Electromagnetic Lumped Parameter Model of HTS Bulks in Magnetic circuits

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Motivation

• New High Temperature Superconductors (HTS):
  – Cooling of HTS became more cheap and accessible (more research):
    • Renew interest in design and optimization of new electrical machines with HTS bulks incorporated;

HTS Horizontal Levitation Bearing for High Speed Motors

In loco HTS Bulk Magnetization
When design an Electrical Machine with HTS bulks incorporated...

- Requires multiple physics and complex geometries
- HTS bulk physics -> non-linear models

• Simulations:
  - 3D: several days / 2D: several hours.
  - If using a multi-objective optimization tools (100 elements and 100 generations) it can take up to several months and years of simulation!

Analytical Models are still important! Is it possible to obtain a Lumped Parameter Model (LPM) for the HTS bulk with enough accuracy?
LPM requires an analytical solution!

\[ \nabla \times \vec{H} = \vec{j} \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{E} = E_0 \left( \frac{\vec{j}}{J_c} \right)^n \]

\[ J_c = \frac{k}{(B_0 + |B(t)|)} \]

No feasible analytical solution!
D.X. Chen and R.B. Goldfarb (1989) obtained a simplified analytical model:

\[ J = J_c \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \nabla \times \vec{H} = \vec{J} \]

\[ \vec{J}_c = \frac{k}{(B_0 + |B(t)|)} \]

It has an analytical solution!

(although, not sensible to \( \frac{\partial \vec{B}}{\partial t} \))
HTS Modelling: $J = J_c + \text{Kim Model}$

- Solution for an axisymmetric 2D problem:

\[ \nabla \times \vec{H} = \vec{J} \]

\[ J_c = \frac{k}{(H_0 + |H(t)|)} \]

\[ \frac{dH_{sc}}{dr} = -J_c = \frac{-\text{sgn}(J_{sc})k}{(H_0 + |H_p(t)|)} \]

\[ H_{sc} = -\text{sgn}(H_p)H_0 \pm \sqrt{H_0^2 - \text{sgn}(J_{sc}H_p)2k(r + c)} \]

\[ J_{sc} = -\text{sgn}(J_{sc})k/\sqrt{H_0^2 - \text{sgn}(J_{sc}H_p)2k(r + c)} \]
HTS Modelling: $J = J_c + \text{Kim Model}$

\[ r_0(t) = a - \frac{[(H_0 + H_p)^2 - H_o^2]}{2k} \]

\[ r_1(t) = a - \frac{[(H_0 + H_m)^2 - (H_0 + H_p)]}{4k} \]
J=Jc + Kim Model: Accuracy

\[ J = J_c + \text{Kim Model} \]

\[ J_c = \frac{J_{c_0}}{(B_0 + |B(t)|)} \]

FEA 2D Simulation

\[ \nabla \times \vec{H} = \vec{j} \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{E} = E_0 \left( \frac{\vec{j}}{J_{c_0}} \right)^n \]

GdBCO

\[ J_{c_0} = 1.8 \times 10^8 \text{ A/m}^2 \]

\[ n = 30 \]

\[ E_0 = 1 \times 10^{-4} \text{ V/m} \]

\[ B_0 = 1 \text{ T} \]
J=Jc + Kim Model: \textbf{Accuracy\\\\

B_p = 1.5T \quad | \quad \tau (time to peak) = variable

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\textbf{GdBCO}
Lumped Parameter Model
Lumped Parameter Model

- \( J = J_c + \text{Kim model} \)

\[
\begin{align*}
H_{sc} &= -\text{sgn}(H_p) H_0 \pm \sqrt{[H_0^2 - \text{sgn}(J_{sc} H_p) 2k(r + c)]} \\
J_{sc} &= -\text{sgn}(J_{sc}) k / \sqrt{[H_0^2 - \text{sgn}(J_{sc} H_p) 2k(r + c)]} \\
r_0 &= a - [(H_0 + H_p)^2 - H_0^2]/2k \\
r_1 &= a - [(H_0 + H_m)^2 - (H_0 + H_i)]/4k
\end{align*}
\]

- Representation by average values.
- Information about induced currents
Lumped Parameter Model

\[ \frac{dH_p}{dt} > 0 \]

\[ H_{sc} = \begin{cases} 
\int_{r_0}^{a} J_{sc0}(r) dr & 0 < r < r_0 \\
\int_{r}^{a} J_{sc0}(r) dr & r_0 < r < a 
\end{cases} \]

\[ r_0 = a - \frac{[(H_0 + H_p)^2 - H_0^2]}{2k} \]

\[ H_{scav} = \frac{1}{a} \int_{0}^{a} H_{sc}(r) dr = \left[ -\sqrt{(H_0 + H_p)^2} + \sqrt{(H_0 + H_p)^2 - 2k(a - r_0)} \right] \frac{r_0}{a} + \left[ -\sqrt{(H_0 + H_p)^2 (a - r_0)} + \frac{[(H_0 + H_p)^2]^{3/2} - [(H_0 + H_p)^2 - 2k(a - r_0)]^{3/2}}{3k} \right] \frac{1}{a} \]
Lumped Parameter Model

\[
\frac{dH_p}{dt} < 0
\]

\[
H_{sc} = \begin{cases} 
\int_{r_0}^{r_1} J_{sc0}(r)dr + \int_{r_1}^{a} J_{sc1}(r)dr & r < r_0 \\
\int_{r}^{r_1} J_{sc0}(r)dr + \int_{r_1}^{a} J_{sc1}(r)dr & r_0 < r < r_1 \\
\int_{r}^{a} J_{sc1}(r)dr & r_1 < r < a \end{cases}
\]

\[
H_{scav} = \frac{1}{a} \int_{0}^{a} H_{sc}(r) \, dr
\]

\[
r_0 = a - [(H_0 + H_p)^2 - H_0^2]/2k
\]

\[
r_1 = a - [(H_0 + H_m)^2 - (H_0 + H_p)]/4k
\]
Lumped Parameter Model

Φ_{sc} – Magnetic flux ($H_{scav}$ as function of $H_p$)

R_{sc} - Magnetic reluctance ($\mu_r = 1$)

HTS keeps trying to maintain its magnetic field unchanged by using superconductor currents.

“Inspired” in LPM of permanent magnets.

Has information about the HTS currents -> Power losses
HTS Levitation Circuit

HTS (3.3x2.5x1.4 cm)

PM (2.5x2.5x1 cm)
HTS Levitation Circuit

PM + HTS bulk

PM only

HTS only

\[ \mu = \mu_0 \]

\[ \phi_{\text{g}} \]

\[ \phi_{\text{m}} \]

\[ \phi_{\text{d}} \]

\[ \phi_{\text{l}} \]

\[ \phi_{\text{sc}} \]

\[ \phi_{\text{c}} \]

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Instituto Superior Técnico
HTS Levitation Circuit

1) **Air instead of HTS**: Distribution of flux due to **only PM**;
HTS Levitation Circuit

\[ \Phi_{sc} = R_{sc} \]

\[ \mu = \mu_0 \]

\[ \phi_g \]

\[ \phi_m \]

\[ \phi_{lv} \]

\[ \phi_{lh} \]

\[ \phi_{sc} \]

\[ B_{\text{air}} \]

\[ B_{s_{sc}} \] (YBCO)

\[ B_{s_{sc}} \] (GdBCO)

\[ \text{gap [mm]} \]
HTS Levitation Circuit

3) HTS Levitation Circuit

- Diagram of the HTS Levitation Circuit with various components labeled.
- Graph showing the relationship between gap [mm] and magnetic field strength $B_{sc}$ [T] for different cases:
  - No SC
  - $Jc0 + \text{Kim}$
  - $\mu_r = 0$

- Another graph illustrating the force $F_z$ [N] as a function of gap [mm] for:
  - LPM ($\mu_r = 0$)
  - 2D FEA ($\mu_r = 0$)
  - 2D FEA YBCO
  - LPM YBCO $\phi_{sc}$

(J = $Jc + \text{Kim}$ model)
Multi-Objective Optimization of the HTS and PM (Generic Algorithms):

- Maximize levitation force;
- Minimize materials required.

V=60 cm³

\[1.4\text{ cm} \quad 3.3\text{ cm} \quad \text{HTS}\]

\[1.2\text{ cm} \quad 2.5\text{ cm} \quad \text{PM} \quad \text{PM}\]

V=36 cm³

\[1.0\text{ cm} \quad 3.2\text{ cm} \quad \text{HTS}\]

\[1.0\text{ cm} \quad 2.0\text{ cm} \quad \text{PM} \quad \text{PM}\]

V=54 cm³

\[4.0\text{ cm} \quad 1.0\text{ cm} \quad \text{HTS}\]

\[1.7\text{ cm} \quad \text{PM} \quad \text{PM}\]

\[2.0\text{ cm} \quad \text{PM}\]
Multi-Objective Optimization of the HTS and PM (Generic Algorithms):

- Maximize levitation force
- Minimize materials required

Computation of Power Losses

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Optimized:

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Computation of Power Losses

\[ P_J = 9 \text{mW} \]
\[ P_J = 6.4 \text{mW} \]
\[ P_J = 6.4 \text{mW} \]
Conclusions

1. HTS electromagnetic lumped parameter model (LPM) can be done using Kim model and $J = J_c$;

2. It outperforms simple models as $\mu_r = 0$.

3. Can be used in optimization algorithms for electrical machines predesign.

4. Can be used to study the power losses inside the HTS bulk.
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Thank you
Lumped Parameter Model

- Other simplified LPM used in the pre-design of magnetic circuits:

\[ \mu_r = 0 \quad \mu_r = 0.2 \]

No information about induced currents!
HTS Modelling: $J = J_c + \text{Kim Model}$
Lumped Parameter Model

\[ \frac{dH_p}{dt} < 0 \]

\[ r_1 = a - \frac{[(H_0 + H_m)^2 - (H_0 + H_p)]}{4k} \]

\[ H_{scav} = \frac{1}{a} \left[ \sqrt{(H_0 + H_m)^2 - 2k(a - r_0)} - \sqrt{(H_0 + H_m)^2 - 2k(a - r_1)} + \sqrt{(H_0 + H_p)^2 - 2k(a - r_1)} - \sqrt{(H_0 + H_p)^2} \right] r_0 \]

\[ \frac{3k}{a} \left[ \left( \frac{(H_0 + H_m)^2 - 2k(a - r_1)}{3k} \right)^{3/2} - \left( \frac{(H_0 + H_m)^2 - 2k(a - r_0)}{3k} \right)^{3/2} \right] + \frac{r_1 - r_0}{a} - \frac{(a - r_1)}{a} + \frac{1}{a} \left[ \left( \frac{(H_0 + H_p)^2}{3k} \right)^{3/2} + \left( \frac{(H_0 + H_p)^2 - 2k(a - r_1)}{3k} \right)^{3/2} \right] \]
J=Jc + Kim Model: Accuracy
HTS Levitation Circuit
### J=Jc + Kim Model: Accuracy

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</tr>
<tr>
<td>2T / 20 ms</td>
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</tr>
<tr>
<td>2.5T / 25 ms</td>
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