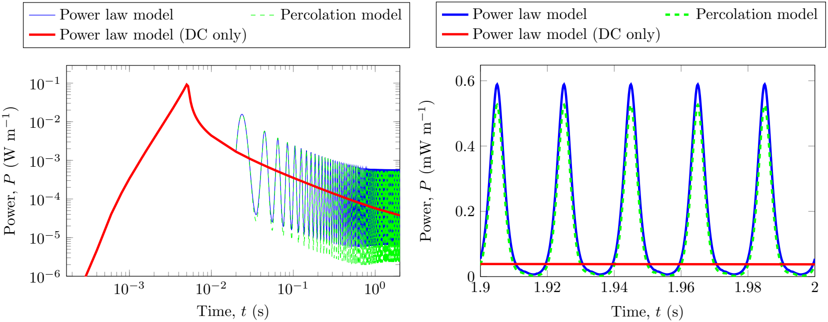
- Strip's critical current: 121.64 A
- DC current: 100 A ramped in 5 ms
- AC ripples: 10 A applied after waiting for 15 ms





Case 3: thin strip with DC transport current + AC ripples

- After 2 s, relaxation losses are still 25-30 % of the AC losses
- After 100 s, they are just 1 %
- On practical time scales (minutes to hours), only the losses associated to the AC component matter



Conclusion

- In DC cases (long time scales, very low electric field):
  - Only the percolation model can correctly predict the magnetic relaxation losses and the critical state profile
  - The CSM should be used with $J_\mathrm{c} = J_{\mathrm{c,min}}$ to give physically relevant results
- With purely AC excitations:
  - Both percolation and power law models are suitable (10-20 % difference in results, compatible with experimental accuracy)
  - The CSM predicts correct result only in a particular range of frequencies, which is coincidentally (?) that of power applications
  - Scaling laws relating the power-law and CSM as a function of frequency have been developed (Chen, Pardo), but only for particular geometries
- In the case of DC + AC ripples:
  - Same conclusions of purely AC cases apply (AC takes over)

More details in Sirois/Grilli/Morandi 2018 IEEE TAS (in press)
https://doi.org/10.1109/TASC.2018.2848219