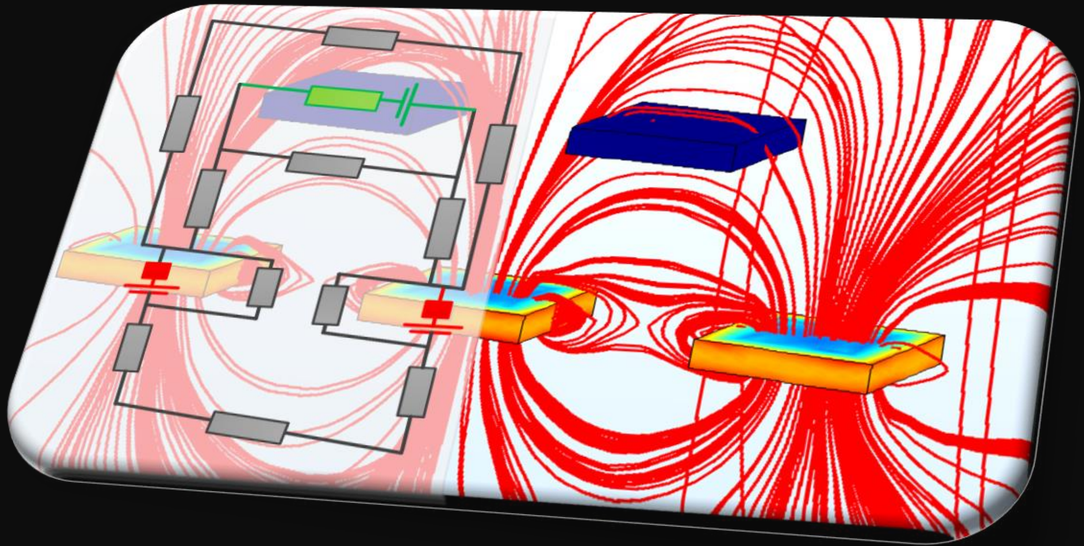
Electromagnetic Lumped Parameter Model of HTS Bulks in Magnetic circuits



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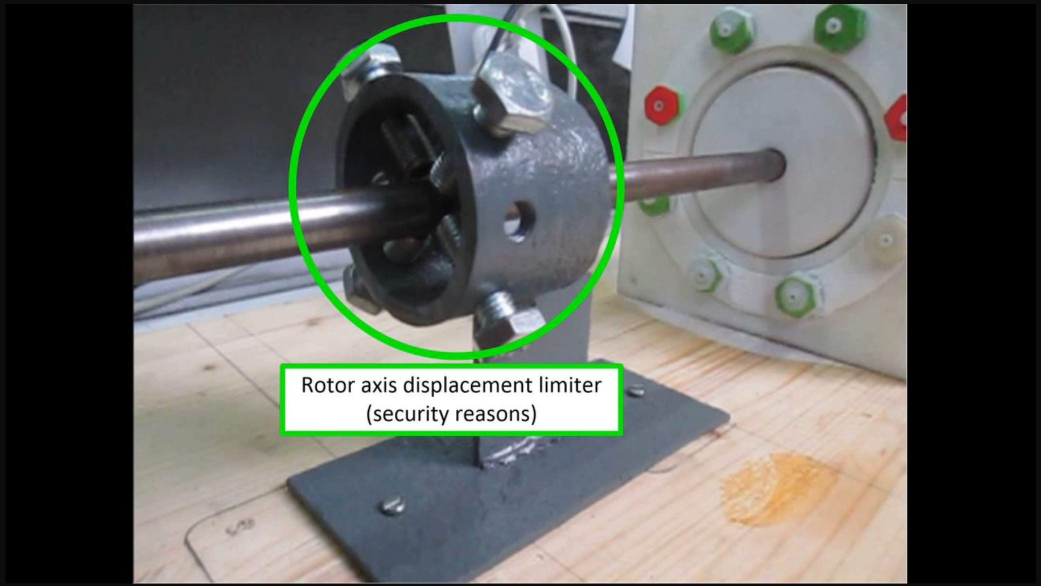
Motivation

- New High Temperature Superconductors (HTS):
  - Cooling of HTS became more cheap and accessible (more research):
    - Renew interest in design and optimization of new electrical machines with HTS bulks incorporated;

HTS Horizontal Levitation Bearing for High Speed Motors



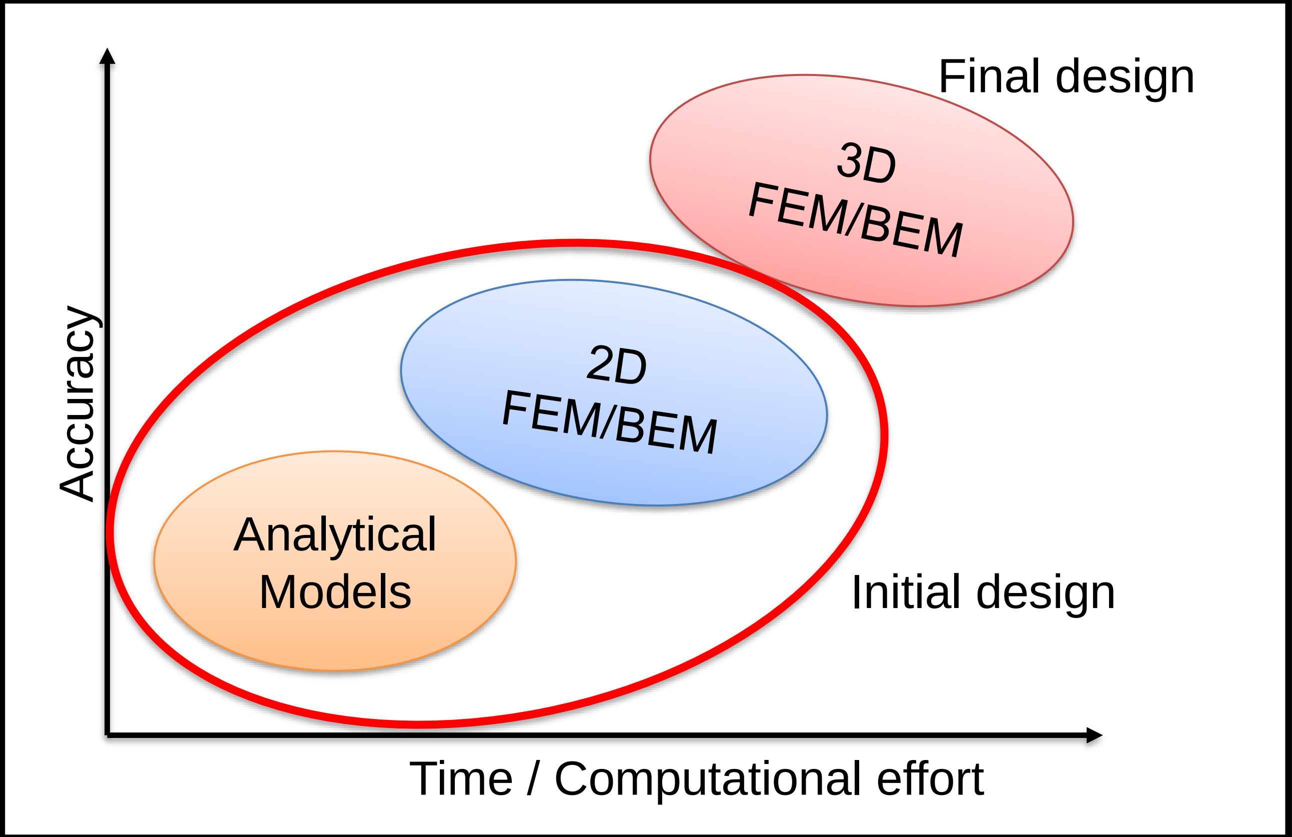
In loco HTS Bulk Magnetization

Motivation

When design an Electrical Machine with HTS bulks incorporated…

- Requires multiple physics and complex geometries
- HTS bulk physics -> non-linear models

- Simulations:
  - 3D: **several days** **/** 2D: **several hours.**
  - If using a multi-objective optimization tools (100 elements and 100 generations) it can take up to several **months and years** of simulation!



Analytical Models are still important! Is it possible to obtain a Lumped Parameter Model (LPM) for the HTS bulk with enough accuracy?

HTS Modelling

- LPM requires an analytical solution!

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$J_c = \frac{k}{(B_0 + |B(t)|)}$$

$$\vec{E} = E_0 \left(\frac{\vec{J}}{J_c} \right)^n$$

No feasible analytical solution!

HTS Modelling

- D.X. Chen and R.B. Goldfarb (1989) obtained a simplified analytical model:

$$J = Jc$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$J_c = \frac{k}{(B_0 + |B(t)|)}$$

It has an analytical solution!

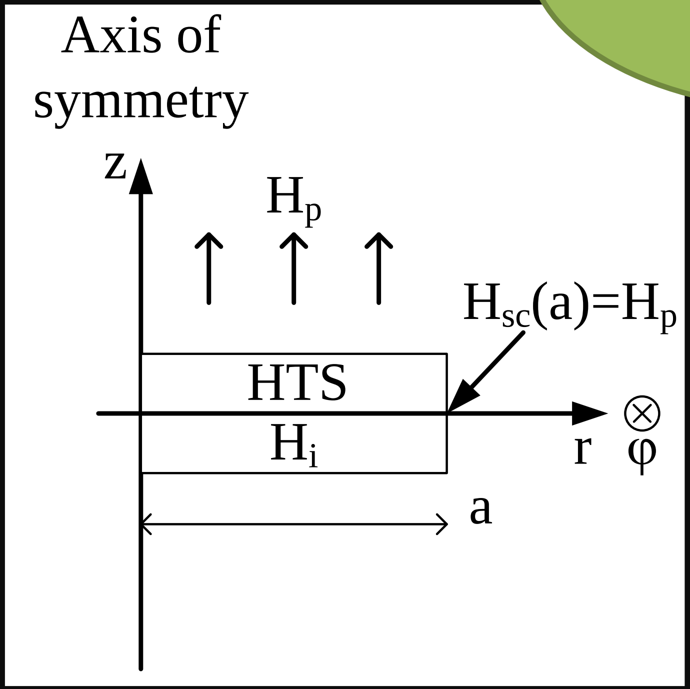
(however, not sensible to $\frac{\partial \vec{B}}{\partial t}$)

HTS Modelling: J=Jc + Kim Model

- Solution for an axisymmetric 2D problem:

$$\nabla \times \vec{H} = \vec{J}$$

$$J_c = \frac{k}{(H_0 + |H(t)|)}$$

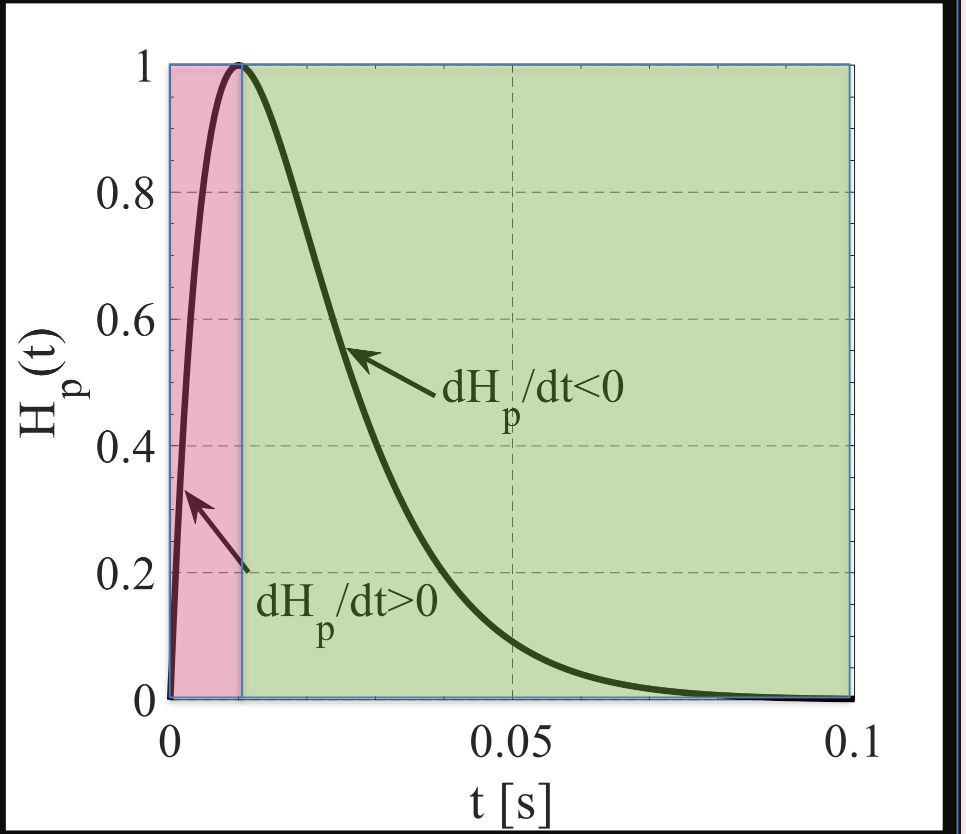


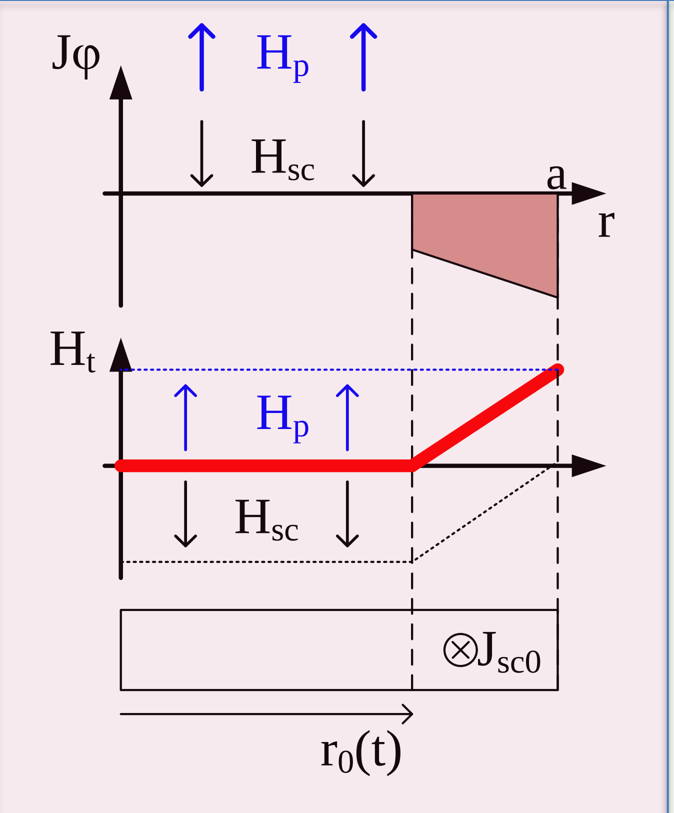
$$\frac{dH_{sc}}{dr} = -J_c = \frac{-\mathrm{sgn}(J_{sc})k}{\left(H_0 + \left|H_p(t)\right|\right)}$$

$$H_{sc} = -\,\mathrm{sgn}\left(H_p\right) H_0 \pm \sqrt{\left[H_0^2 - \mathrm{sgn}\left(J_{sc}H_p\right) 2k(r + c)\right]}$$

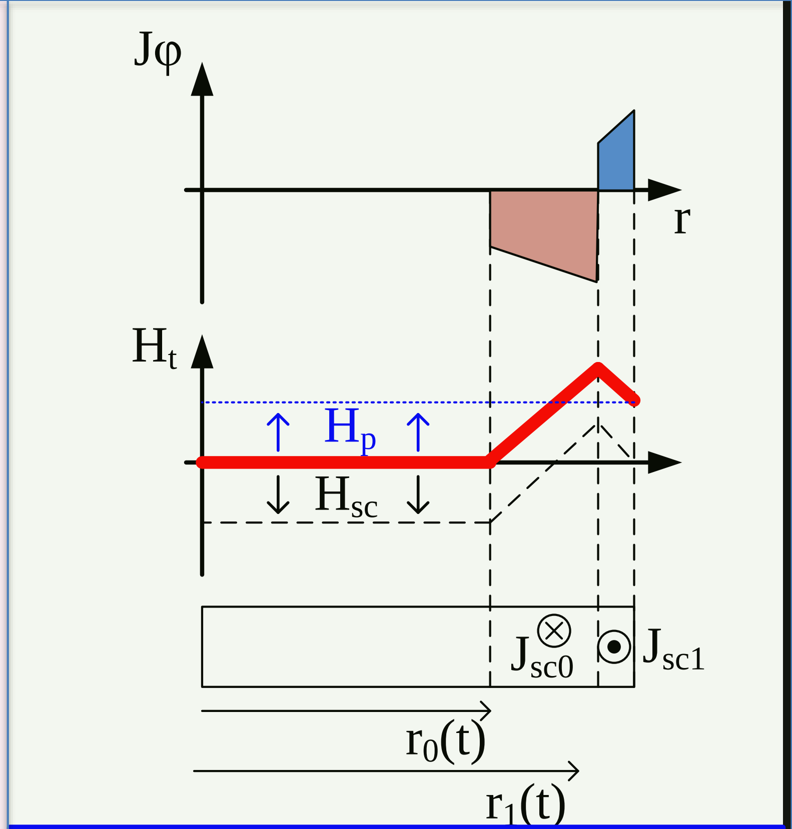
$$J_{sc} = -\mathrm{sgn}(J_{sc})k/\sqrt{\left[H_0^2 - \mathrm{sgn}\left(J_{sc}H_p\right) 2k(r + c)\right]}$$

HTS Modelling: J=Jc + Kim Model



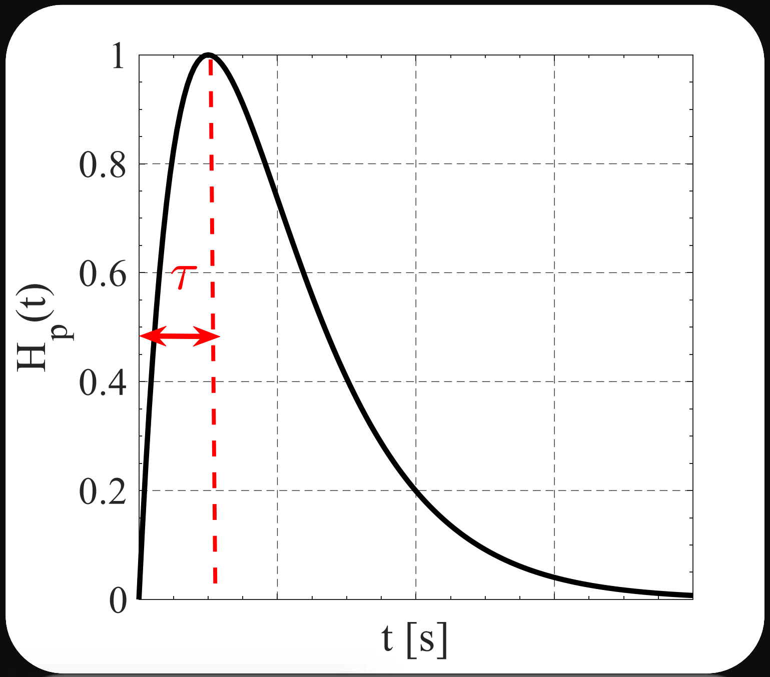


$$r_0(t) = a - [(H_0 + H_p)^2 - {H_0}^2]/2k$$



$$r_1(t) = a - [(H_0 + H_m)^2 - (H_0 + H_p)]/4k$$

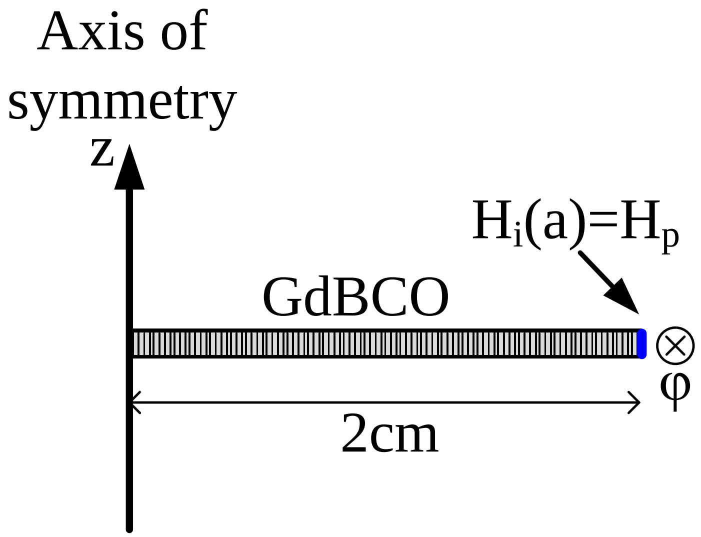
J=Jc + Kim Model: Accuracy



GdBCO

Jc_0=1.8 ×10^8 A/m^2
n=30
E_0=1×10^{-4} V/m
B0=1 T

FEA 2D Simulation



$$\nabla \times \vec{H} = \vec{J}$$

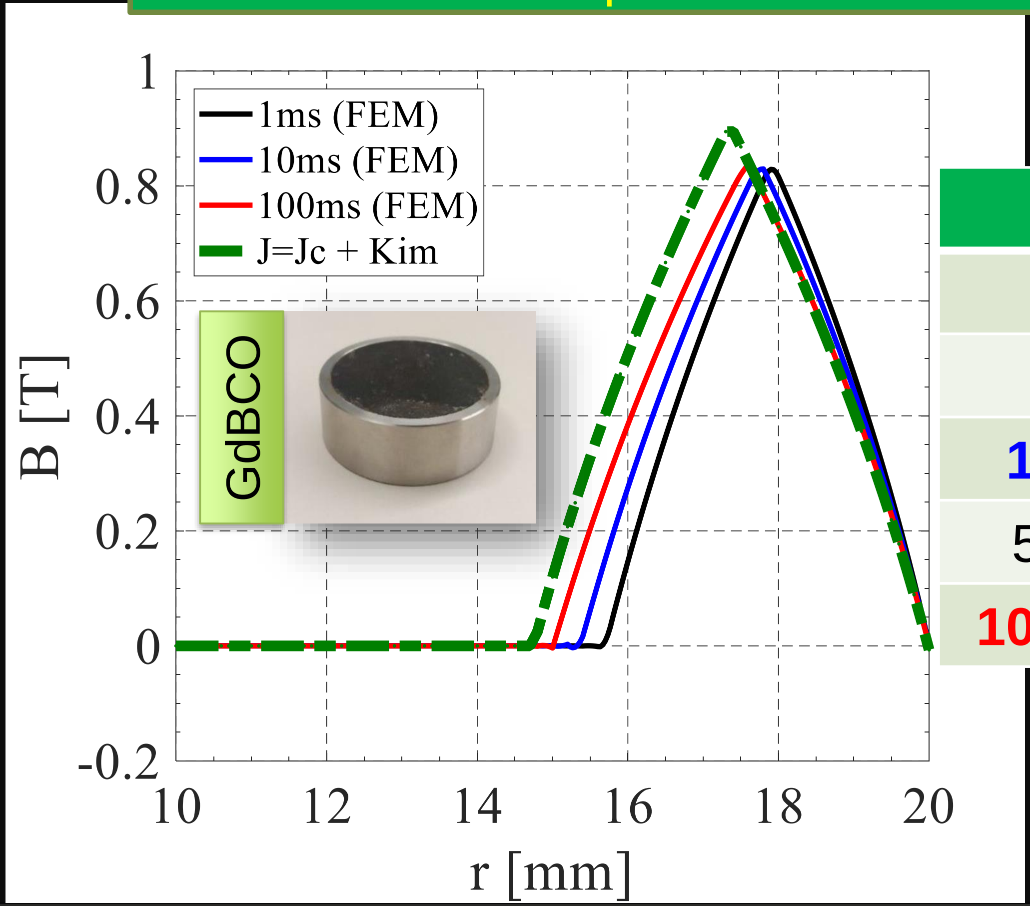
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E} = E_0 \left(\frac{\vec{J}}{J_{c0}} \right)^n$$

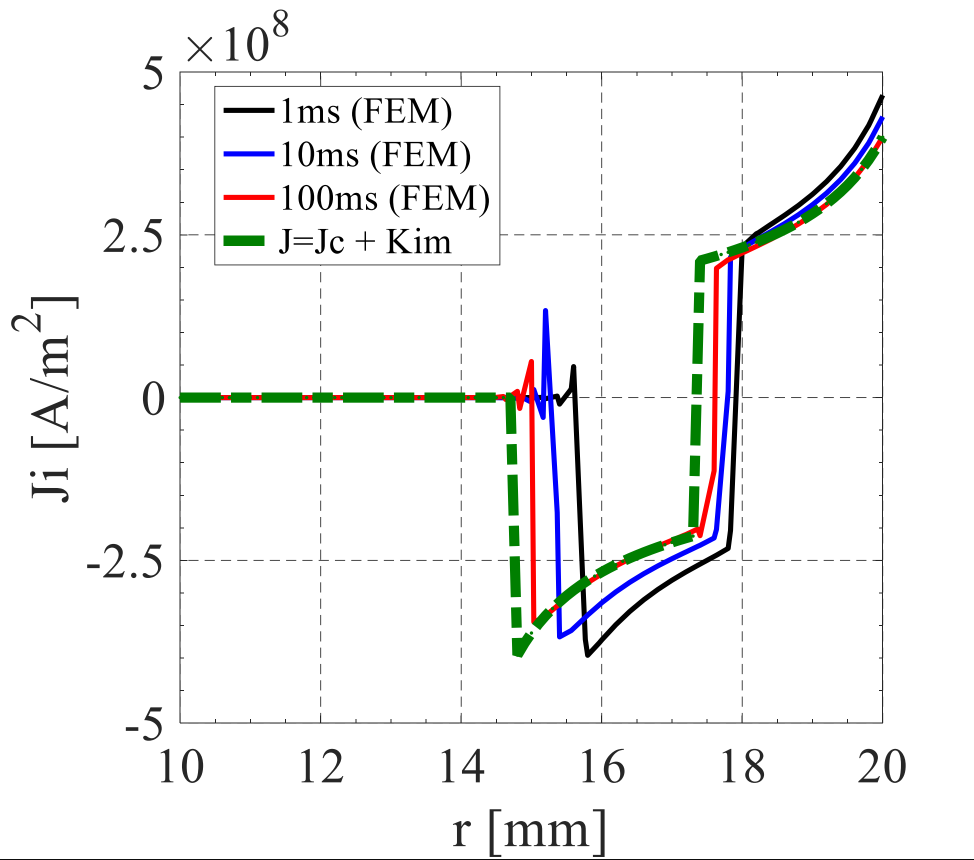
$$J_c = \frac{J_{c_0}}{(B_0 + |B(t)|)}$$

J=Jc + Kim Model: Accuracy

B_p=1.5T | $\tau\ (time\ to\ peak) = variable$



τ	Error B_{av}
1 ms	**18,9%**
5 ms	15,8%
10 ms	**14,5%**
50 ms	11,6%
100 ms	**10,3%**



Lumped Parameter Model

Lumped Parameter Model

- *J=Jc + Kim model*

$$H_{sc} = -\operatorname{sgn}(H_p)\, H_0 \pm \sqrt{[H_0^2 - \operatorname{sgn}(J_{sc}H_p)\, 2k(r+c)]}$$

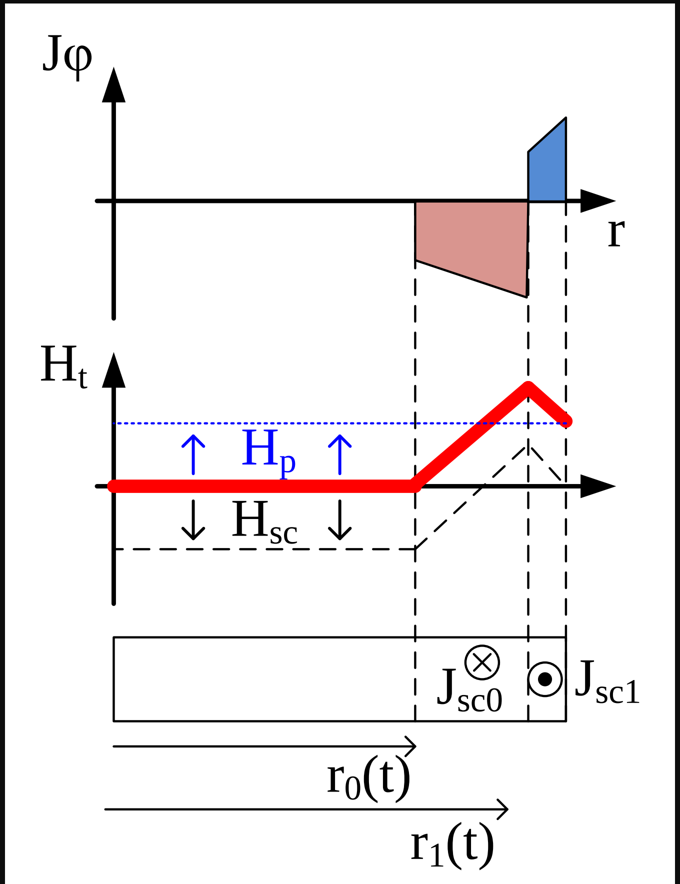
$$J_{sc} = -\operatorname{sgn}(J_{sc})k/\sqrt{[H_0^2 - \operatorname{sgn}(J_{sc}H_p)\, 2k(r+c)]}$$

$$r_0 = a - [(H_0 + H_p)^2 - {H_0}^2]/2k$$

$$r_1 = a - [(H_0 + H_m)^2 - (H_0 + H_i)]/4k$$

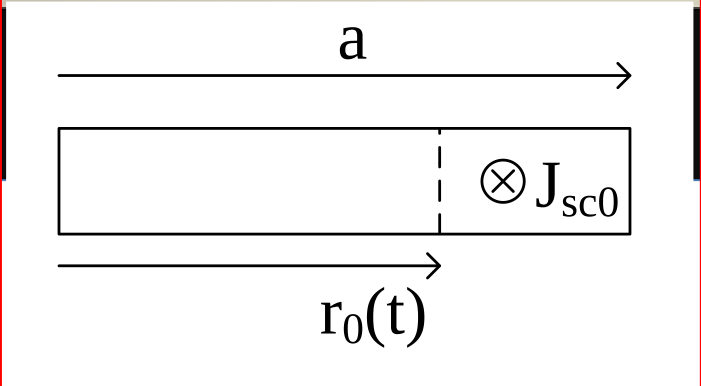
Representation by average values.

- Information about induced currents ✔



Lumped Parameter Model

$$\frac{dH_p}{dt} > 0$$



$$H_{sc} = \begin{cases} \int_{r0}^{a} J_{sc0}(r)dr & 0 < r < r_0 \\ \int_{r}^{a} J_{sc0}(r)dr & r_0 < r < a \end{cases}$$

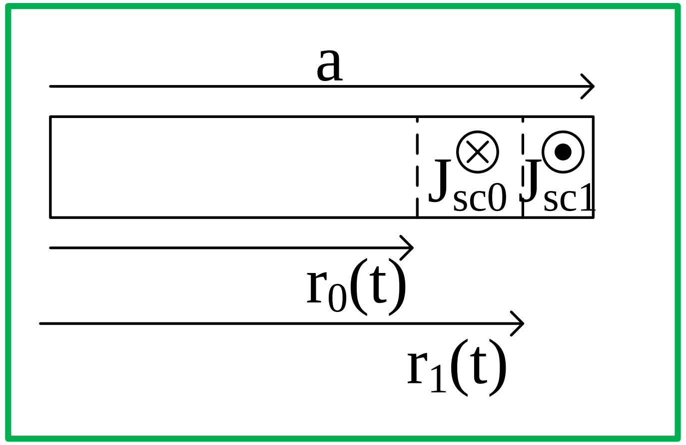
$$r_0 = a - \left[\left(H_0 + H_p\right)^2 - H_0{}^2\right]/2k$$

$$H_{sc_{av}} = \frac{1}{a}\int_0^a H_{sc}(r)\,dr = \left[-\sqrt{\left(H_0 + H_p\right)^2} + \sqrt{\left(H_0 + H_p\right)^2 - 2k(a - r_0)}\right]\frac{r_0}{a} + \\ + \left[-\sqrt{\left(H_0 + H_p\right)^2}(a - r_0) + \frac{\left[\left(H_0 + H_p\right)^2\right]^{3/2} - \left[\left(H_0 + H_p\right)^2 - 2k(a - r_0)\right]^{3/2}}{3k}\right]\frac{1}{a}$$

Lumped Parameter Model

$$\frac{dH_p}{dt} < 0$$

$$H_{sc} = \begin{cases} \int_{r0}^{r1} J_{sc0}(r)dr + \int_{r1}^{a} J_{sc1}(r)dr & r < r_0 \\ \int_{r}^{r1} J_{sc0}(r)dr + \int_{r1}^{a} J_{sc1}(r)dr & r_0 < r < r_1 \\ \int_{r}^{a} J_{sc1}(r)dr & r_1 < r < a \end{cases}$$

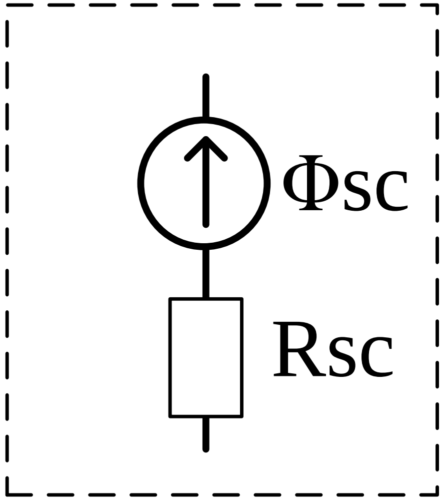


$$\boxed{H_{sc_{av}} = \frac{1}{a}\int_0^a H_{sc}(r)\,dr}$$

$$\boxed{r_0 = a - [\left(H_0 + H_p\right)^2 - H_0{}^2]/2k}$$

$$\boxed{r_1 = a - [(H_0 + H_m)^2 - \left(H_0 + H_p\right)]/4k}$$

Lumped Parameter Model



Φsc – Magnetic flux ($H_{sc_{av}}$ as function of H_p)

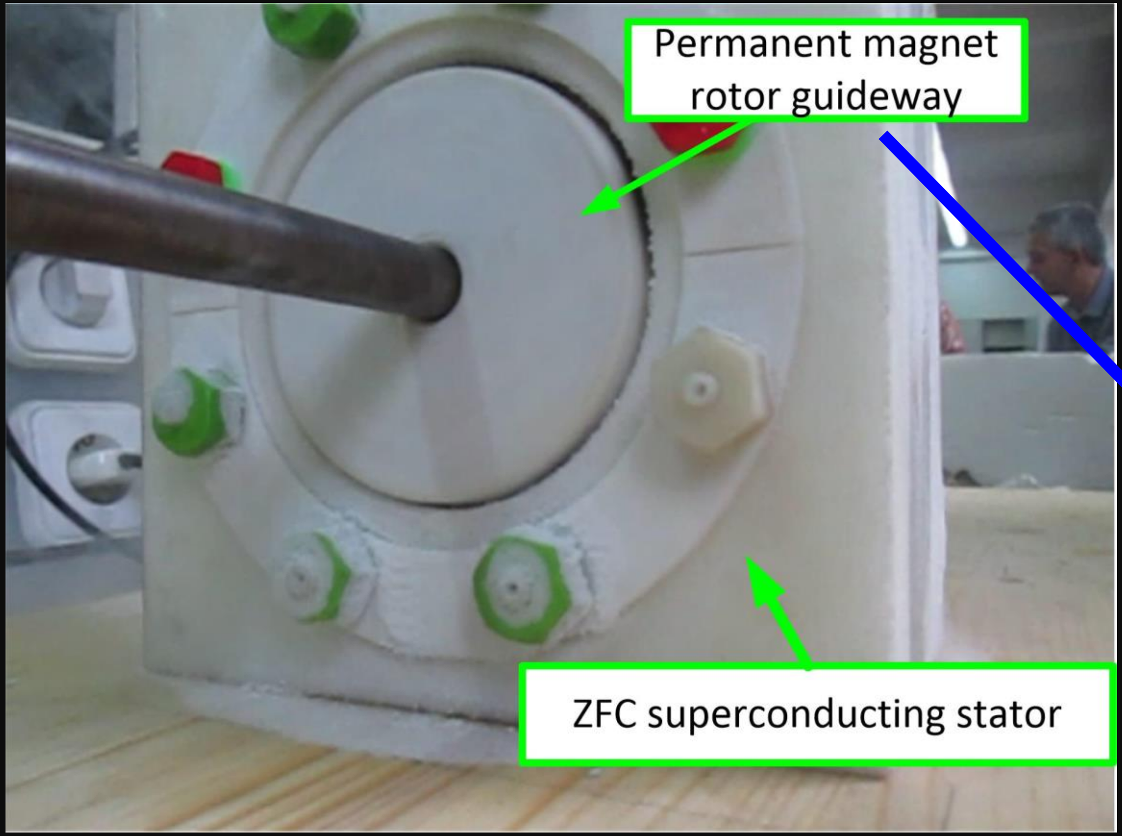
Rsc - Magnetic reluctance ($\mu_r = 1$)

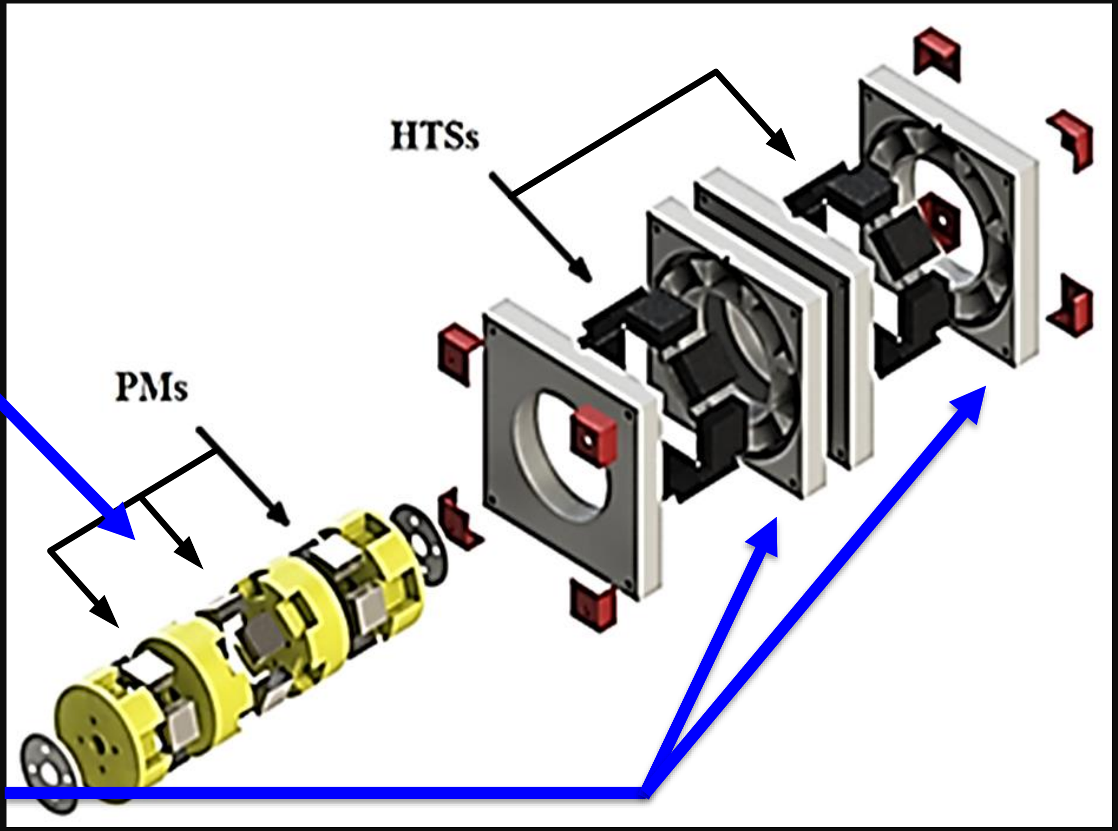
HTS keeps trying to maintain its magnetic field unchanged by using superconductor currents.

“Inspired” in LPM of permanent magnets.

Has information about the HTS currents -> Power losses

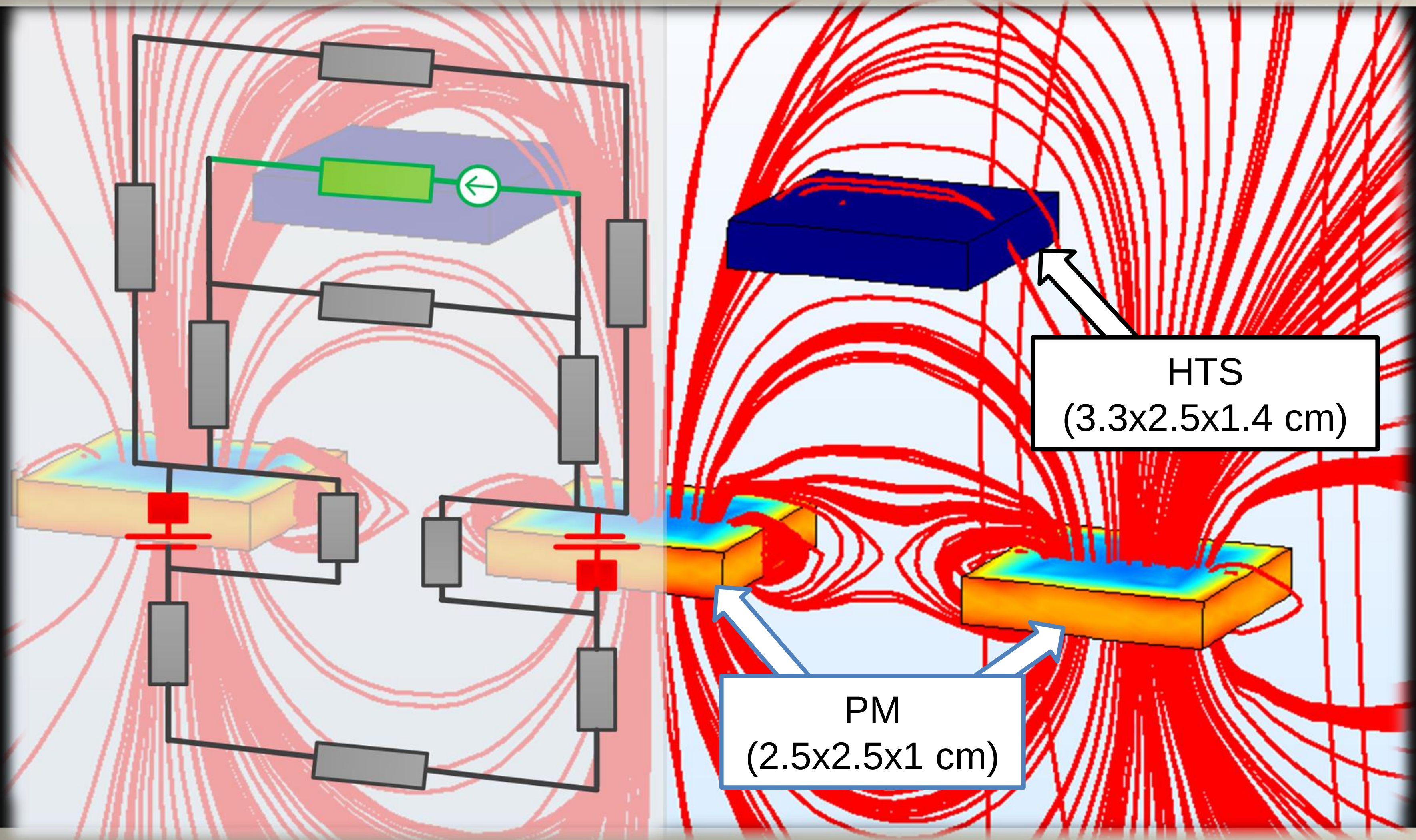
HTS Levitation Circuit





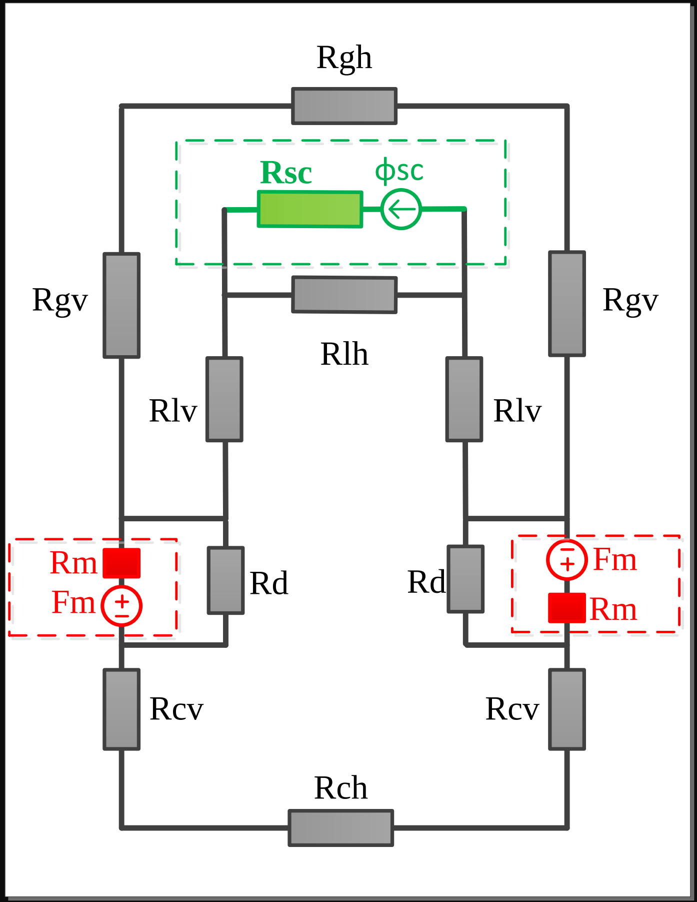
A. J. Arsénio, et al, "Prototype of a Zero-Field-Cooled YBCO Bearing With Continuous Ring Permanent Magnets," in *IEEE Transactions on Applied Superconductivity*, vol. 28, no. 4, pp. 1-7, June 2018.

HTS Levitation Circuit

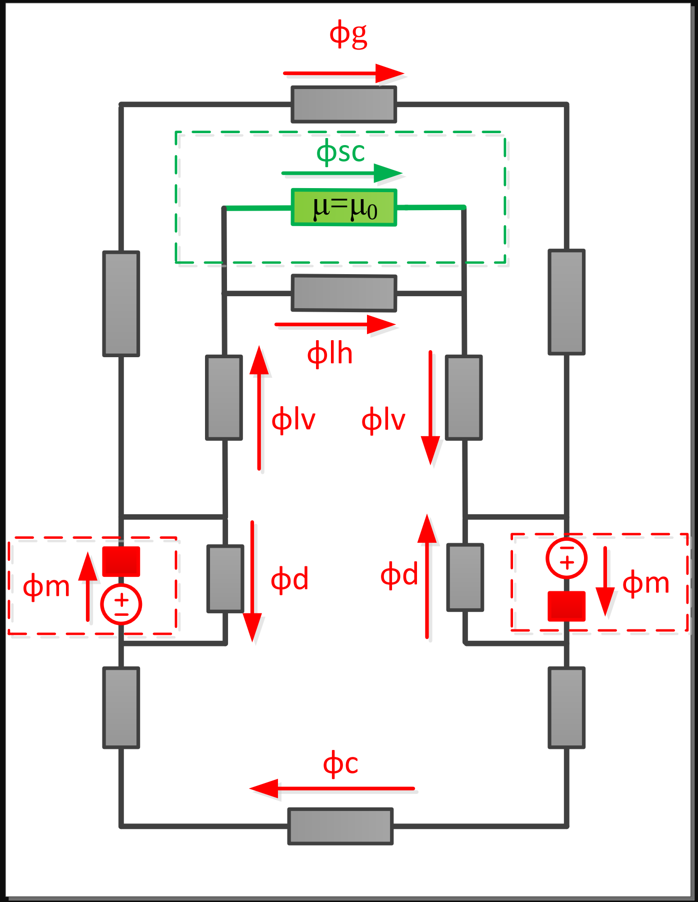


HTS Levitation Circuit

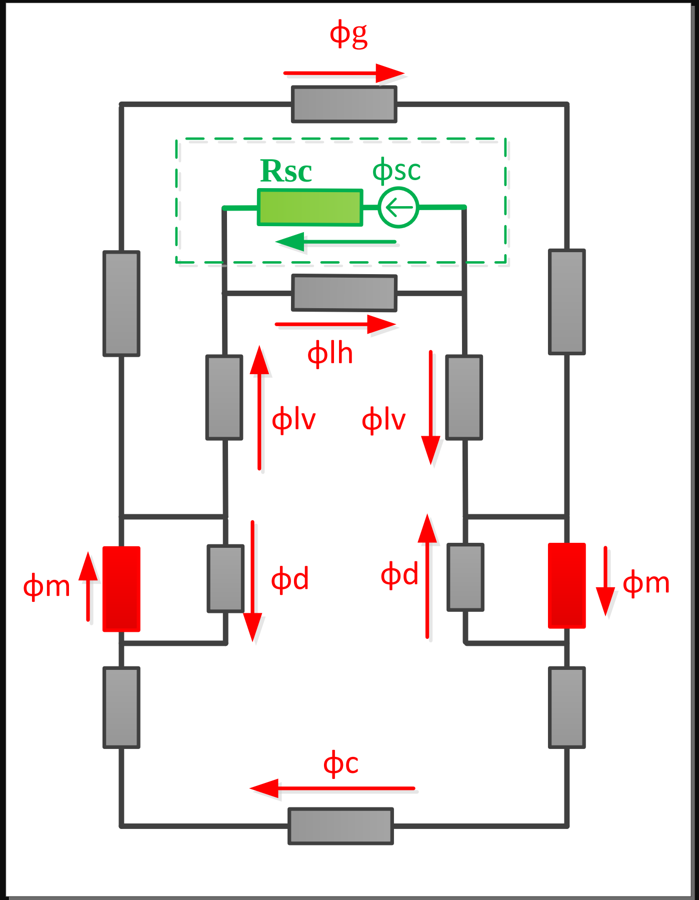
PM + HTS bulk



PM only

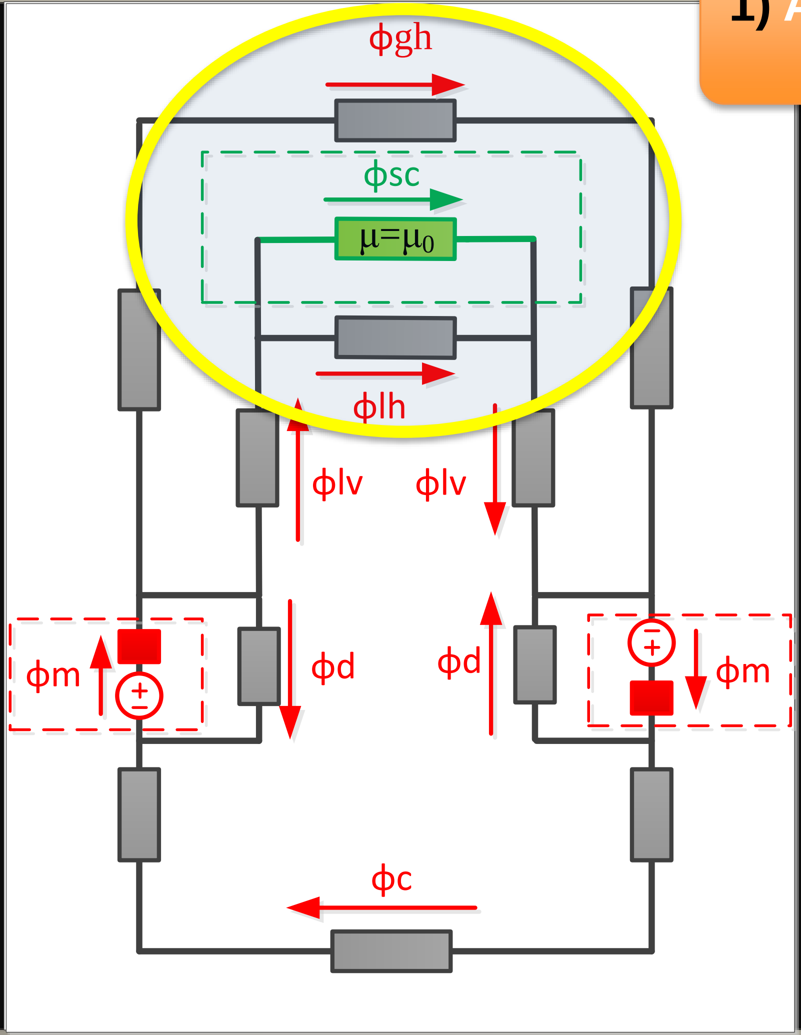


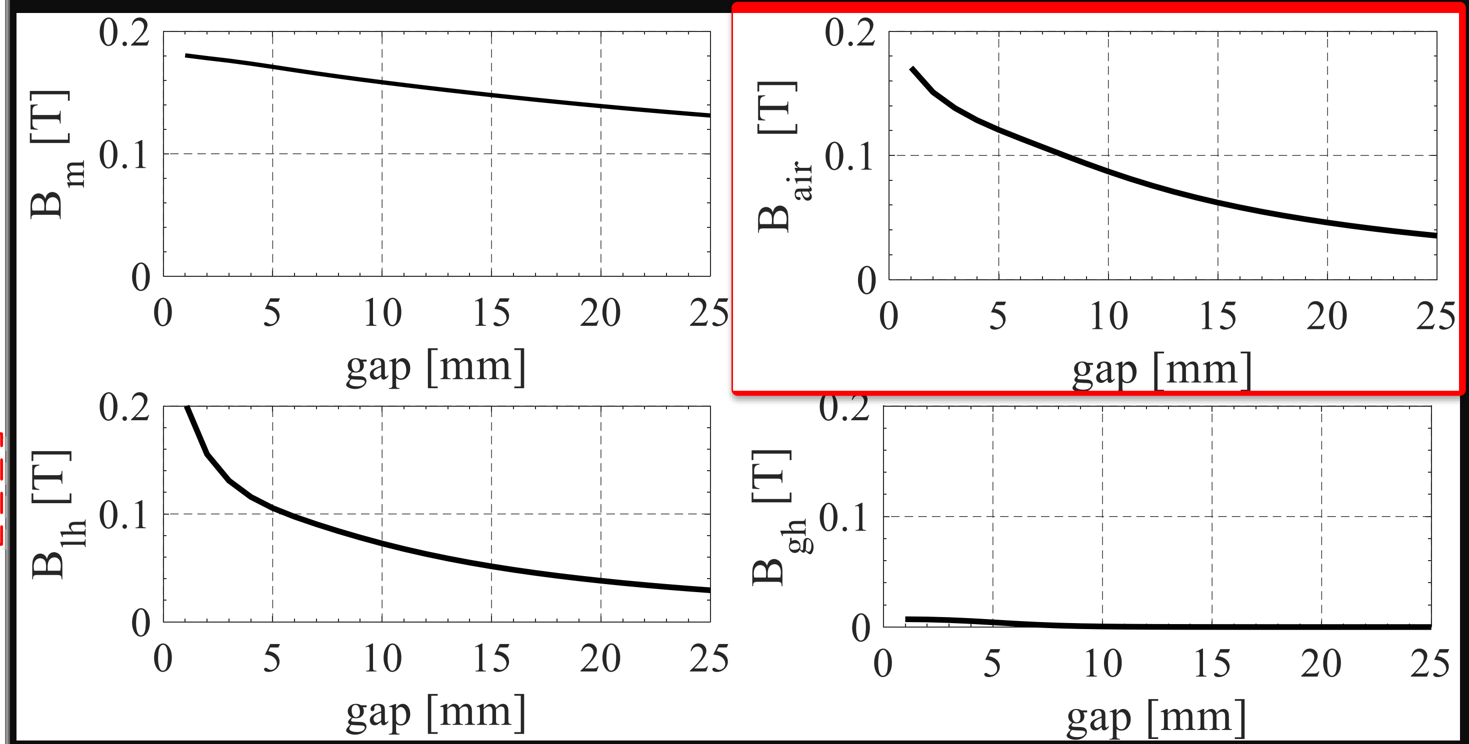
HTS only



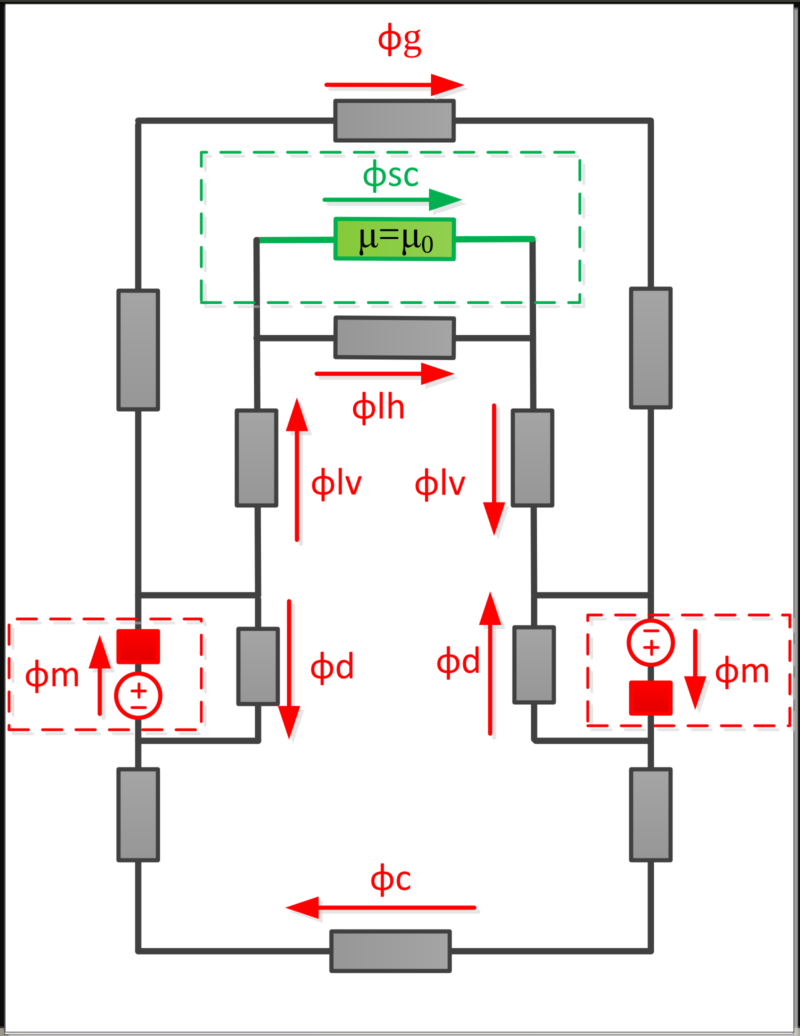
HTS Levitation Circuit

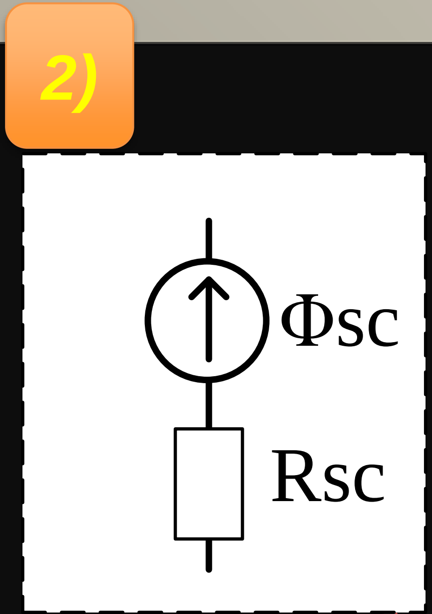
1) Air instead of HTS: Distribution of flux due to only PM;

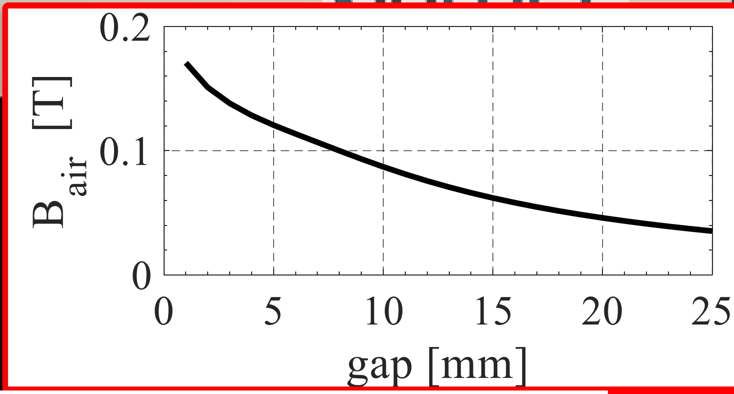


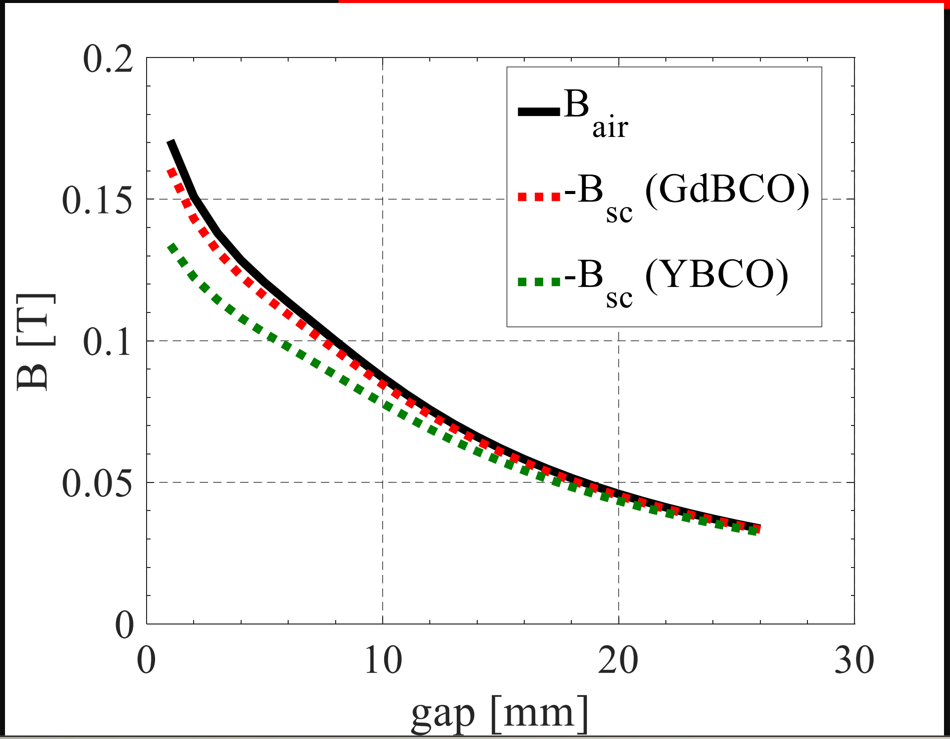


HTS Levitation Circuit



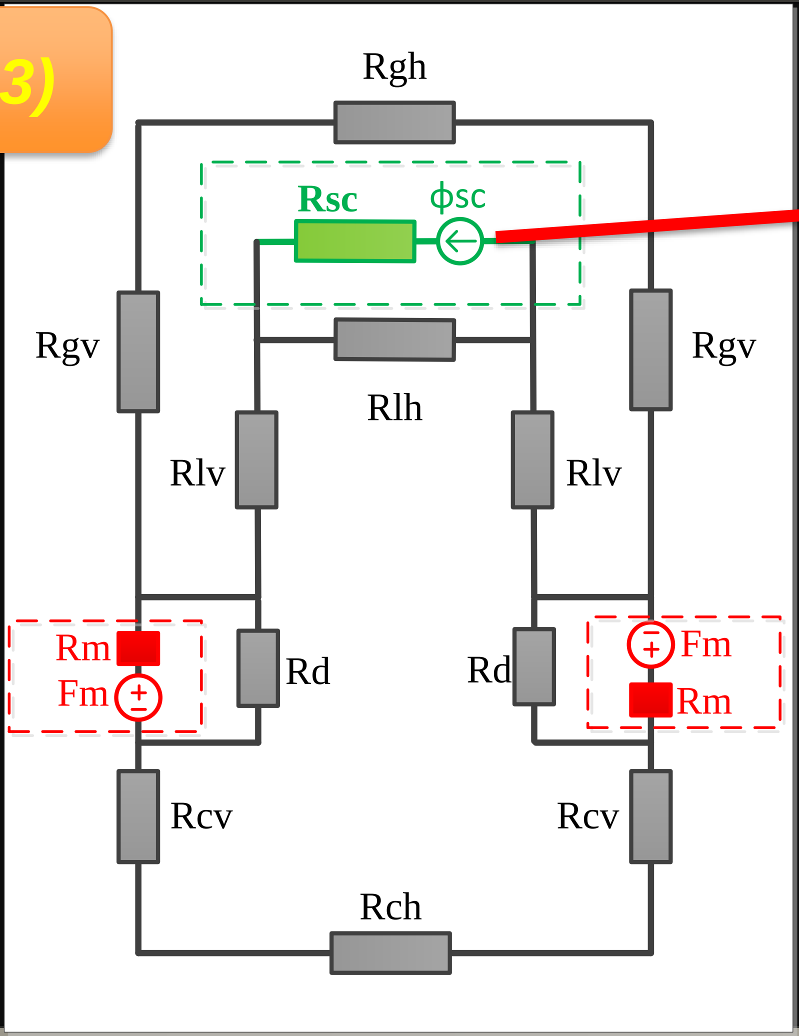


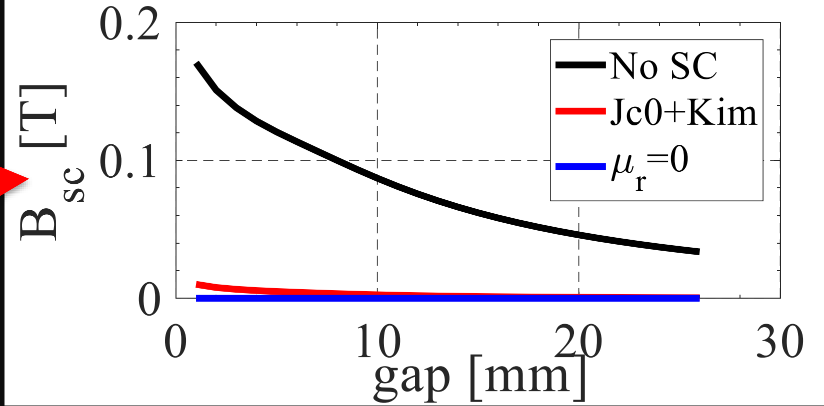


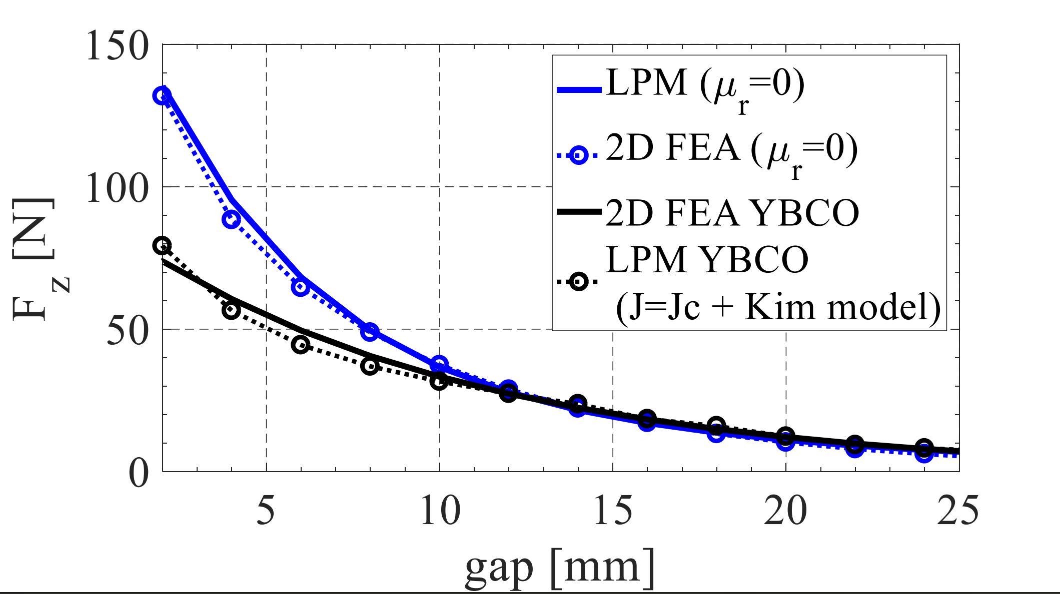


HTS Levitation Circuit

3)



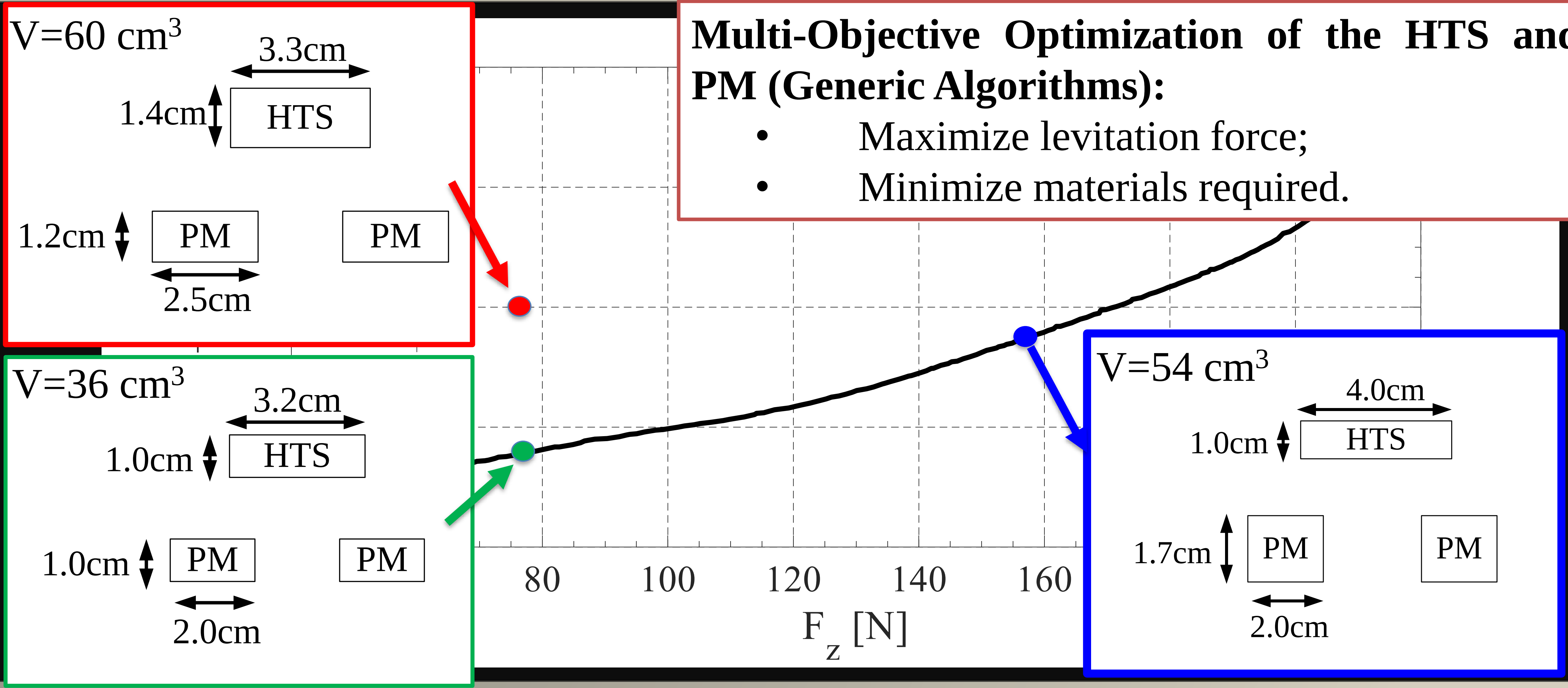




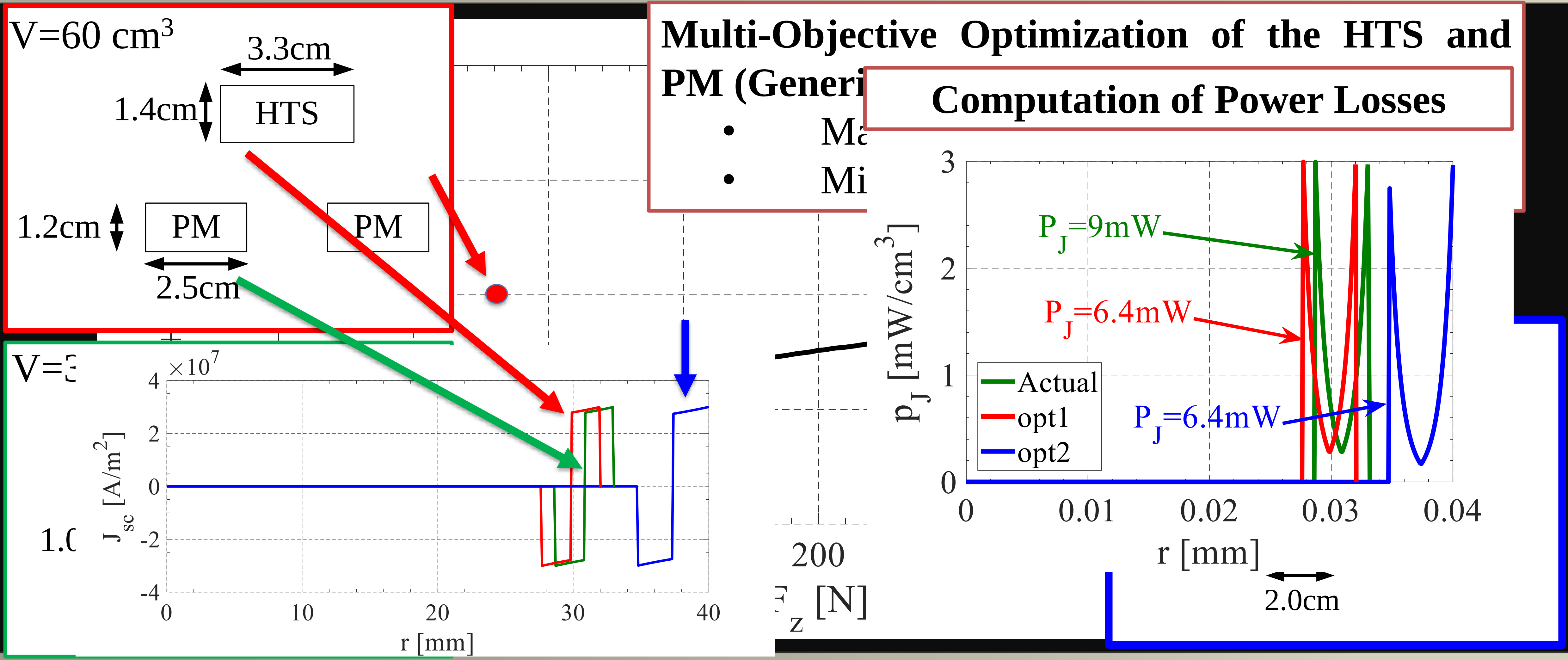
Current On-Going Work

Multi-Objective Optimization of the HTS and PM (Generic Algorithms):

- Maximize levitation force;
- Minimize materials required.



Current On-Going Work



Conclusions

1. HTS electromagnetic lumped parameter model (LPM) can be done using Kim model and J=Jc;

2. It outperforms simple models as $\mu_r = 0$.

3. Can be used in optimization algorithms for electrical machines predesign.

4. Can be used to study the power losses inside the HTS bulk.

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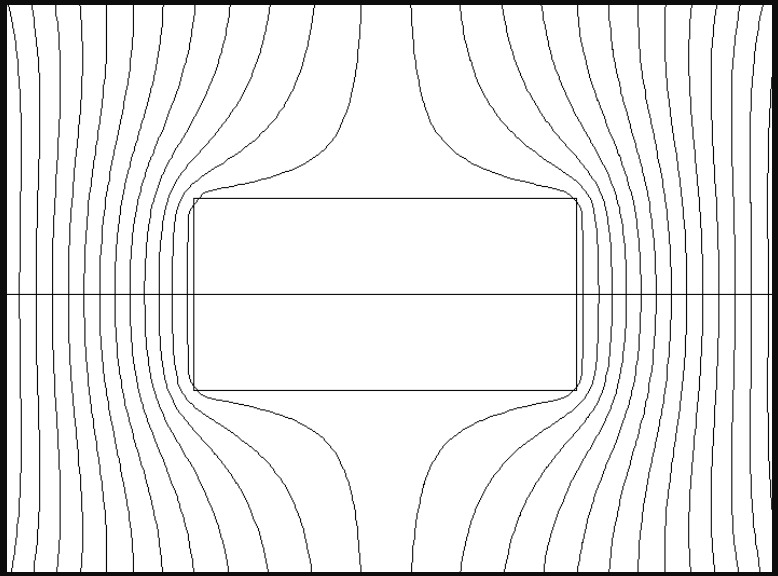
Thank you

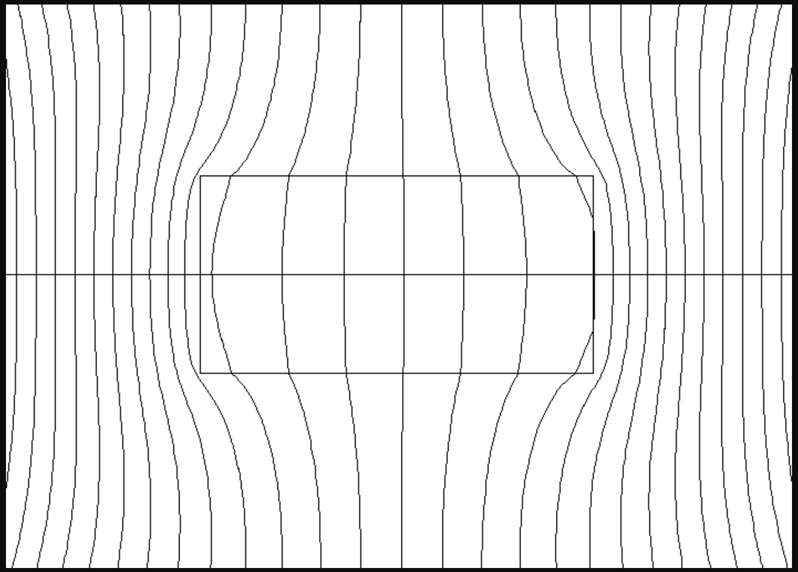
Lumped Parameter Model

- *Other simplified LPM used in the pre-design of magnetic circuits:*

$\mu_r=0$

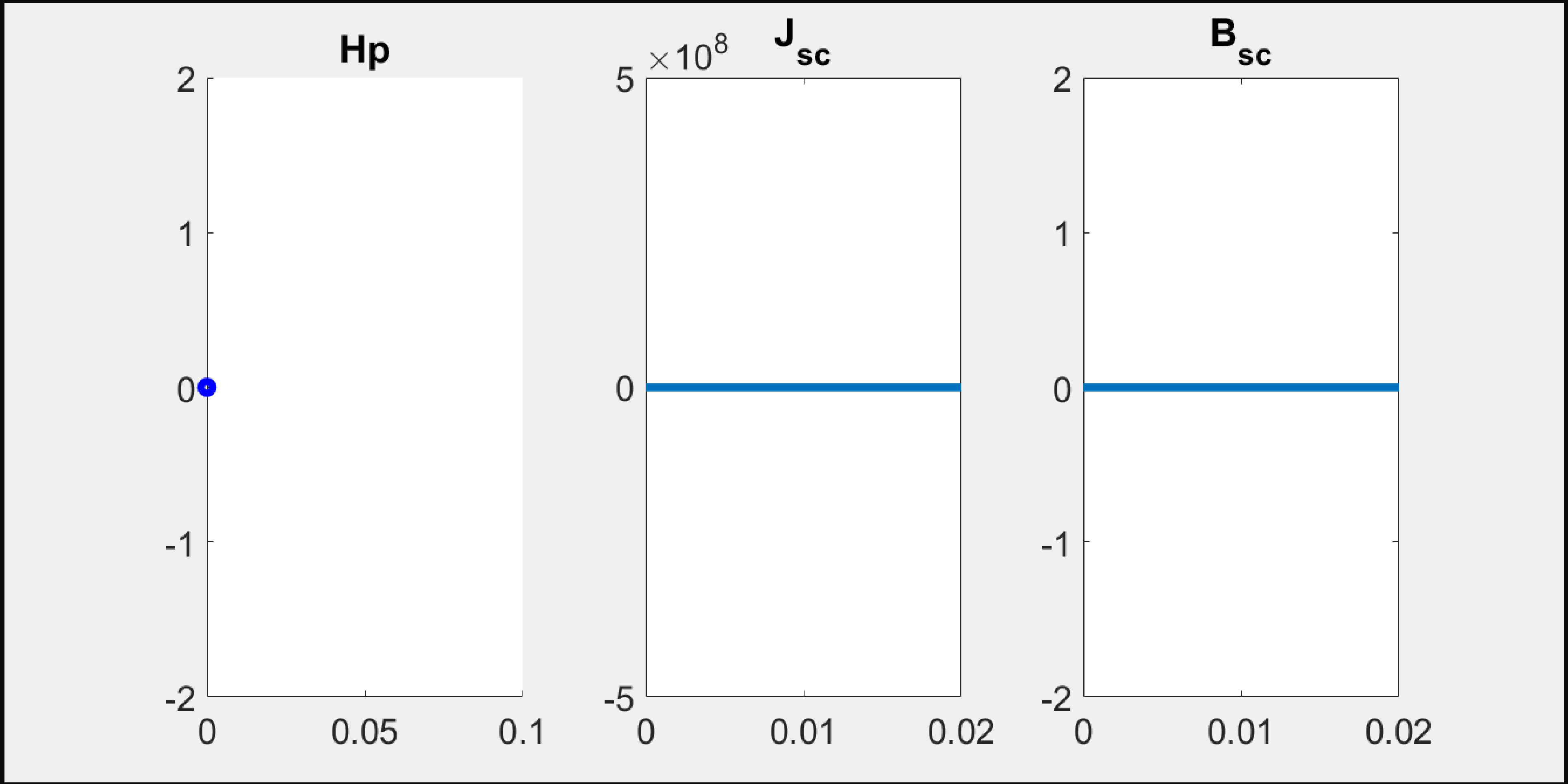


$\mu_r=0.2$



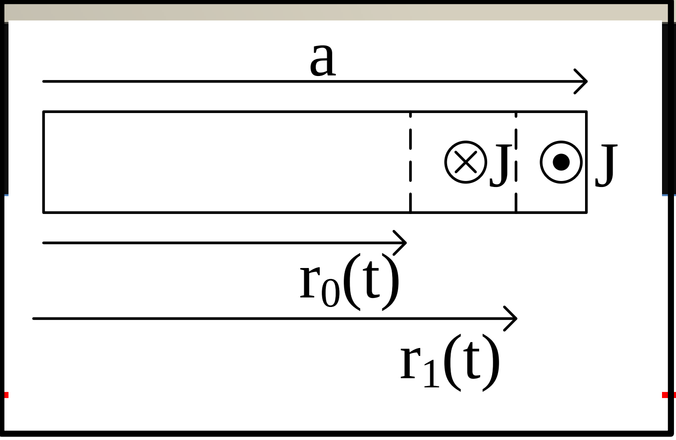
No information about induced currents!

HTS Modelling: J=Jc + Kim Model



Lumped Parameter Model

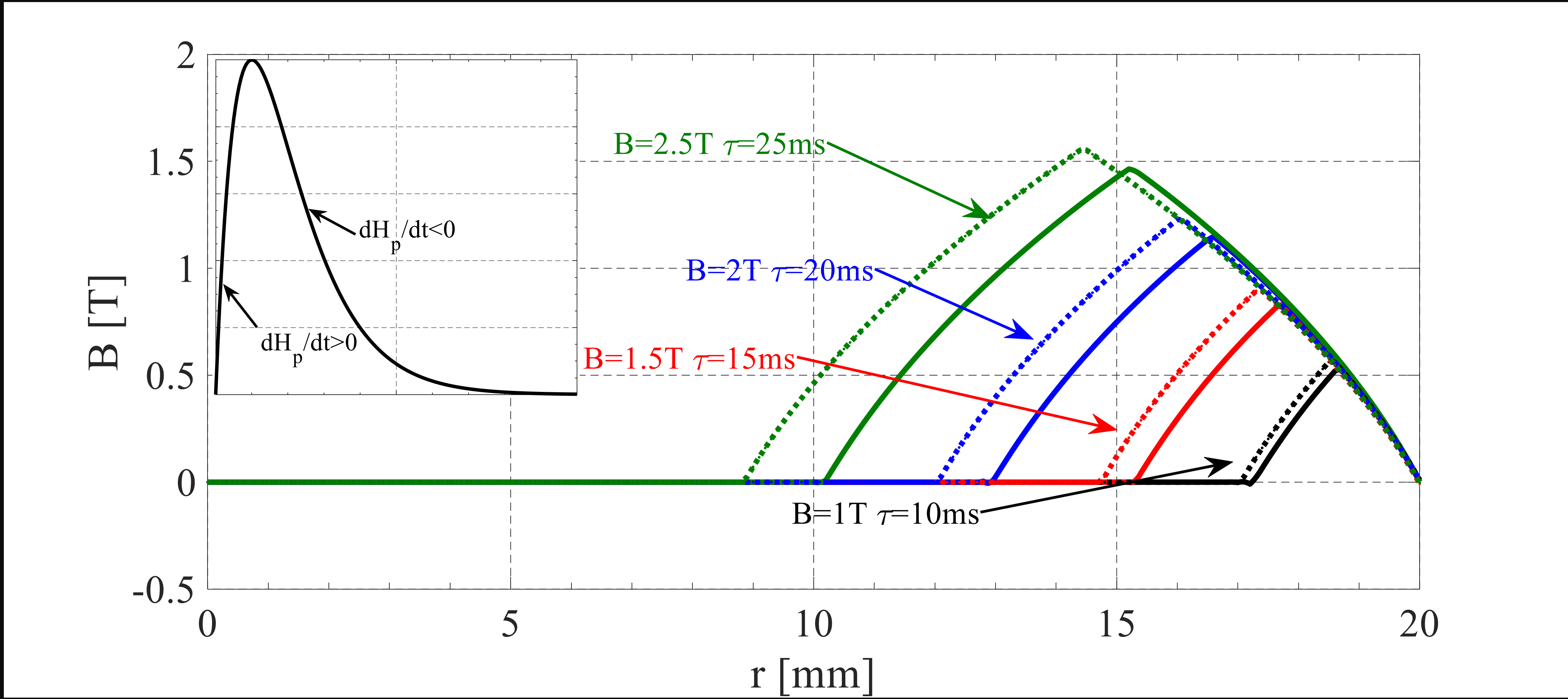
$$\frac{dH_p}{dt} < 0$$



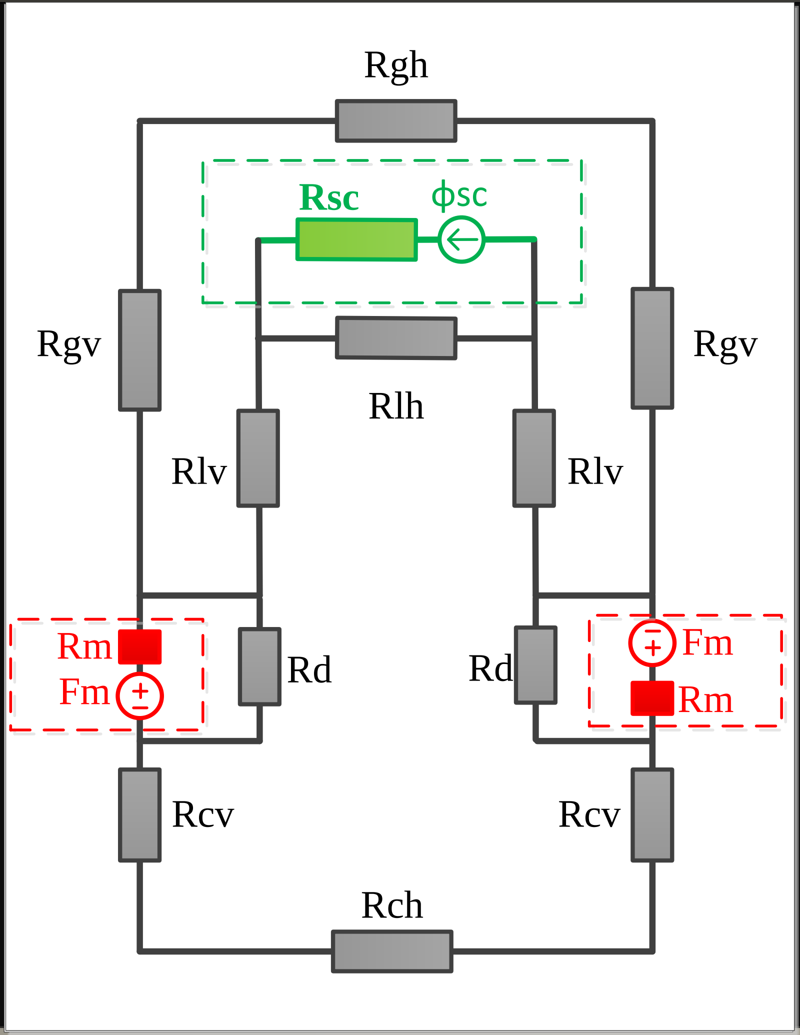
$$r_1 = a - \left[(H_0 + H_m)^2 - \left(H_0 + H_p\right)\right]/4k$$

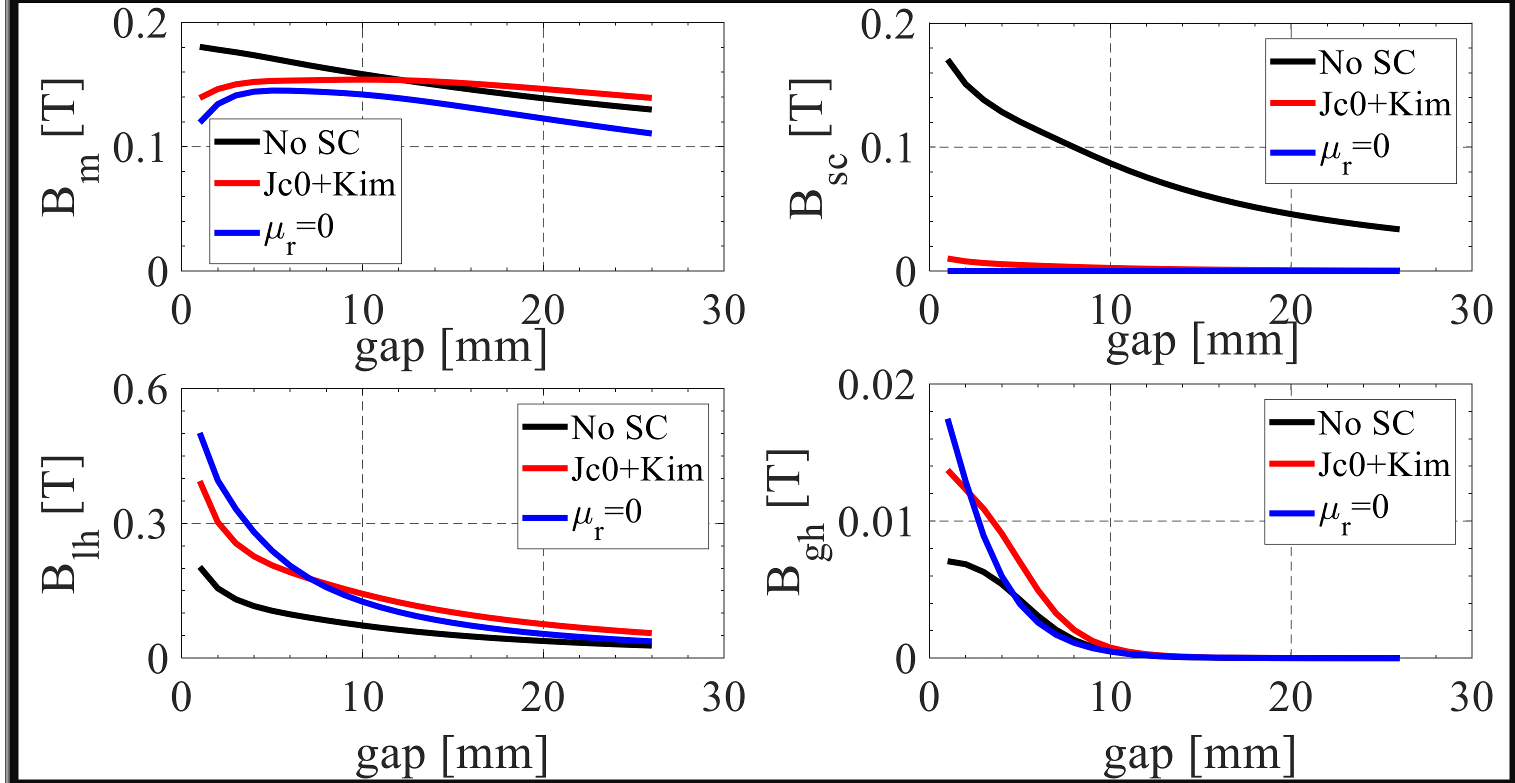
$$\begin{aligned}
&H_{sc_{av}} \\
&= \left[\sqrt{(H_0 + H_m)^2 - 2k(a - r_0)} - \sqrt{(H_0 + H_m)^2 - 2k(a - r_1)} + \sqrt{\left(H_0 + H_p\right)^2 - 2k(a - r_1)} - \sqrt{\left(H_0 + H_p\right)^2}\right]\frac{r_0}{a} + \\
&+ \left[\frac{\left[(H_0 + H_m)^2 - 2k(a - r_1)\right]^{3/2} - \left[(H_0 + H_m)^2 - 2k(a - r_0)\right]^{3/2}}{3k}\right]\frac{1}{a} + \\
&+ \left[-\sqrt{(H_0 + H_m)^2 - 2k(a - r_1)} + \sqrt{\left(H_0 + H_p\right)^2 - 2k(a - r_1)} - \sqrt{\left(H_0 + H_p\right)^2}\right]\frac{(r_1 - r_0)}{a} - \sqrt{\left(H_0 + H_p\right)^2}\,\frac{(a - r_1)}{a} + \\
&+ \left[-\frac{\left[\left(H_0 + H_p\right)^2\right]^{3/2} + \left[\left(H_0 + H_p\right)^2 - 2k(a - r_1)\right]^{3/2}}{3k}\right]\frac{1}{a}
\end{aligned}$$

J=Jc + Kim Model: Accuracy



HTS Levitation Circuit





J=Jc + Kim Model: Accuracy

τ	Error B_{av}
1 ms	18,9%
5 ms	15,8%
10 ms	14,5%
50 ms	11,6%
100 ms	10,3%

B/τ	Error B_{av}
1T / 1 ms	10,78%
1.5T / 15 ms	13,77%
2T /20 ms	14,08%
2.5T / 25 ms	12,81%