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« Calculation of the local current density in HTS coils using a volume integral formulation »

HTS2018 MODELLING

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Context

HTS magnets made of insulated REBCO tapes

- **<u>Design stage</u>** : how to estimate properly the critical current of the coil ?
- **<u>Test stage</u>** : how to efficiently protect it when submitted to a current ramp ?

Current ramp ⇒ Transient behaviour observed on the voltage signal

An early detection can be compromised because of the transient voltage whose order of magnitude is the same as the one of an early dissipative voltage

Problem definition

Characteristics :

Formulation requirements

- Physics : Electromagnetic (at constant temperature) → Maxwell equations
- HTS specificities : nonlinear E(J) and anisotropic J_c → Nonlinear algorithm
- Transient simulation over long duration (several minutes of current ramp) → Stability
- Two dynamics : → Time discretization : adaptive time stepping methods
  - Current distribution variations: slow
  - Voltage variations: slow / fast

→ Focus on the impact of the dynamique current distribution on the critical current and the transient voltage induced when ramping

Outline

1. Formulation presentation

2. Estimation of critical current in HTS coils

3. Evaluation of « nominal » transient voltage during coil current ramping

Volume Integral formulation based on the generalization of the Partial Element Equivalent Circuit (PEEC) method

No mesh in inactive regions (air)

Conservation of current density strongly ensured

Equivalent circuit generated easily from any FEM mesh tool

External circuit coupling

Full matrices

Axisymetric conditions to be included in the integral formulations

Model implemented in an internal simulation platform (Mipse)

1. Formulation

Integral equation (1)

- Maxwell Equation's **+** constitutive law (conducting region) : $\rho(\boldsymbol{J})\boldsymbol{J} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \boldsymbol{V}$

- Biot-Savart law : $\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_{\Omega_c} \frac{\boldsymbol{J}}{r} d\Omega_c + \boldsymbol{A_0}$

- Integral formulation :

$$\rho(\boldsymbol{J})\boldsymbol{J} + \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_{\Omega_c} \frac{\boldsymbol{J}}{r} d\Omega_c = -\frac{\partial \boldsymbol{A_0}}{\partial t} - \nabla V$$

$$\boxed{\begin{cases} \nabla \cdot \boldsymbol{J} = 0 & on\ \Omega_c \\ \boldsymbol{J} \cdot \boldsymbol{n} = 0 & on\ the\ boundary \end{cases}}$$

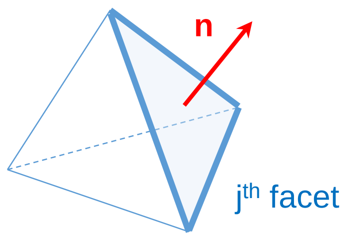
1. Formulation

Integral equation (2)

- Interpolation :

  (Whitney facet elements[1])

$$J = \sum_j w_j I_j$$



- Assembly :

$$\left([R] + \frac{\partial}{\partial t}[L]\right)[I] = [U]$$

$$R_{ij} = \int_{\Omega_c} w_i \cdot \rho(J) \cdot w_j \, d\Omega_c$$

Resistivity matrix
(Sparse)

[1] A. Bossavit, « Whitney forms: a class of finite elements for three-dimensional computations in electromagnetism », *IEE Proc. Phys. Sci. Meas. Instrum. Manag. Educ. Rev.*, vol. 135, nº 8, p. 493, 1988

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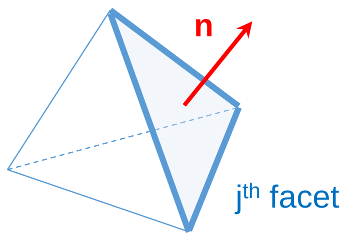

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Resistivity matrix
(Sparse)

$$L_{ij} = \frac{\mu_0}{4\pi} \int_{\Omega_c} w_i \int_{\Omega_c} \frac{w_j}{r} \, d\Omega_c$$

Inductance matrix
(Full)

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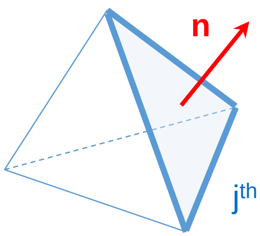


- Assembly :

$$\left([\boldsymbol{R}] + \frac{\partial}{\partial t}[\boldsymbol{L}]\right)[\boldsymbol{I}] = [\boldsymbol{U}]$$

$$R_{ij} = \int_{\Omega_c} w_i \cdot \rho(J) \cdot w_j \, d\Omega_c$$

Resistivity matrix
(Sparse)

$$L_{ij} = \frac{\mu_0}{4\pi} \int_{\Omega_c} w_i \int_{\Omega_c} \frac{w_j}{r} \, d\Omega_c$$

Inductance matrix
(Full)

$$U_i = -\int_{\Omega_c} w_i \nabla V \, d\Omega_c - \frac{\partial}{\partial t} \int_{\Omega_c} w_i \, A_0 d\Omega_c$$

External sources vector

[1] A. Bossavit, « Whitney forms: a class of finite elements for three-dimensional computations in electromagnetism », *IEE Proc. Phys. Sci. Meas. Instrum. Manag. Educ. Rev.*, vol. 135, nº 8, p. 493, 1988

→ Generation of an equivalent electrical circuit

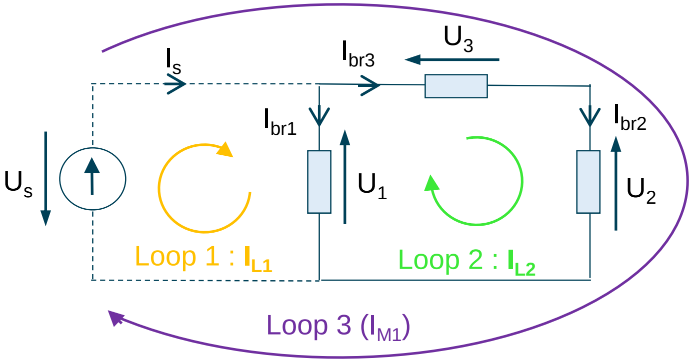
- Kirchhoff's circuit laws :

$$[M][U] = [0]$$
$$[M]^t[I_L] = [I_B]$$

Incident matrix : $[M] = [\underbrace{M_{RL}}\ \underbrace{M_s}]$

Circuit generated based on the initial geometry

External branches (sources)



Determination of independant loops[2]

- Equation to be solved :

$$\begin{pmatrix} [M_{RL}]\left([R] + \frac{\partial}{\partial t}[L]\right)[M_{RL}]^t & [M_S] \\ [M_S]^t & [0] \end{pmatrix} \begin{pmatrix} [I_L] \\ [U_S] \end{pmatrix} = \begin{pmatrix} -[M_{RL}][U_{ext}] \\ [I_S] \end{pmatrix}$$

[2] T.-S. Nguyen, J.-M. Guichon, O. Chadebec, G. Meunier, and B. Vincent, "An independent loops search algorithm for solving inductive PEEC large problems," Progr. Electromagn. Res., vol. 23, pp. 53–63, Jan. 2012

Nonlinear behaviour → Newton-Raphson algorithm

$$\mathbf{J}\cdot\Delta\mathbf{X} = -\mathcal{R}$$

$$\mathcal{R} = \begin{pmatrix} [M_{RL}]([R] + \frac{1}{\Delta t}[L])[M_{RL}]^t & [M_S] \\ [M_S]^t & [0] \end{pmatrix} \begin{pmatrix} [I_M]^k \\ [U_S]^k \end{pmatrix} - \begin{pmatrix} [M_{RL}](\frac{1}{\Delta t}[L])[M_{RL}]^t & [0] \\ [0] & [Id] \end{pmatrix} \begin{pmatrix} [I_M]^n \\ [I_s]^{n+1} \end{pmatrix}$$

$$\mathrm{J} = \begin{pmatrix} [M_{RL}]\left([R] + \frac{1}{\Delta t}[L]\right)[M_{RL}]^t + [M_{RL}]\left(\frac{\partial [R]}{\partial [I_M]}\right)[M_{RL}]^t[I_M]^k & [M_S] \\ [M_S]^t & [0] \end{pmatrix}$$

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$$\mathrm{J} = \begin{pmatrix} [\boldsymbol{M_{RL}}]\left([\boldsymbol{R}] + \frac{1}{\Delta t}[\boldsymbol{L}]\right)[\boldsymbol{M_{RL}}]^t + [\boldsymbol{M_{RL}}]\left(\frac{\partial[\boldsymbol{R}]}{\partial[\boldsymbol{I_M}]}\right)[\boldsymbol{M_{RL}}]^t[\boldsymbol{I_M}]^k & [\boldsymbol{M_S}] \\ [\boldsymbol{M_S}]^t & [0] \end{pmatrix}$$

$$R_{ij} = \int_{\Omega_c} w_i \cdot \boldsymbol{\rho(J)} \cdot w_j \, d\Omega_c \qquad \left(\frac{\partial R}{\partial I_M}\right)_{ij} = \int_{\Omega_c} w_i \cdot \frac{\boldsymbol{\partial\rho(J)}}{\partial J} \cdot w_j \, d\Omega_c$$

→ **Power Law** & its first derivative

1. Formulation

Axisymmetric Green Kernel

Axisymetric geometry using Volume Integral Method : how ?

Axisymetric geometry using Volume Integral Method : how ?

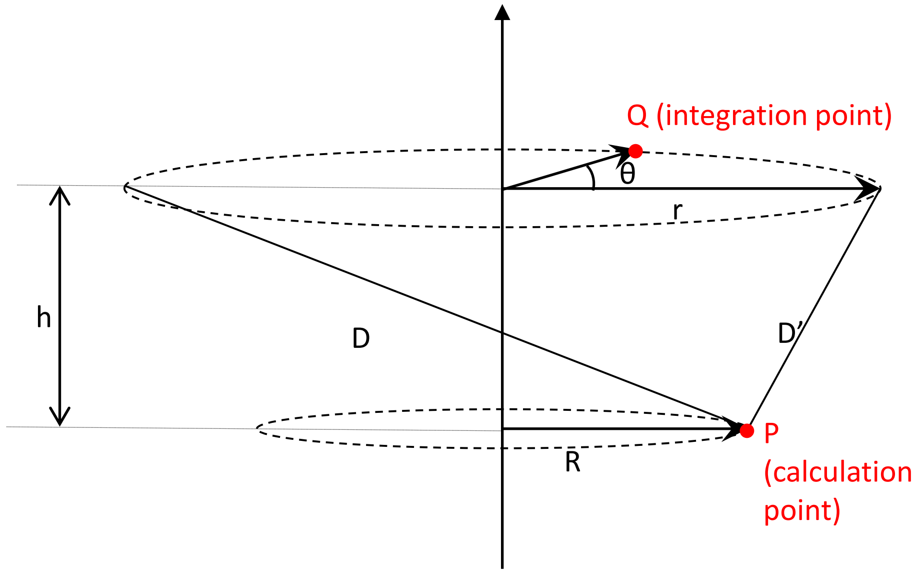
By integrating the 3D Green Kernel[3] !

$$G_{3D} = \frac{1}{4\pi\, PQ}$$

$$G_{2D\,axi} = \int_0^{2\pi} G_{3D} r\cos\theta \, d\theta$$

...

$$G_{2D\,axi} = \frac{D}{4\pi\, R}\left((2-k^2)J_1(k) - 2J_2(k)\right)$$



With :

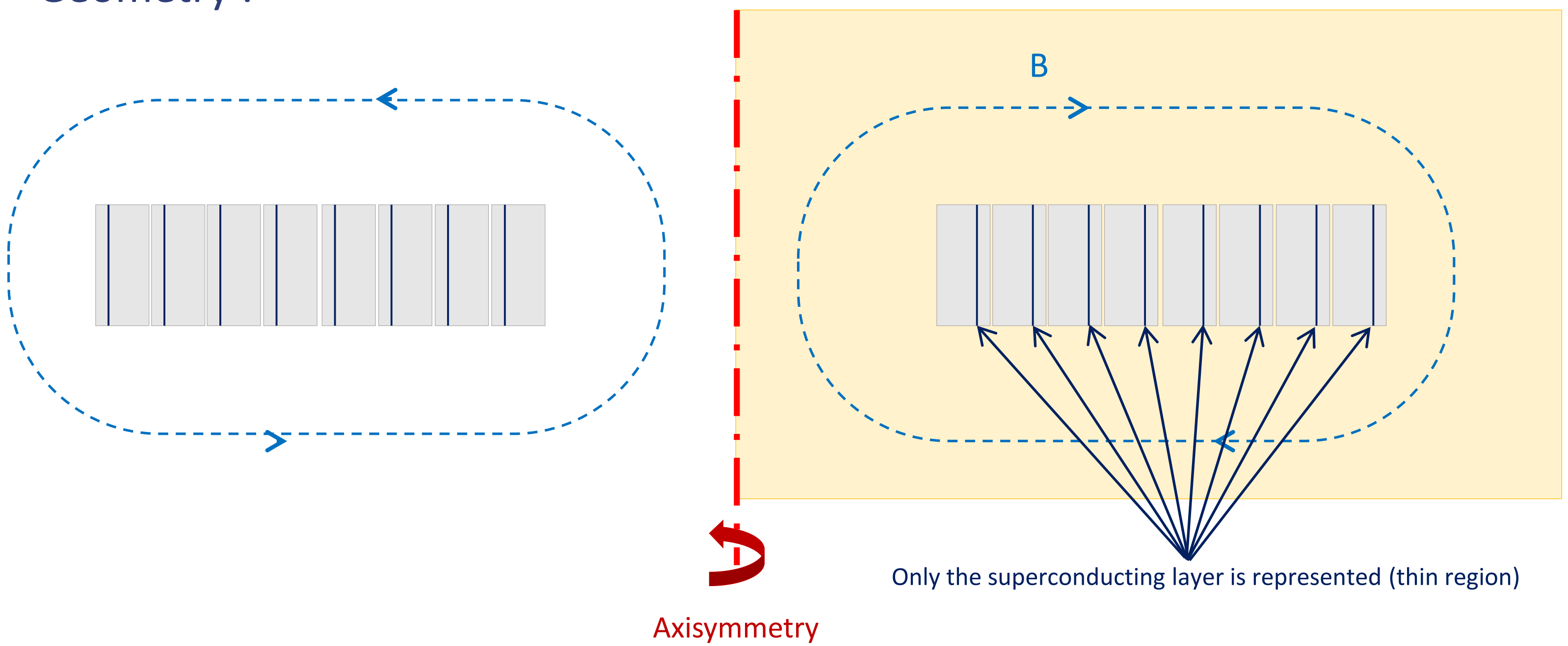
- J_1 : complete elliptic integrals of first order
- J_2 : complete elliptic integrals of second order
- $k = 4rR/D^2$

[3] L. J. Gray, M. Garzon, V. Mantič, et E. Graciani, « Galerkin boundary integral analysis for the axisymmetric Laplace equation », Int. J. Numer. Methods Eng., vol. 66, no 13, p. 2014-2034, june 2006

1. Formulation

Model specificities

- Geometry :



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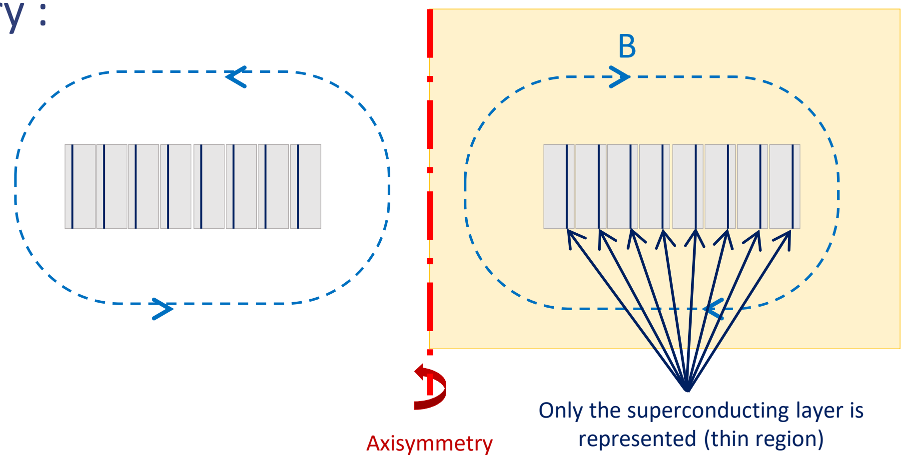
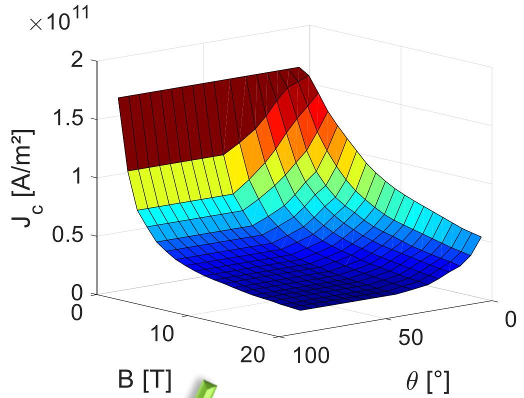


- Anisotropy of J_c :

Short sample characterization map[4]



Newton-Raphson convergence → *Current distribution* → Induction Map → **Dynamic J_c**

[4] T. Benkel et al., « REBCO Performance at High Field With Low Incident Angle and Preliminary Tests for a 10-T Insert », IEEE Trans. Appl. Supercond., vol. 26, no 3, p. 1-5, avr. 2016

2. HTS coil : I_c estimation

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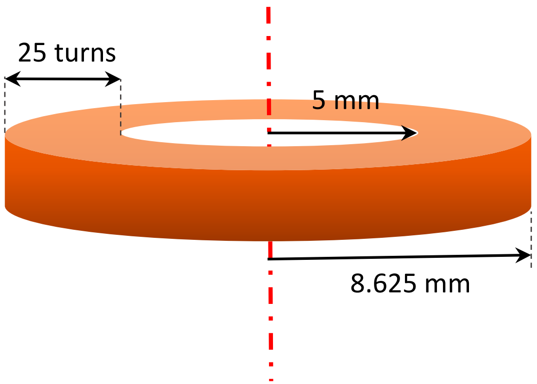
Test case description

Coil Description :

- Single pancake
- 12 mm width REBCO tape
- 25 turns
- Inner radius : 5 mm
- Outer radius : 8.625 mm



Properties :

- n value (Power Law) : 25
- J_c anisotropy → from measurements

Silver	5 µm
Copper	30 µm
REBCO	2 µm
Hastelloy	60 µm
Buffer	2 µm
Insulator	40 µm

Thickness of each layer of the tape

Discretization :

- Each turn is divided into 100 elements → 2500 elements

2. HTS coil : I_c estimation

Static / Transient study

→ Magnetostatic study : hypothesis of homogeneous current distribution

- Estimation using an average value of B along the turn width
- Estimation using an integration of J_c over the tape width

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Static / Transient study

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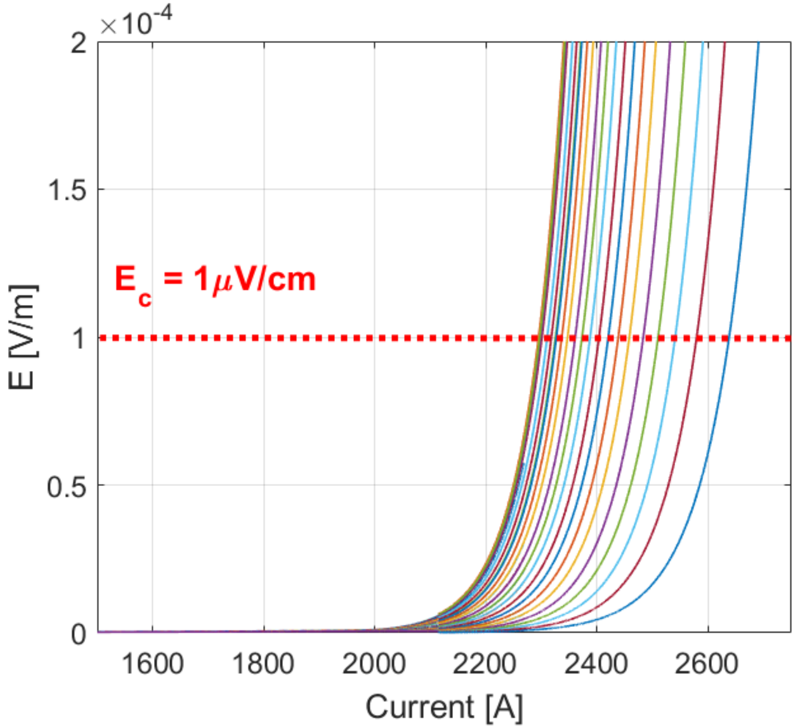
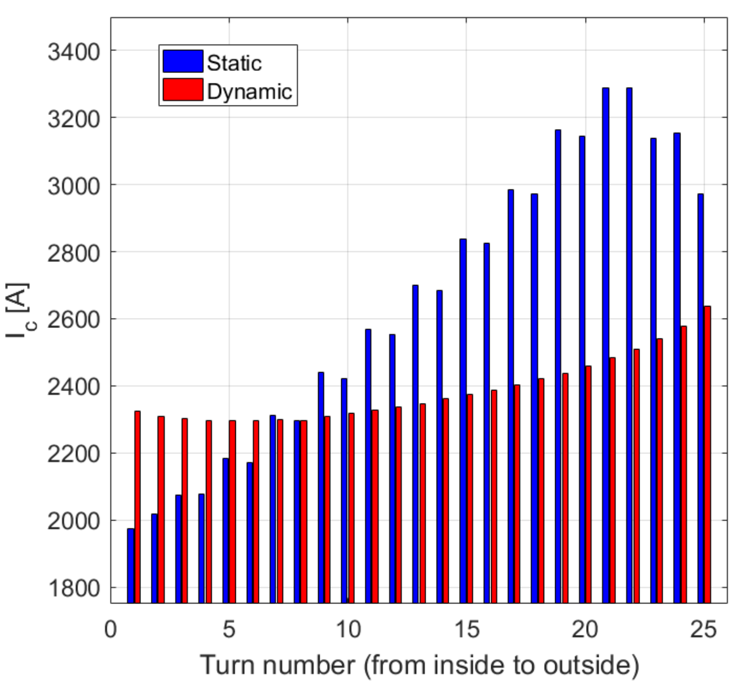
→ Dynamic study : inhomogeneous current distribution and anisotropic J_c





3. HTS coil : transient voltage

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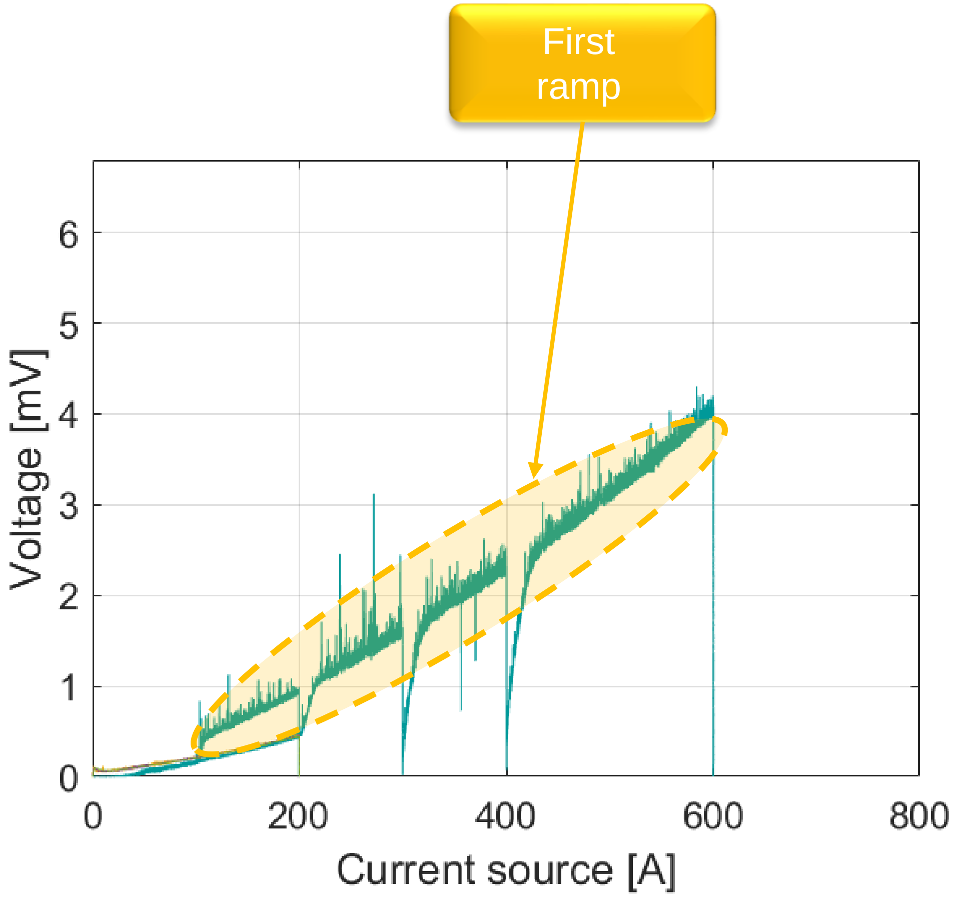
Experimental results & Motivations

Voltage measurements of a double pancake during a current ramp



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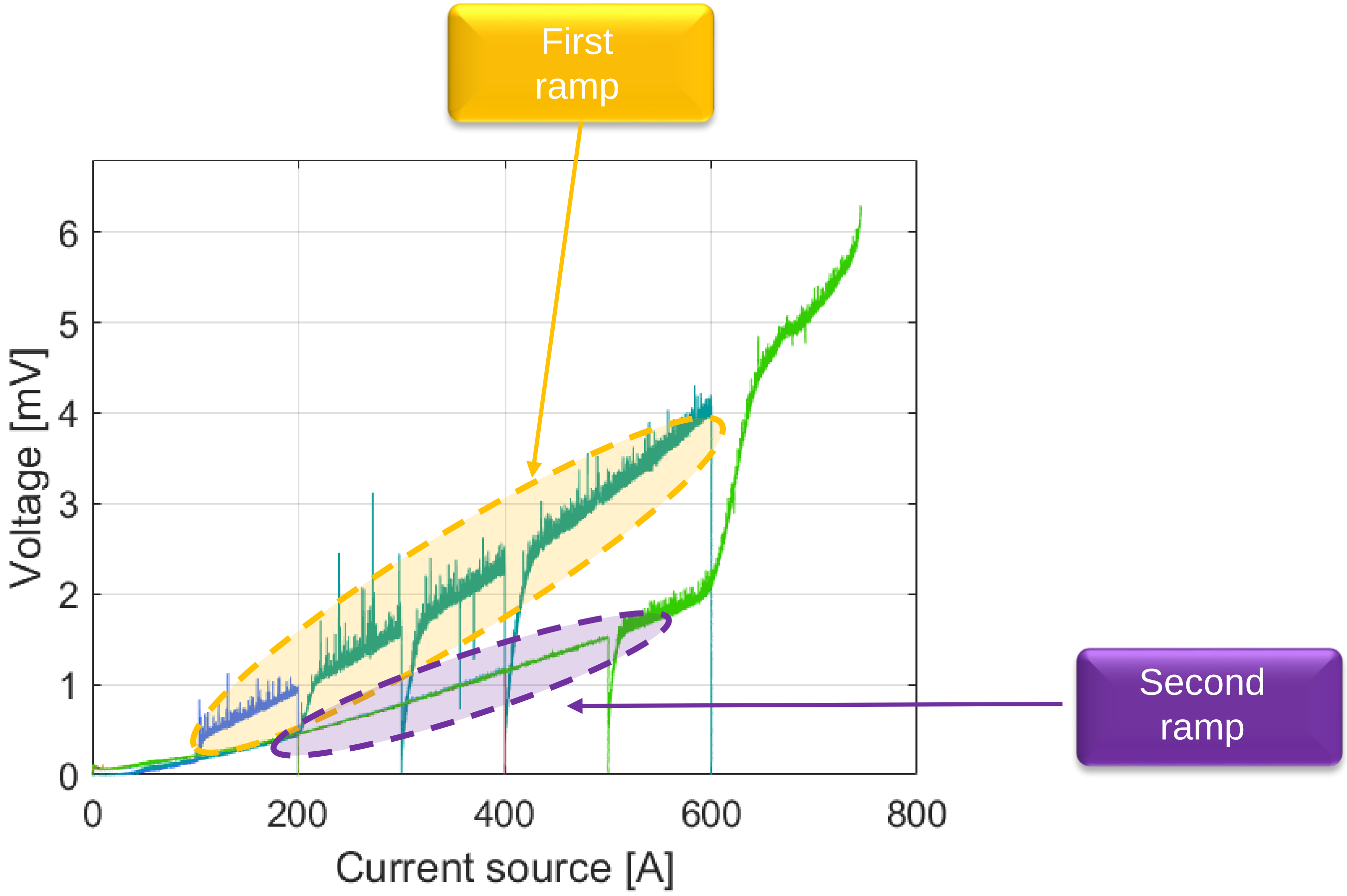
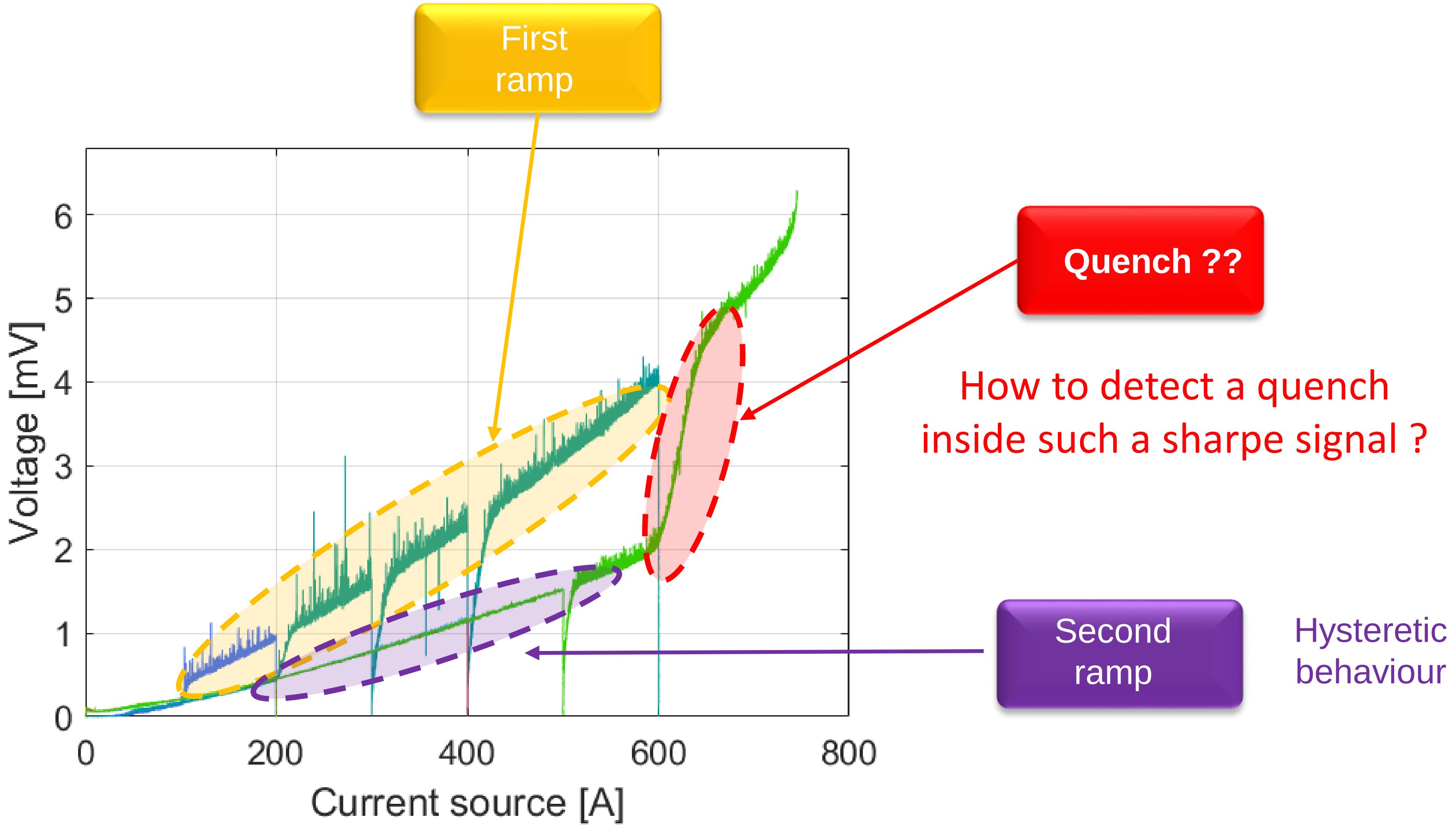


Hysteretic behaviour

Voltage measurements of a double pancake during a current ramp

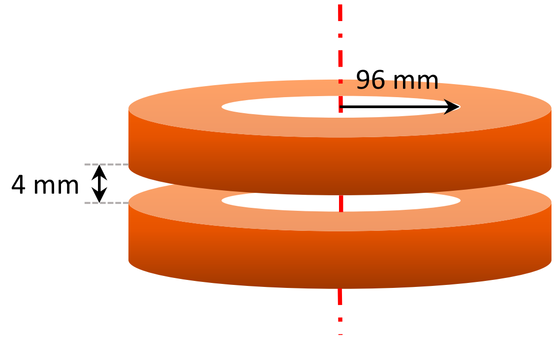


3. HTS coil : transient voltage

Comparison to experimental data : Coil description

Coil Description :

- Double pancake
- 12 mm width REBCO tape (SuperOx)
- Top : 150 turns / Bottom : 168 turns
- Inner radius : 96 mm



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Thickness of each layer of the tape

Discretization :

- Each turn is divided into 50 elements → 15900 elements

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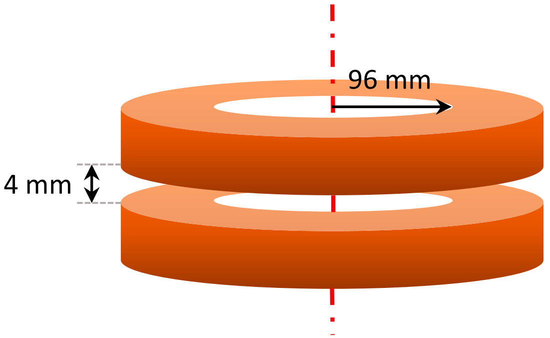

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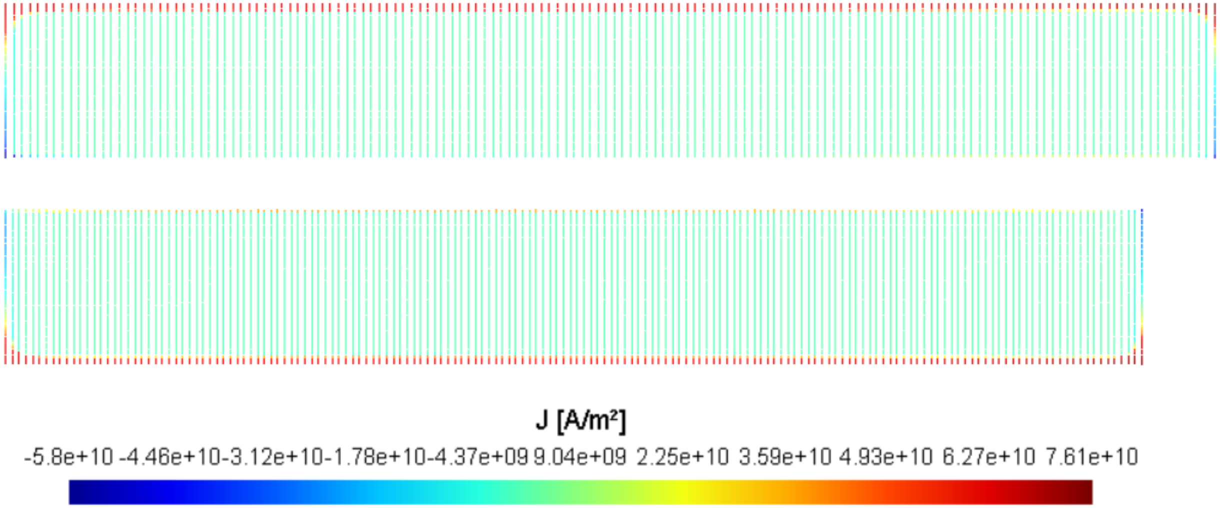
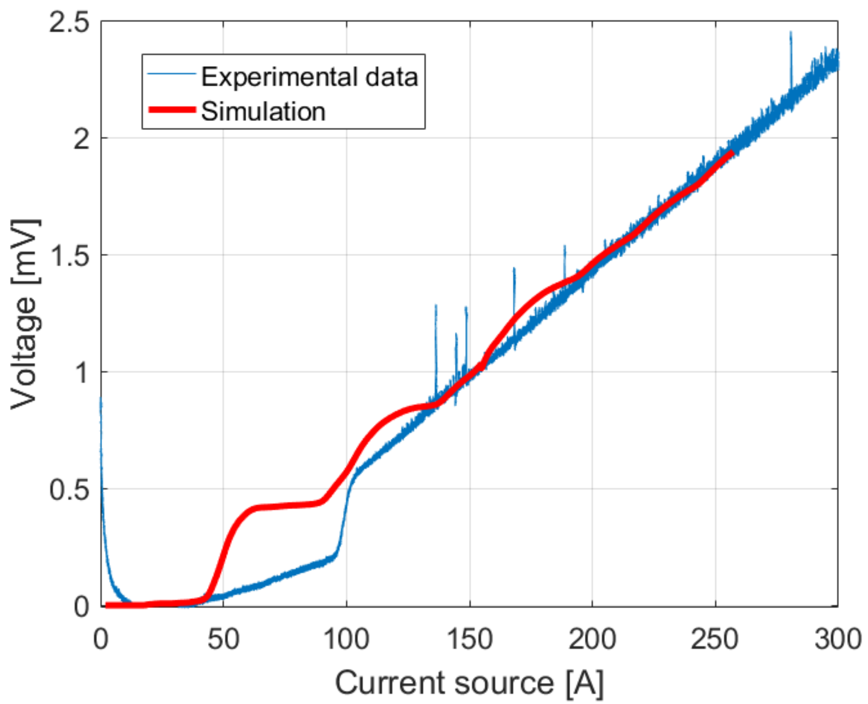
Exemple of current density distribution (cross-section view)



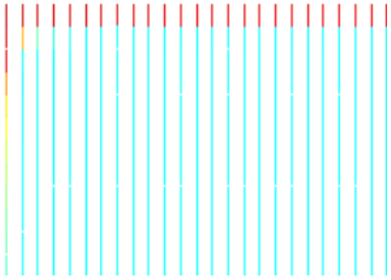
Scenario : 1st current ramp (2 A/s) until 200 A



Model slope : 0.0084 A/mV
Experimental slope : 0.0093 A/mV

Error : ~ **3.4 %**

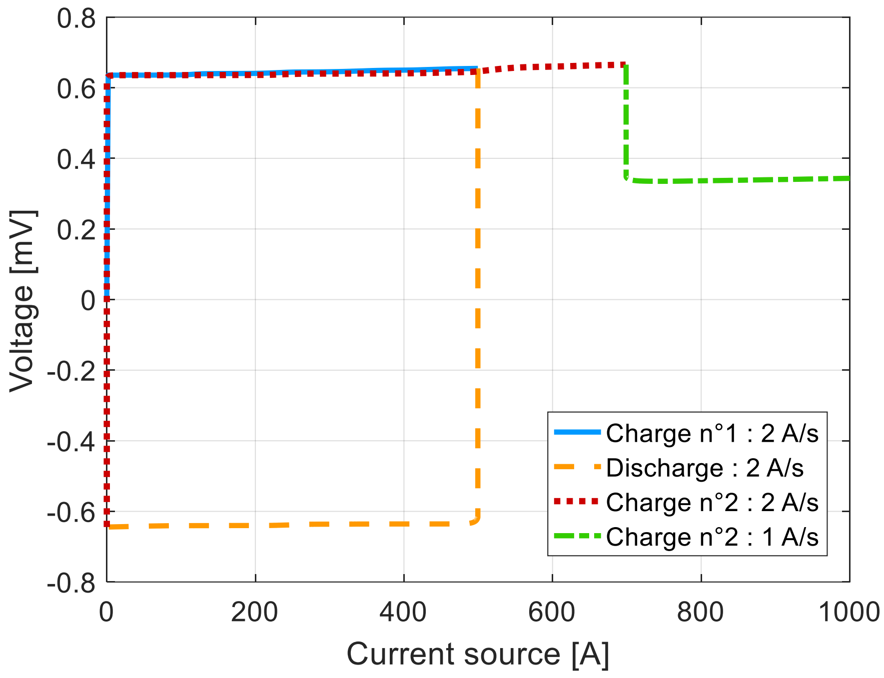
Current density constant per element
→ Current penetration impacts the voltage value at the very beginning



3. HTS coil : transient voltage

Tests on a smaller problem

Does the model reproduce the phenomena observed during a current ramp (charge / discharge / higher current …) ?



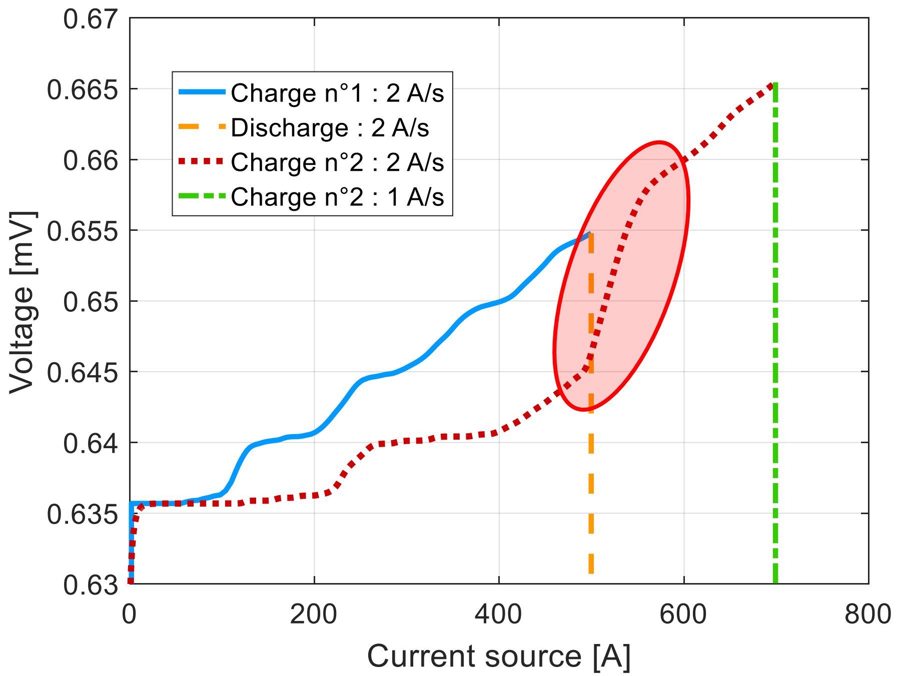
Scenario :

1. Charge : 2 A/s until 200 A
2. Discharge : 2A/s
3. Charge : 2 A/s until 700 A
4. Charge : 1 A/s from 700 to 1000 A

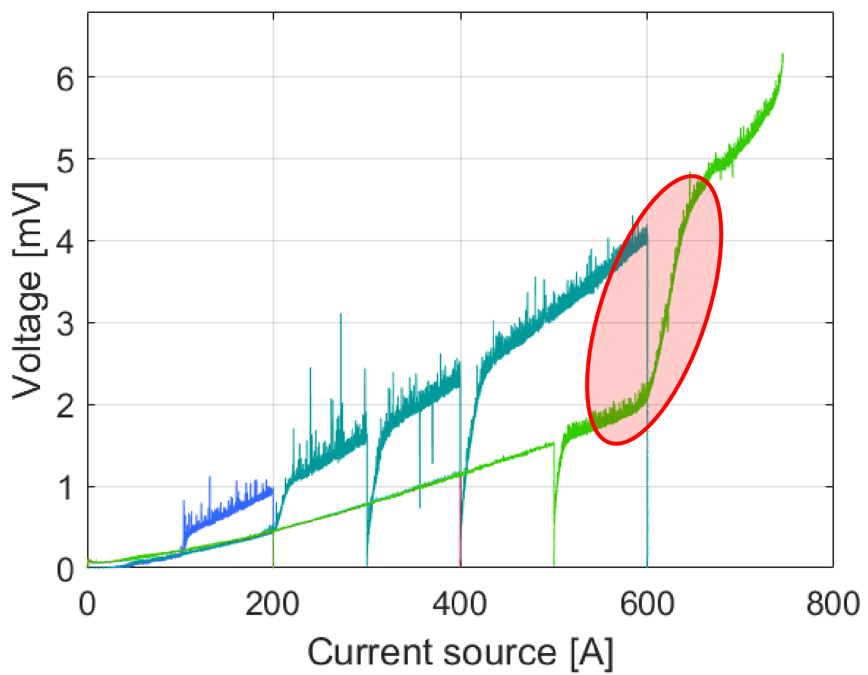
3. HTS coil : transient voltage

Tests on a smaller problem

Quench detection : computation of the transient voltage behaviour to remove it from the voltage and monitor only the dissipative component



Simulation results on a 25-turns coil



Experimental data from a double pancake coil

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Conclusions

- The model respects the requirements:
  - Long duration simulations (slow ramping of 0.2 A/s) : high stability
  - Nonlinear convergence

- Future improvements:
  - Full matrix storage → Matrix compression technics (FMM, …)
  - Computation time → auto-adaptive time stepping methods to be implemented to speed-up the calculation

- First results are in good agreement with the behaviour experimentally observed (quantitative match with a double pancake and qualitative results on a smaller test case) → validations on more cases

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Thank you for your attention !