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Computational fluid dynamics approach to determine current distribution in cylindrical bulk HTS

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1. Background

Target of this work

Problem on bulk HTS undulator

2. CFD approach

Fluid model for shielding current

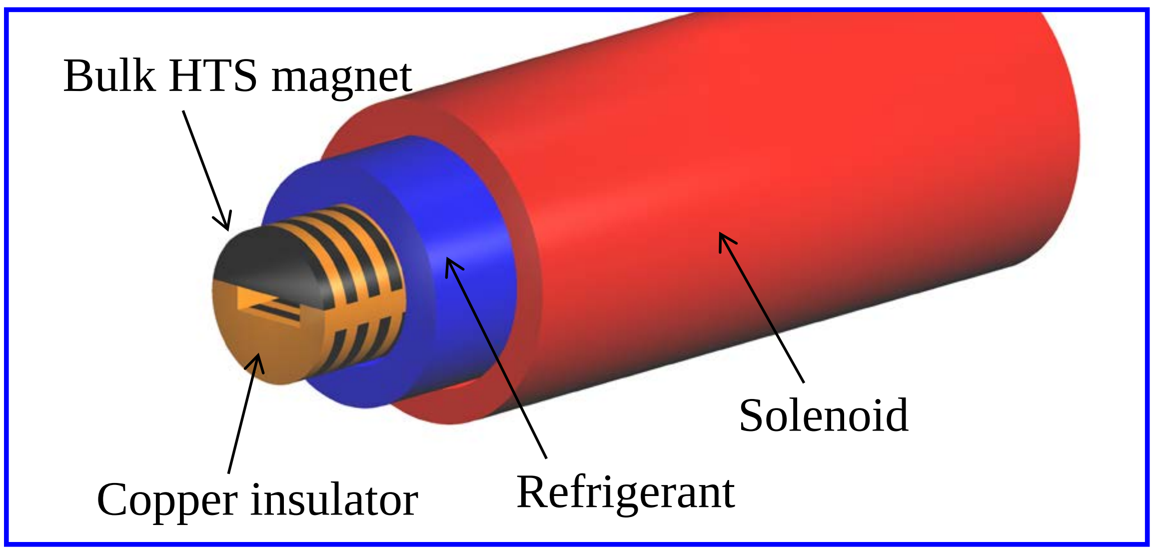
NS equation and MPS method

3. Summary

Future of the CFD approach

Target of this work

To determine SC current distribution in bulk HTS array very precisely



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **17**, 022401 (2014)

Magnetic property of a staggered-array undulator using a bulk high-temperature superconductor

Ryota Kinjo,* Kenta Mishima, Yong-Woon Choi, Mohamed Omer, Kyohei Yoshida, Hani Negm, Konstantin Torgasin, Marie Shibata, Kyohei Shimahashi, Hidekazu Imon, Kensuke Okumura, Motoharu Inukai, Heishun Zen, Toshiteru Kii, Kai Masuda, Kazunobu Nagasaki, and Hideaki Ohgaki

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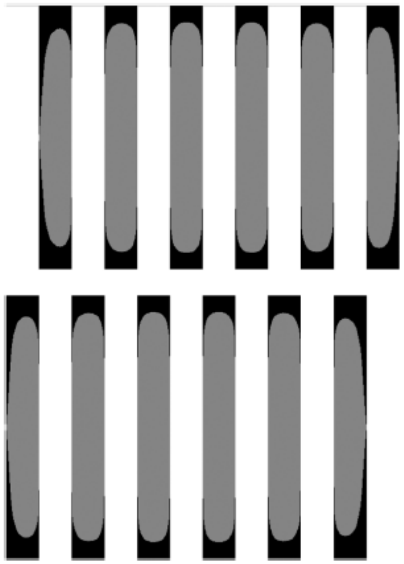


FIG. 5. Cross-sectional view of current distribution (EM method). The black area indicates where the current is flowing. The gray area indicates where the current is not flowing.

Difficulty on modelling the HTS array

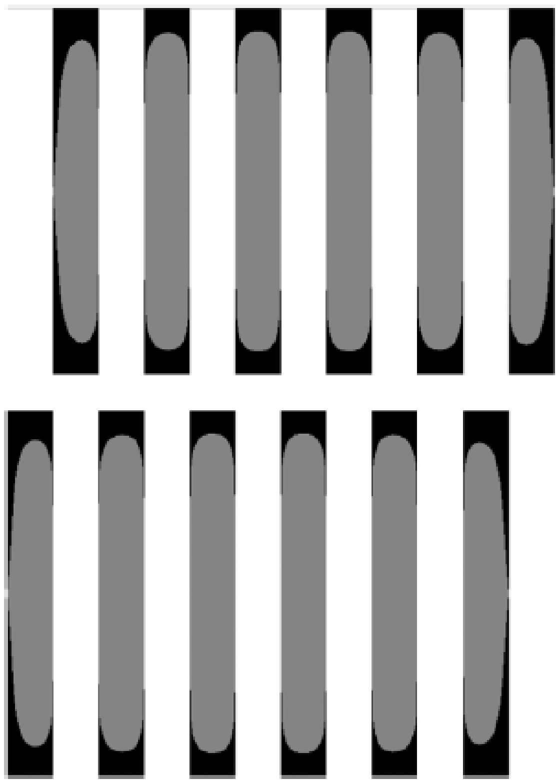


FIG. 5. Cross-sectional view of current distribution (EM method). The black area indicates where the current is flowing. The gray area indicates where the current is not flowing.

Poor symmetricity

Bulk HTSs are partially magnetized.

Current fronts are largely deflected.

Required field accuracy is few %.

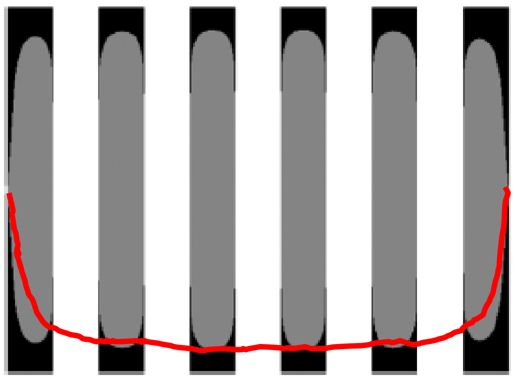
Energy minimization method is not so bad, but

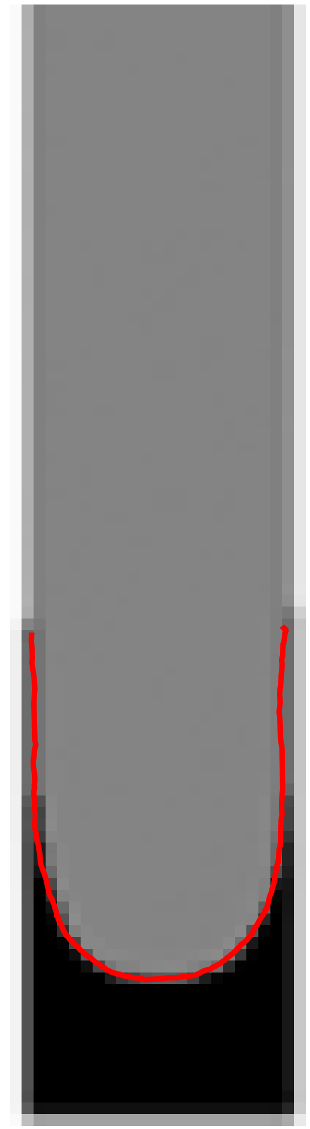
- not suitable for scaling up to real machine.
- requires "*a priori*" current path.

Undulator:

- >200 pieces of bulk (>100 period)
- Shape of the SC loop current path might be modified. (Non *a priori*)

Is there any way to calculate current/flux front?



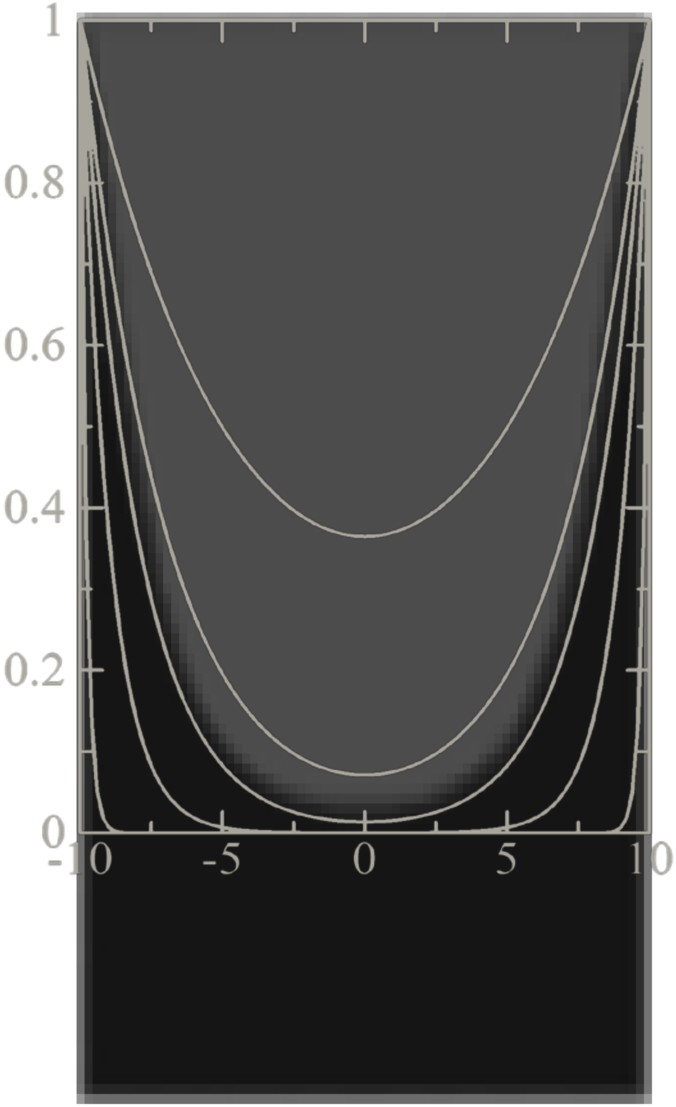


Usual solution

EM problem

- FEM
- Integral equation
- Energy Minimization
- Variational Formulation
- .
- .
- .

Is there any way to calculate flux front?

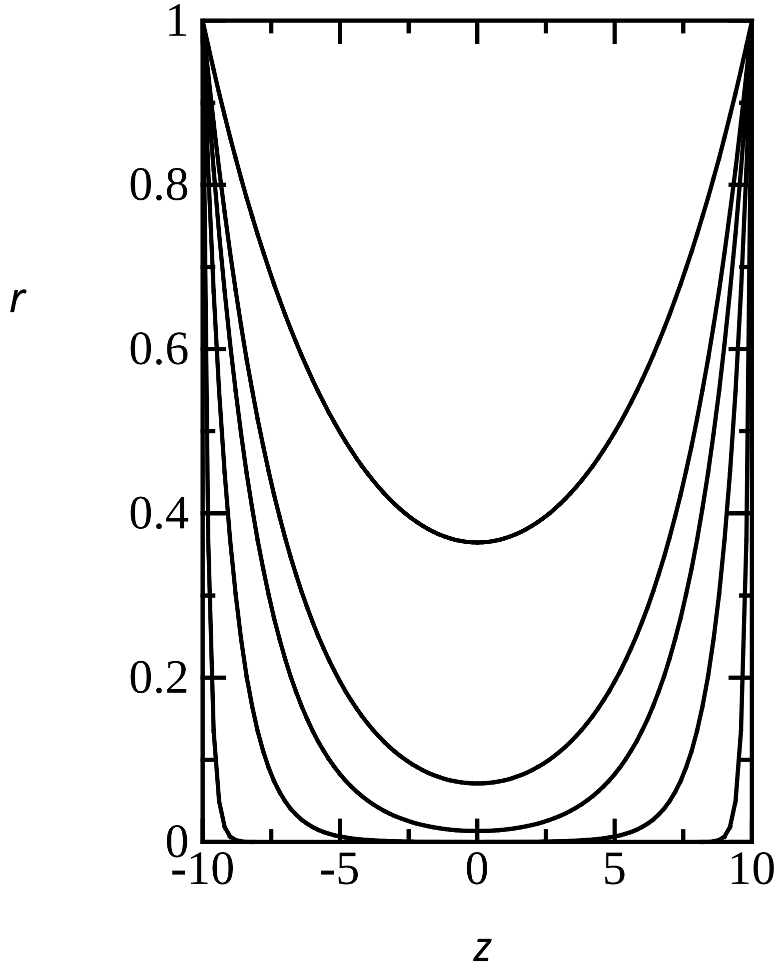


Typical current distribution
in bulk HTS thin cylinder

Simplest case:
Cylindrical column

Cross section of the interface
is similar to the famous curves.

Meniscus, Catenary, etc.



$r = \cosh(z/v)\,/\,\cosh(10/v)$

Catenary

Horizontal tension along the line is constant.

Shape of cross section is similar to normalized catenary.
(Height of both ends is normalized to unity.)

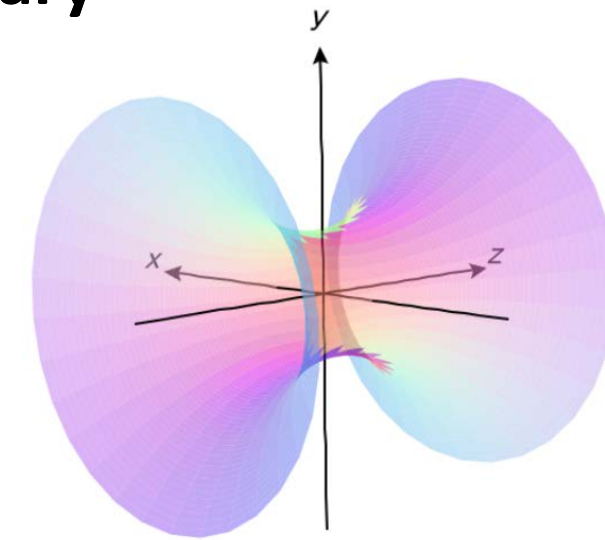
Catenoid? -Rotated catenary-

Catenoid: minimal surface

Soap film attached to twin circular rings will take the shape of a catenoid.

$$x = t\cosh(v/t)\,/\,\cosh(1/t) * \sin(u)$$
$$y = t\cosh(v/t)\,/\,\cosh(1/t) * \cos(u)$$
$$z = t$$

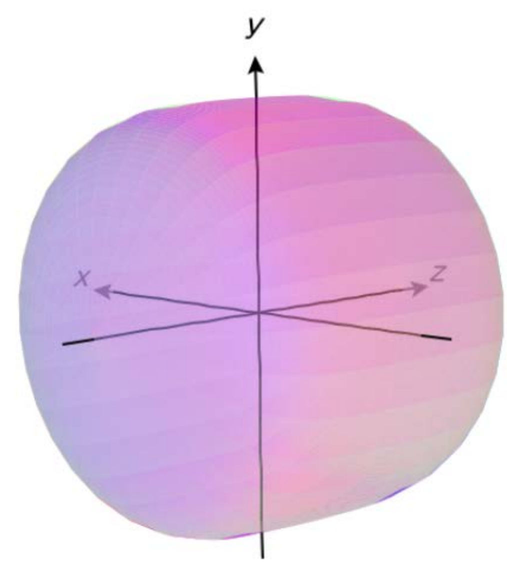
rotated about the *z*-axis



Flux front seems to take similar surface.

$$x = (1 - \cosh(v/t)\,/\,\cosh(1/t)) * \sin(u)$$
$$y = (1 - \cosh(v/t)\,/\,\cosh(1/t)) * \cos(u)$$
$$z = t$$

rotated about the *line* (z’-axis)
which is determined by two end points of normalized catenary



Is there any reason to apply catenary or catenoid like surface?

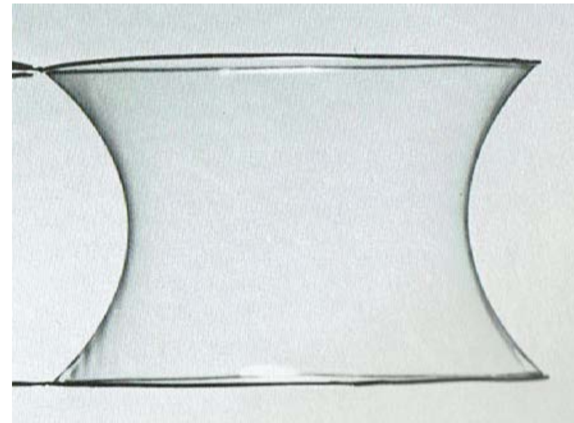
catenary

Chain between poles

Horizontal component of the tension along the line is constant.

Potential energy is minimized.

catenoid

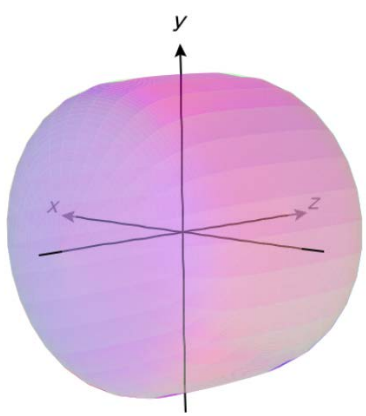


Soap film attached to twin circular rings

Surface tension is constant.

Surface free energy is minimized.

Bulk HTS



Flux front looks like liquid surface.

Tension of the balloon???

Free energy???

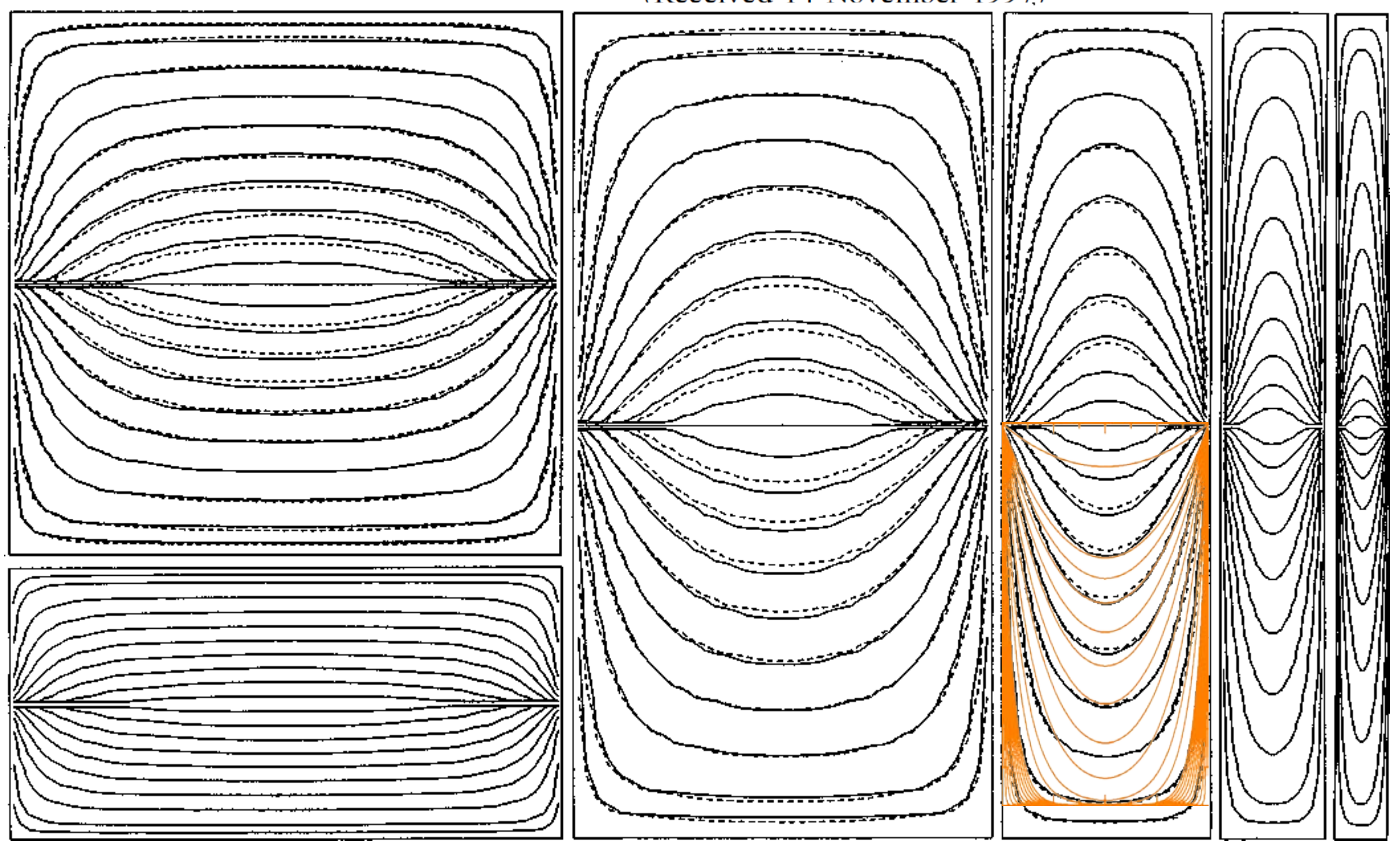
Comparison with various numerical results (1)

PHYSICAL REVIEW B VOLUME 58, NUMBER 10 1 SEPTEMBER 1998-II

Superconductor disks and cylinders in an axial magnetic field. I. Flux penetration and magnetization curves

Ernst Helmut Brandt
Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany

(Received 14 November 1997)

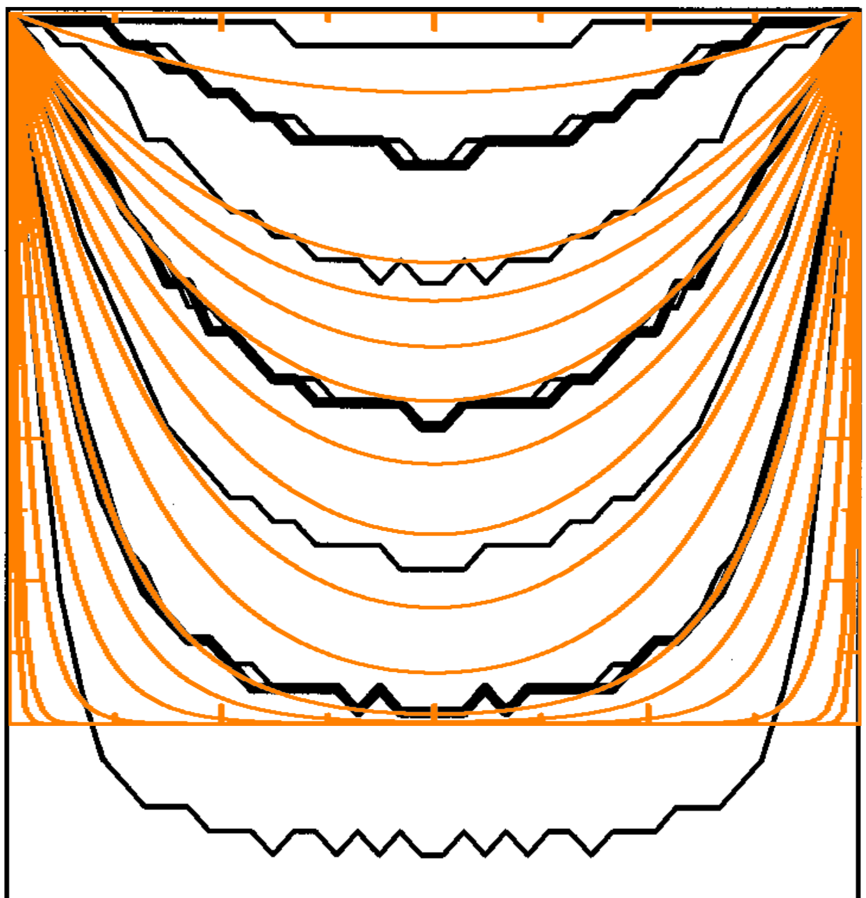


Comparison with various numerical results (2)

PHYSICAL REVIEW B, VOLUME 64, 214506

Magnetic properties of finite superconducting cylinders. I. Uniform applied field

Alvaro Sanchez[1] and Carles Navau[1,2]
[1]*Grup d'Electromagnetisme, Departament de Física, Universitat Autònoma Barcelona, 08193 Bellaterra (Barcelona), Catalonia, Spain*
[2]*Escola Universitària Salesiana de Sarrià, Rafael Batlle 7, 08017 Barcelona, Catalonia, Spain*
(Received 13 March 2001; published 1 November 2001)



Comparison with various numerical results (3)

PHYSICAL REVIEW B **67**, 104517 (2003)

Magnetic properties of arrays of superconducting strips in a perpendicular field

Enric Pardo,[1] Alvaro Sanchez,[1] and Carles Navau[1,2]
[1]*Grup d'Electromagnetisme, Departament de Física, Universitat Autònoma Barcelona, 08193 Bellaterra (Barcelona), Catalonia, Spain*
[2]*Escola Universitària Salesiana de Sarrià, Passeig Sant Joan Bosco 74, 08017 Barcelona, Catalonia, Spain*
(Received 30 July 2002; revised manuscript received 13 November 2002; published 31 March 2003)

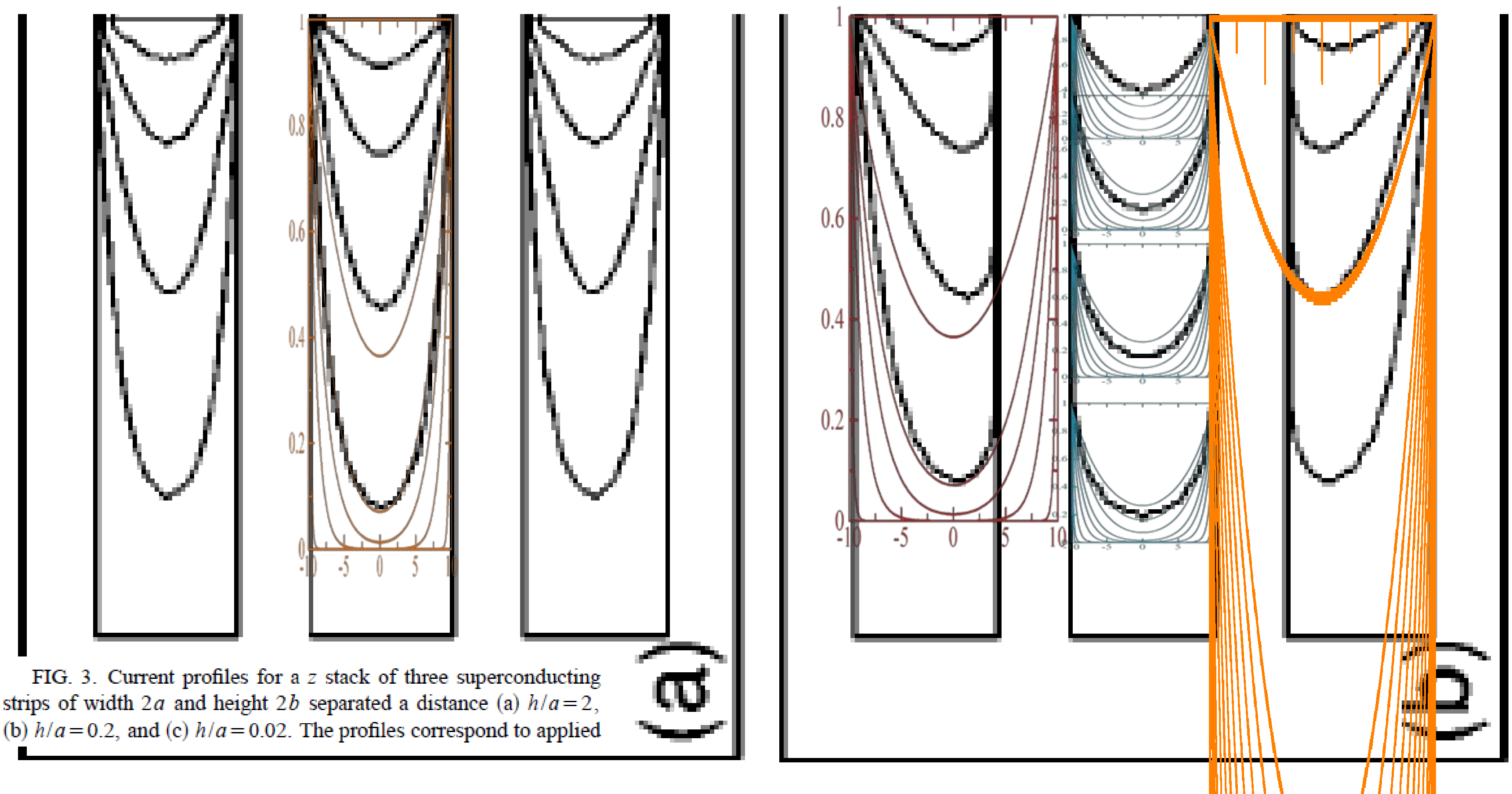


FIG. 3. Current profiles for a z stack of three superconducting strips of width $2a$ and height $2b$ separated a distance (a) $h/a=2$, (b) $h/a=0.2$, and (c) $h/a=0.02$. The profiles correspond to applied

Comparison with various numerical results (4)

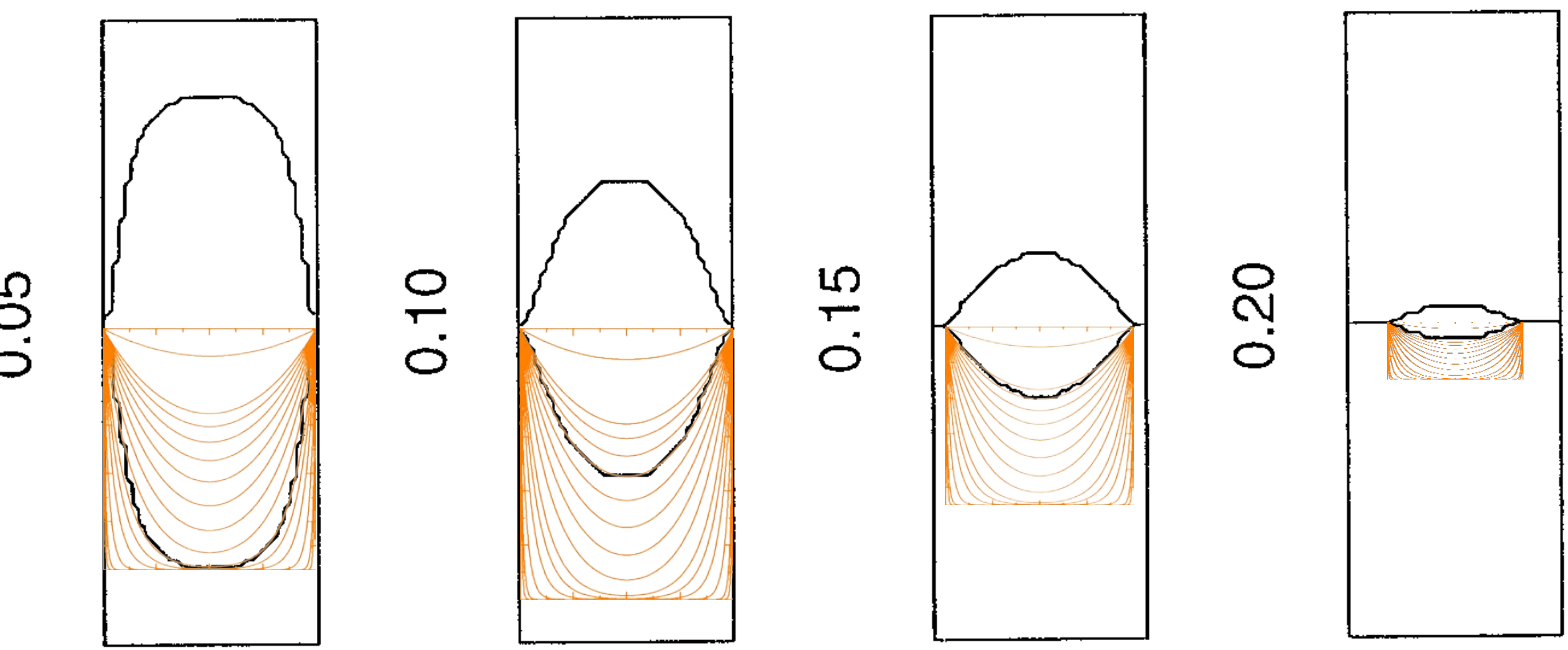
JOURNAL OF COMPUTATIONAL PHYSICS 129, 190–200 (1996)
ARTICLE NO. 0243

The Bean Model in Superconductivity: Variational Formulation and Numerical Solution

Leonid Prigozhin*

Mathematical Institute, OCIAM, University of Oxford, Oxford, United Kingdom

Received February 21, 1996



Comparison with various numerical results (5)

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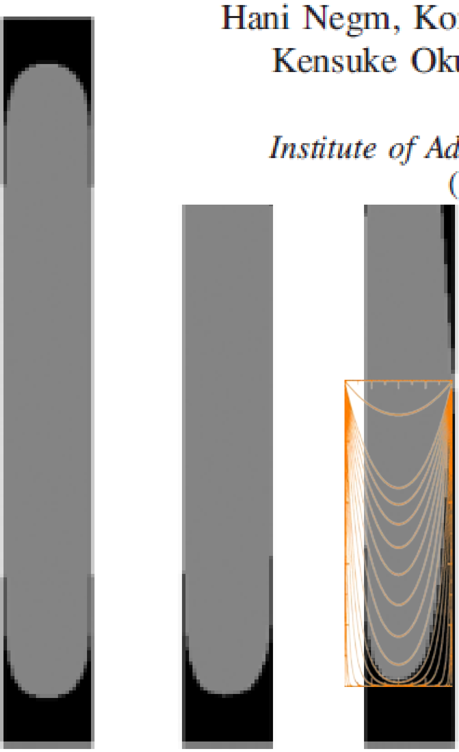

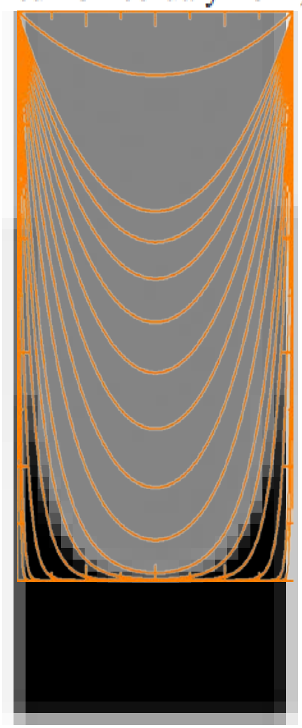

of current distribution (EM
es where the current is flowing

Comparison with various numerical results (6)

JOURNAL OF COMPUTATIONAL PHYSICS 129, 190–200 (1996)
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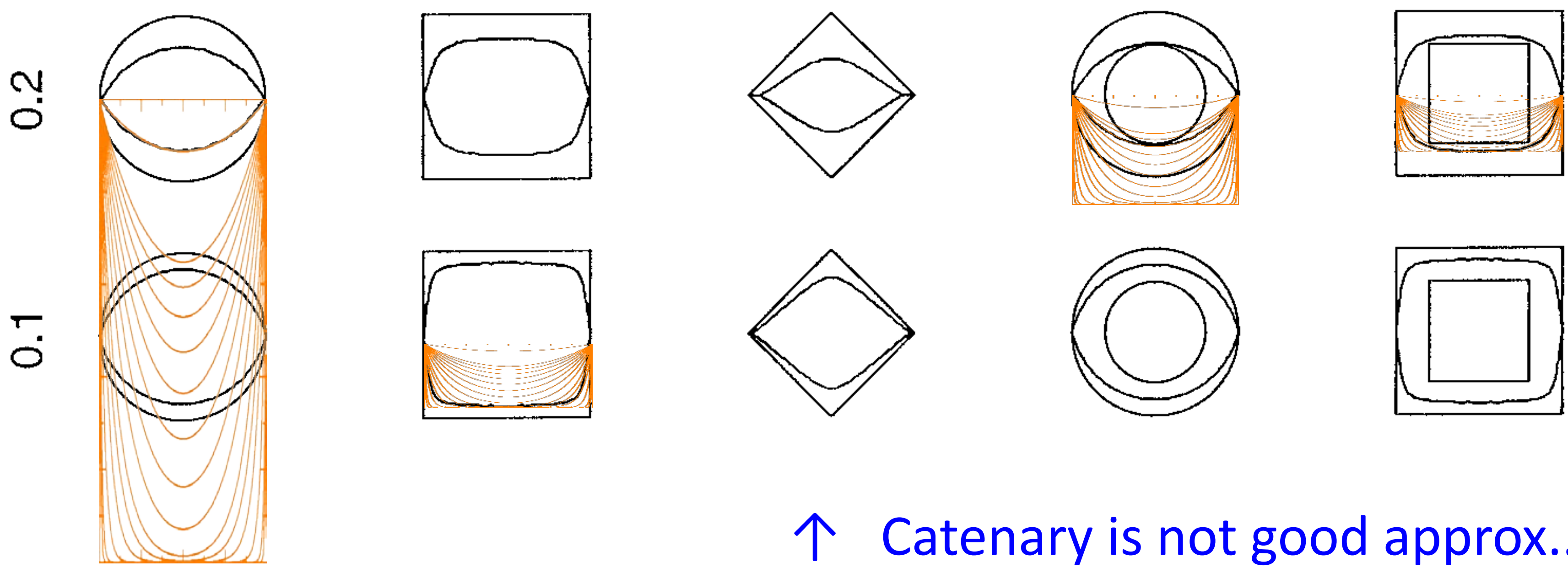

↑ Catenary is not good approx..

↓ Catenary is not good approx..

Possible free energy or force for SC current

Maupertuis' principle (principle of least action)

- "Nature is thrifty in all its actions“
- The physical trajectory that connects two endpoints is an extremum of the action integral.

For example: GL theory

free energy density

f(s) =Bs^2+Ds^4+Fs^6+… s: order parameter

free energy

$$F_s = F_n(0) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}|(-i\hbar\nabla + 2e\boldsymbol{A})\Psi|^2 + \frac{1}{2\mu_0}(rot\boldsymbol{A})^2$$

Kinetic energy Magnetic energy

$$+\mathrm{U}_{\text{gravitational potential}} + \mathrm{U}_{\text{surface/internace}} + \cdots$$

Usually “Magnetic energy” of the HTS system is minimized.

Can not escape from Maxwell equation even for CFD modelling

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   - Target of this work
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   - Fluid model for shielding current
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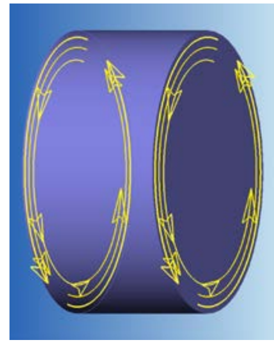
   - Future of the CFD approach

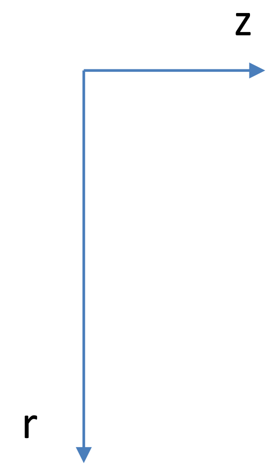
CFD approach

If we use the analogy of water in a cup…

H_2O molecule



Supercurrent loop



Current in a HTS

- Moving Cooper pairs which have barycentric velocity
- Speed is very slow & Cooper pairs is incompressible

r-θ plane　$\mathrm{rot}\, v = 0$ irrotational flow　$v = \mathrm{grad}\, \varphi$ potential flow

Incompressible irrotational flow.

r-z plane　No current in the steady state.

Assumption: *Incompressible and irrotational in r-z*

Supercurrent loop → Ring particle　3D problem → 2D problem

2D problem

z

Navier–Stokes equations

D: Density, P: Pressure, u: Velocity in *r-z plane*

$$\rho \frac{D_{\boldsymbol{u}}}{D_t} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$ External force

μ : Viscosity coefficient

Assumption:

Incompressible and irrotational in r-z → $\mu = 0$

External force → This term is related to magnetic energy.

Density: Density of electron pairs in *r-θ plane*

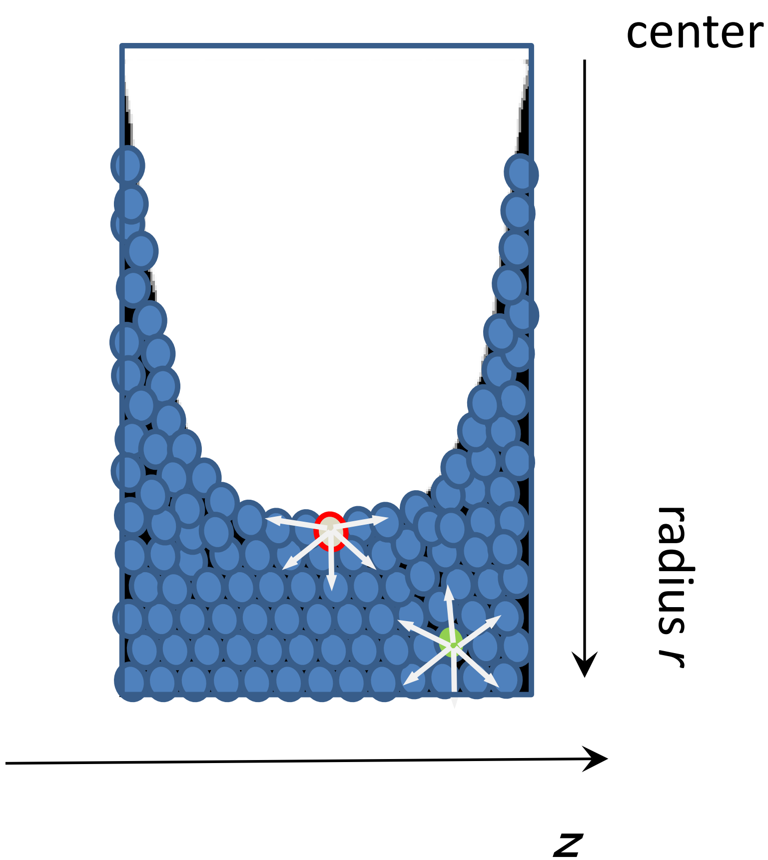
Pressure: Origin to move ring particles

Velocity: Velocity of ring particle ~~Velocity of cooper pair in *r-θ plane*~~

In this work, use of 2D MPS (Moving Particle Simulation) with ring particle as point particle is proposed.

Moving Particle Simulation

Koshizuka S, Tamako H, Oka Y,
"A particle method for incompressible viscous flow with fluid fragmentation."
Comput Fluid Dynamics J 4:29-46 (1995)



Navier–Stokes equation

$$\rho \frac{D_{\boldsymbol{u}}}{D_t} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$

Equation of continuity

$$\frac{D_\rho}{D_t} + \rho \nabla \cdot \boldsymbol{u} = 0$$

Procedure

Initial particles
Calculate force
Move Particles
Calculate density
Calculate force
Move Particles
.
.
Finally all particles stop

Incompressible

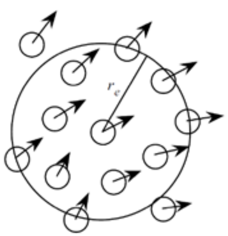
$$\frac{D_\rho}{D_t} = 0$$

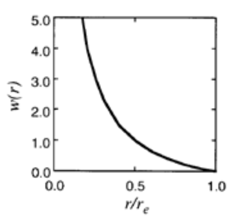
$$\rho \nabla \cdot \boldsymbol{u} = 0$$

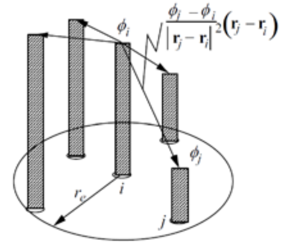
One method for NS equations using Particle

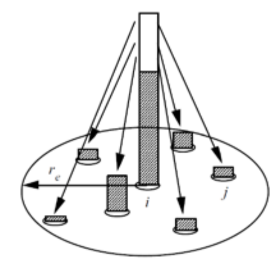
Feature of MPS [Koshizuka]
  Mesh less, Suitable for a surface/interface changing problem
    Simple discretization: weighted average of difference

Treatment of particles
    Movement in “effective range of interaction” is calculated.
      Weighting function is used





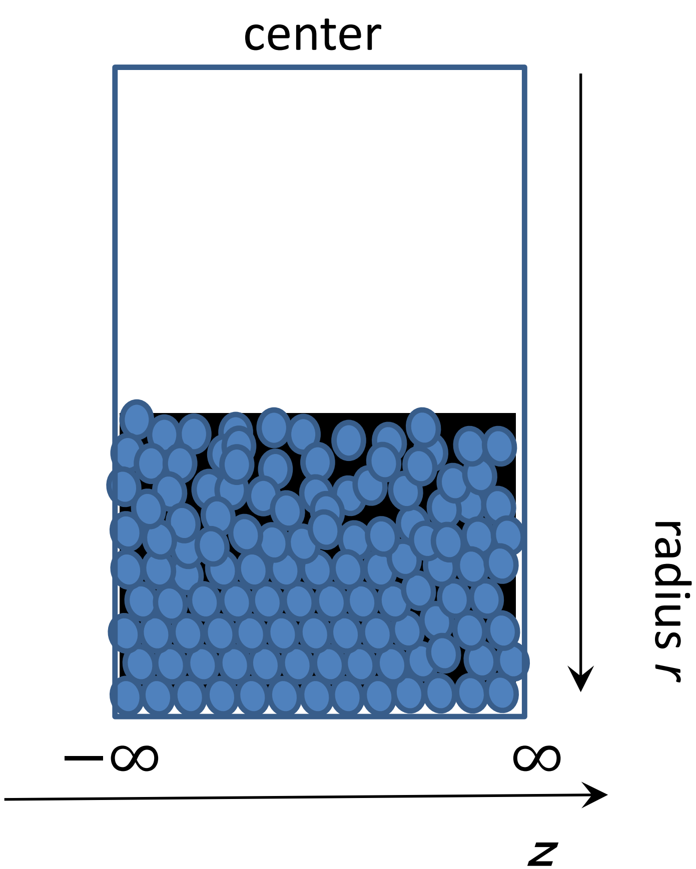




It is similar to **van der Waals** force among particles.
    Various type of the weighting functions are proposed
    and under development.

Moving Particle Simulation (2)

Trivial solution for infinite long cylinder (1D problem)



Magnetic field is changed

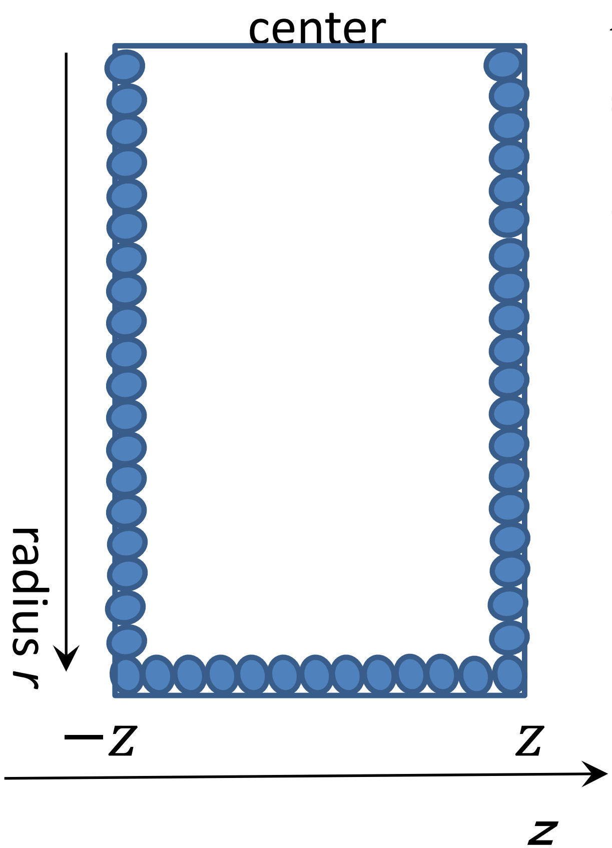
Particle appears at the bottom (thin layer in 2D)

Particles are arranged from bottom to top.

For finite system

How can we treat particles?

Interface between current and wall



Free energy at the surface of the HTS

$$F_s = F_n(0) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}|(-i\hbar\nabla + 2e\boldsymbol{A})\Psi|^2 + \frac{1}{2\mu_0}(rot\boldsymbol{A})^2 + \mathrm{U}_{\text{gravitational potential}} + \mathrm{U}_{\text{surface/internace}} + \cdots$$

Field is changed → Order parameter changes
= Shielding loop current appears

Initial condition:
Particles are placed at the interface and pinned.

However,
for an HTS array, field on HTS surface
should be calculated in each step…..

Force among loop currents (I)

~ Particle and force model like treatment ~

Free energy

$$F_s = F_n(0) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{2m^*}|(-i\hbar\nabla + 2e\boldsymbol{A})\Psi|^2 + \frac{1}{2\mu_0}(rot\boldsymbol{A})^2$$
$$+\mathrm{U}_{\text{gravitational potential}} + \mathrm{U}_{\text{surface/internace}} + \cdots$$

$$U = \sum_i \frac{1}{2} L_{ii} {I_i}^2 + \sum_{i,j} \frac{1}{2} M_{ij} I_i I_j \qquad M_{12} = \frac{\mu}{4\pi} \oint\!\!\oint_{c1,c2} \frac{\cos\theta \, ds_1 ds_2}{x} \qquad F = \frac{dU}{dx}$$

Magnetic energy

Neumann's formula

x : distance between loops

$$\rho \frac{D_{\boldsymbol{u}}}{D_t} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$

If force is calculated separately and added one by one,
same result with the other solutions will be obtained.
But, computing cost is high!

Moving Particle Simulation (5)

Force among loop currents (II)

$$\rho \frac{D_{\boldsymbol{u}}}{D_t} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$

External force term is separated into

① Van der Waals force like treatment with neighbors

$$U = \sum_{i,j} \frac{1}{2} M_{ij} I_i I_j \qquad M_{12} = \frac{\mu}{4\pi} \oiint_{c1,c2} \frac{\cos\theta \, ds_1 ds_2}{x} \qquad F = \frac{dU}{dx}$$

② Hoop force for single loop,

$$U = \sum_{i} \frac{1}{2} L_{ii} {I_i}^2 \qquad F = \frac{dU}{dx}$$

Then the forces can be included

in **weighting function** of the MPS method.

Treatment for self inductance

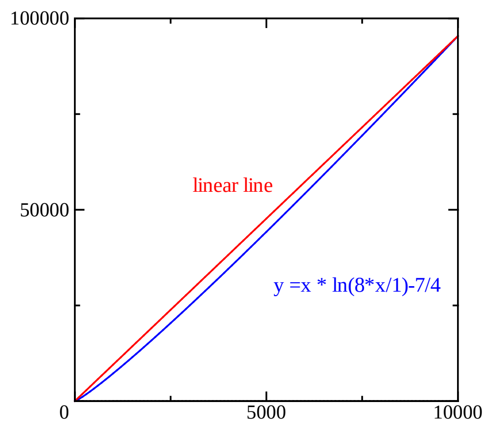
Hoop force and Self inductance

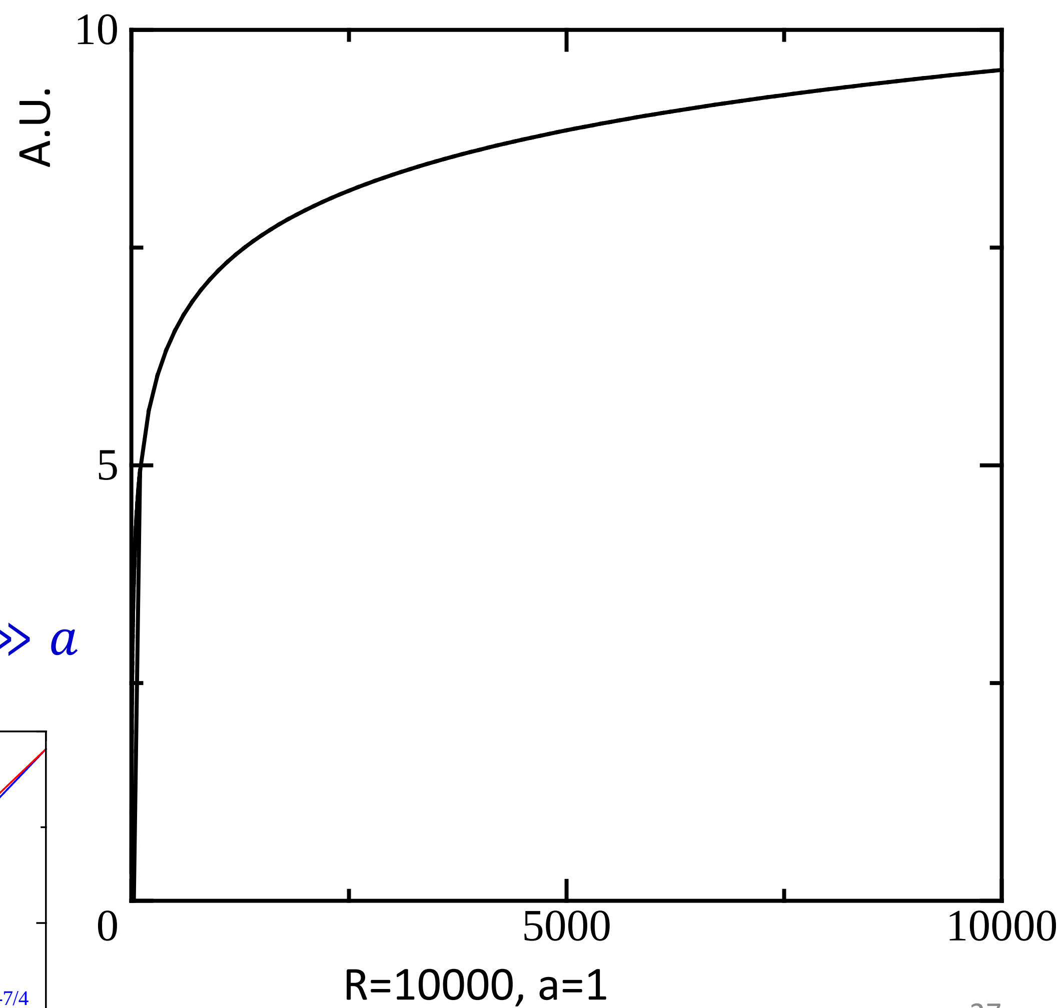
$$F = \frac{dU}{dx}$$

$$U = \sum_i \frac{1}{2} L_i {I_i}^2$$

$$L_i \cong \mu_0 R \left(ln \frac{8R}{a} + \frac{l_i}{2} \right) \quad \text{for} \quad R \gg a$$

This term is almost liner but small non-linearity is existing.





y = ln(8*x/1)-7/4

Treatment for mutual inductance

$$U = \sum_{i,j} \frac{1}{2} M_{ij} I_i I_j \qquad M_{12} = \frac{\mu}{4\pi} \oiint_{c1,c2} \frac{\cos\theta \, ds_1 ds_2}{x} \qquad F = \frac{dU}{dx}$$

here $M_{12} = k\sqrt{L_1}\sqrt{L_2}$, k: coupling coefficient $0 \leq \mathrm{k} \leq 1$

Mutual inductance for horizontally separated rings has similar tendency for self inductance except for coupling coefficient.

Coupling coefficient is small for far rings, and large for near rings.

However, effective range is larger than **van der Waals** force among typical fluid particles.

The weighting function for the HTS modelling is under development....

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Summary

CFD approach for solving a optimal location problem for supercurrent is introduced.

“Free energy” for CFD approach and the other typical approach were compared and checked the equivalence.

Self and mutual inductance model can be naturally treated in weighting function in MPS method.

Future of this model and treatment

Find the good weighting function and the others
for solving problem
Extended to 3D, array, and
detailed comparison to the other solvers.

Multi physics (Coupled analysis)
Motion of flux (flux creep, flux jump)
Thermal, Extra magnetic field,

Property of material Jc-B-θ, hysteresis.

Treatment Viscosity term and local vortex

Marangoni convection, expression of pining force etc.

Obrigado pela sua atenção!

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