Reduced Basis Method for electro-thermal behavior of an HTS

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Abstract

In recent years, a growing interest into superconductor modeling for research, energy and transport applications led to the development of efficient numerical methods. The electro-thermal behavior of high-temperature superconductor (HTS) tapes is one of the most interesting and critical aspects; commercial and home-made software implementations are paving the way for the use of superconductor modeling. However, the simulated quantities of interest (e.g. temperature) depend on the solution of parametric partial differential equations. The parameters, namely length or width of an HTS tape, thermal, electrical or mechanical properties of the tapes, can vary in a wide range. A sensitivity analysis for these parameters can therefore be computationally very demanding with those conventional software implementations. In this contribution, we discuss the possibility of using Reduced Order Models (ROM) to perform sensitivity analyses. These techniques enable accurate and rapid numerical simulations obtained by low-dimensional parametric models for various complex applications. In particular, we aim to apply a particular ROM (namely the Reduced Basis Method) to model thermal and electro-thermal behavior of a commercial HTS tape in case of quench.

Motivation: Flexibility and Efficacy

- **Reduced Basis (RB)** it's an example of computational reduction technique of parametrized Multiphysics problem
- When is interesting to evaluate a problem in **several scenario** one needs an accurate and rapid method.
  - Given ***input parameters*** we are interested in evaluate an ***output value*** *(e.g. T,B)* or into an ***objective function.***
- Example of parameters can be:
  - *Geometrical parameters* (i.e. thickness of the SC tape layers, width, length, etc)
  - *Properties/material parameters* (i.e. material resistivity, heat capacity, etc)

Facing with the difficulties to apply RB to Superconducting Modelling

- Among the difficulties of apply the RB to superconductors there are:
  - Highly *non-linear* dependence of the parameters
  - Dependence of the parameters by the solved variables, also called **non-affine problem** (e.g. $\rho = \rho(J,B,T)$)
- Examples of **Non-linear, Non-Affine** and **Time-dependent** problems can be found in [1][2]

A non-linear electro-thermal model for resistive High Fields Magnets [1]

- The design and optimization of these magnets require from an engineering point of view the prediction of "quantities of interest" as Temperature and Magnetic field .
- These quantities are expressed as functionals of physical properties such as heat transfer coefficients, water temperature and flowrate, and geometric variables.
- To evaluate these implicit input-output relationships, solutions of a multi-physics model involving electro-thermal, magnetostatics, electro-thermal-mechanical and thermo-hydraulics are requested. This model is non-linear as the material properties depend on temperature

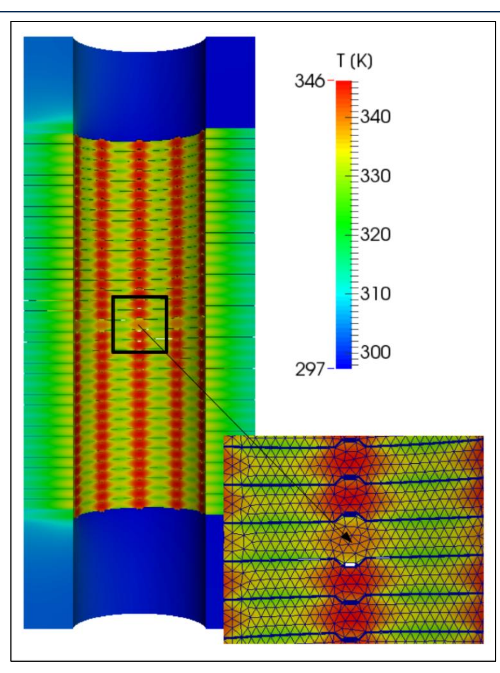
Coupled Electro-Thermal

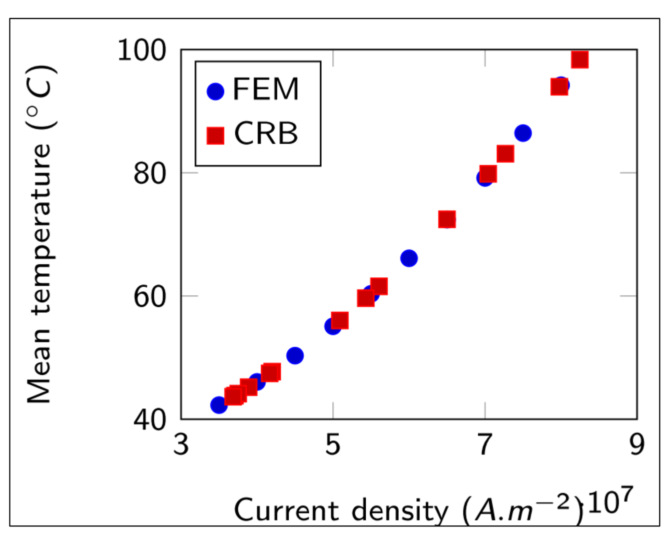
$$\begin{cases} -\vec{\nabla}\cdot\left(k(T)\vec{\nabla}T\right) = \sigma(T)\vec{\nabla}V\cdot\vec{\nabla}V \\ -\vec{\nabla}\cdot\left(\sigma(T)\vec{\nabla}V\right) = 0 \end{cases}$$

Application to a Polyhelix Magnet [1]

Example of Parameters (inputs)	
Electrical Conductivity	$\sigma(T)$
Thermal Conductivity	K(T)
Applied Potential	Vd
Heat transfer coeff	h

Control Quantities (outputs)	
Temperature	$T(k(T),\sigma(T))$
Potential	$V(k(T),\sigma(T))$





- The aim is to monitor the temperature to find the max $\vec{J}$ without thermal damages
- The acceleration factor (ration of the time) is about 17.5

	HF (16 cores)	*RB* (any core)
Time[min]	~35	~2

FEM: High fidelity and Reduced Basis

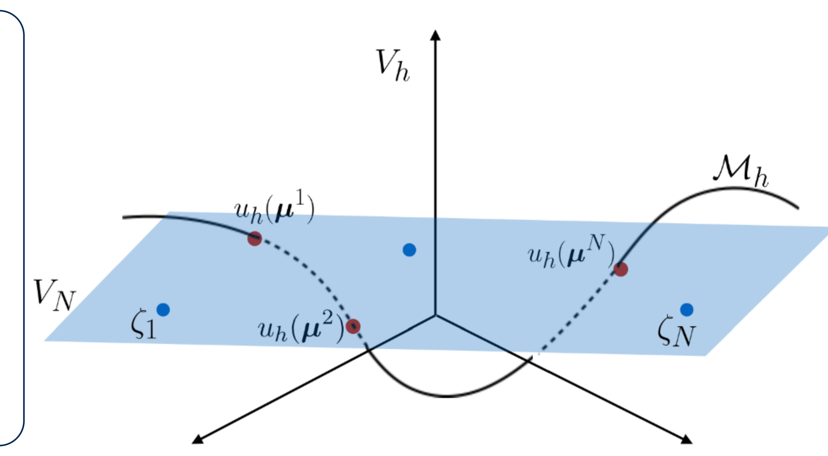
e.g. Elliptic Galerkin Problem

For $\mu \in P$,

Find $u_h(\mu) \in V_h \subset V : a(u_h(\mu), v_h, \mu) = F(v_h) \quad \forall v_h \in V_h$

Denoting a basis of V_h with $\{\varphi_j, j = 1,2,\dots,N_h\}$ *we have* $A\underline{u} = \underline{f}$

A is the stiffness matrix where $a_{ij} = a(\varphi_j, \varphi_i)$,
$\underline{f}$ vector with components $f_j = F(\varphi_j)$



e.g. Elliptic Galerkin Projection

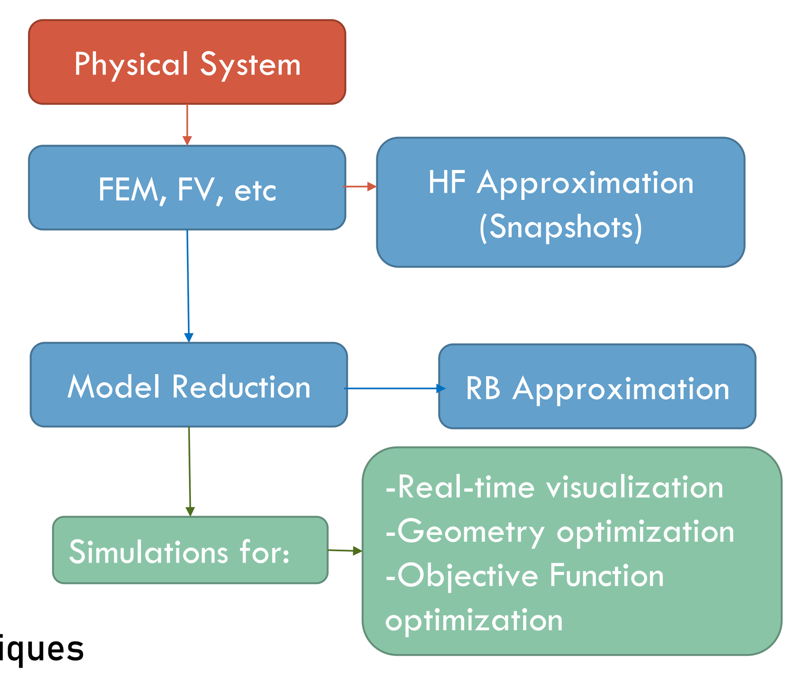
For $\mu \in P$,

Find $u_h(\mu) \in V_N \subset V_h : a(u_N(\mu), v_N, \mu) = F(v_N) \quad \forall v_N \in V_N$

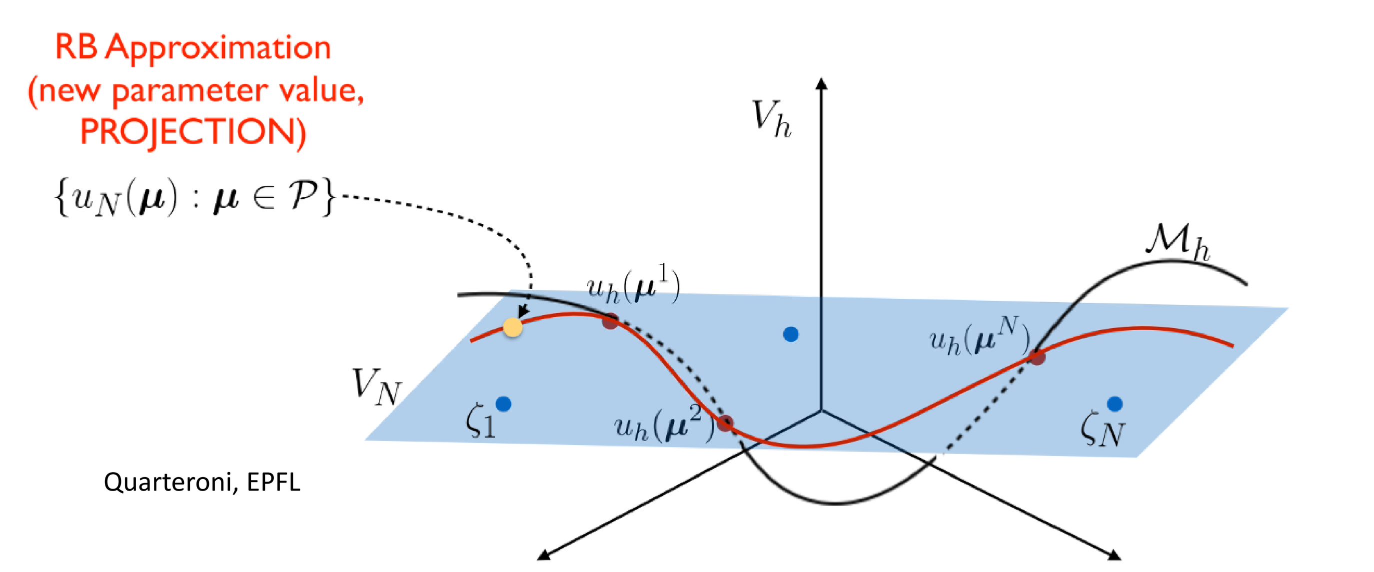
Offline Snapshots computed with HF techniques

On the Snapshots are applied algorithm(Greedy or POD) in order to get the RB basis

RB space functions $\rightarrow V_N = span\{\zeta_1, \zeta_2, \dots, \zeta_N\} \subset V_h$



1. The problem is solved with high fidelity techniques
   - **OFFLINE phase** – *snapshot* calculation
2. Set of basis solution of lower dimension
   - **RB construction** with Proper Orthogonal Decomposition (POD)
3. Projection of the problem on RB space and solving
   - **ONLINE phase** – Solving the problem for various parameter/geometries

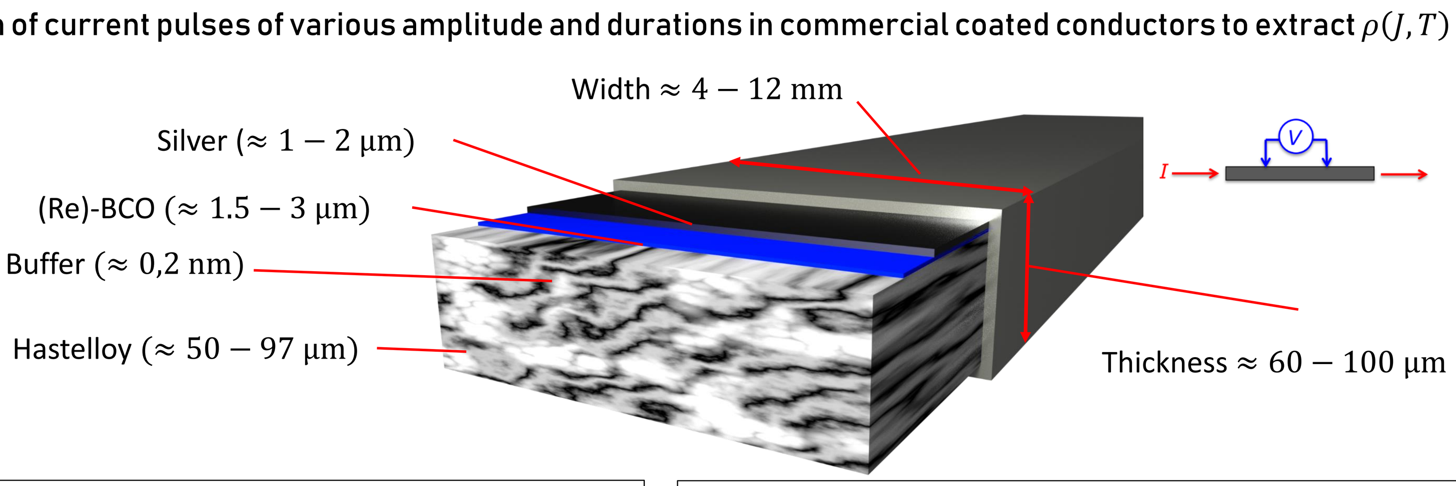


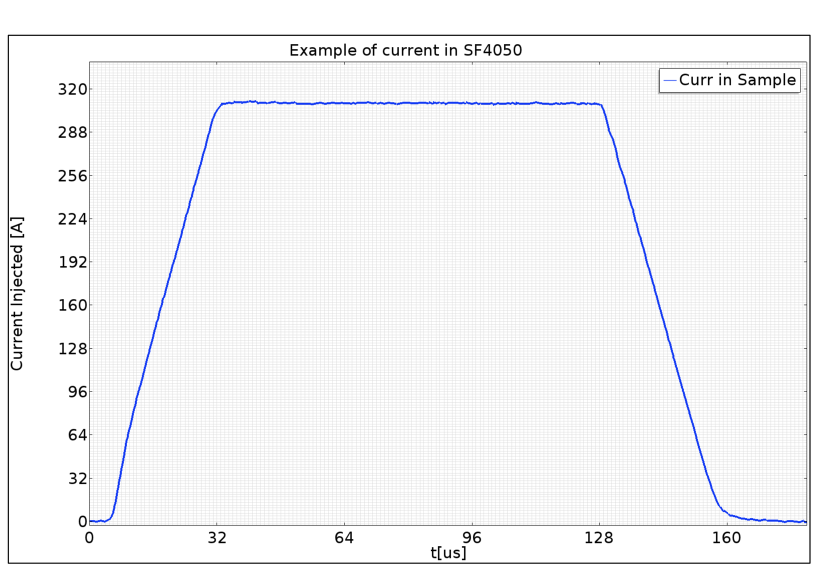
Quarteroni, EPFL

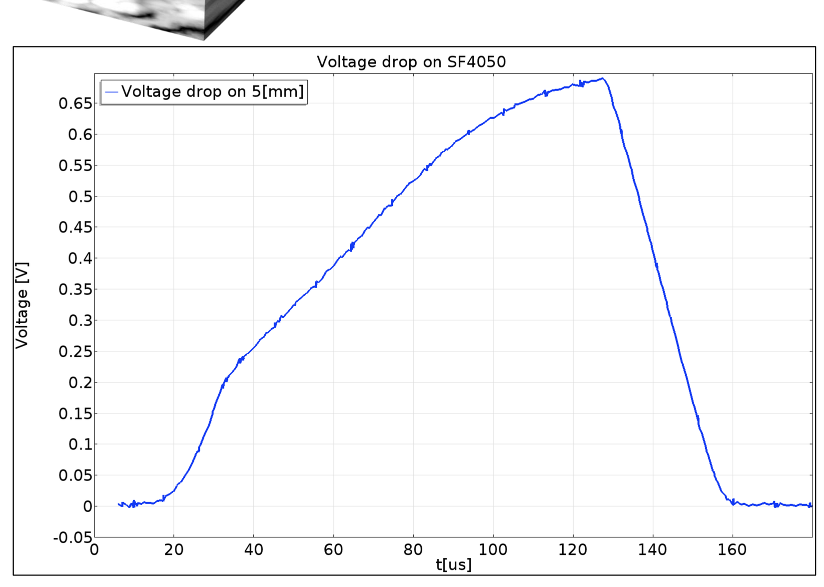
First Application on a Case Study: Thermal Model for YBCO Resistivity

Experiment: Resistivity Measurements through pulsed current measurements

- Injection of current pulses of various amplitude and durations in commercial coated conductors to extract $\rho(J,T)$







- Calculate the resistivity is a difficult task:
  - Complex current interlayer sharing
  - Violent heating, despite the pulsed current, give non-isothermal measurements
  - FEM modelling is necessary, but the iterative and parametric simulations carried out might be time consuming

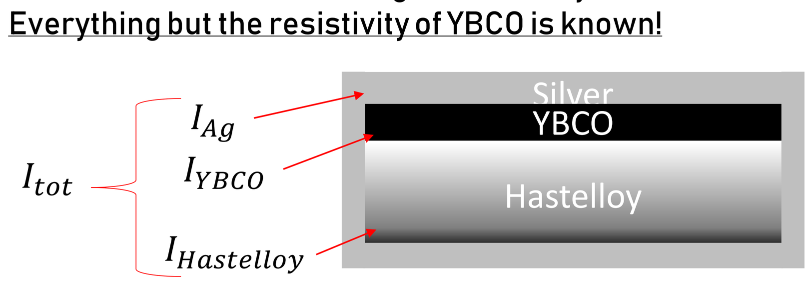
Simulation: Resistivity through pulsed current measurements

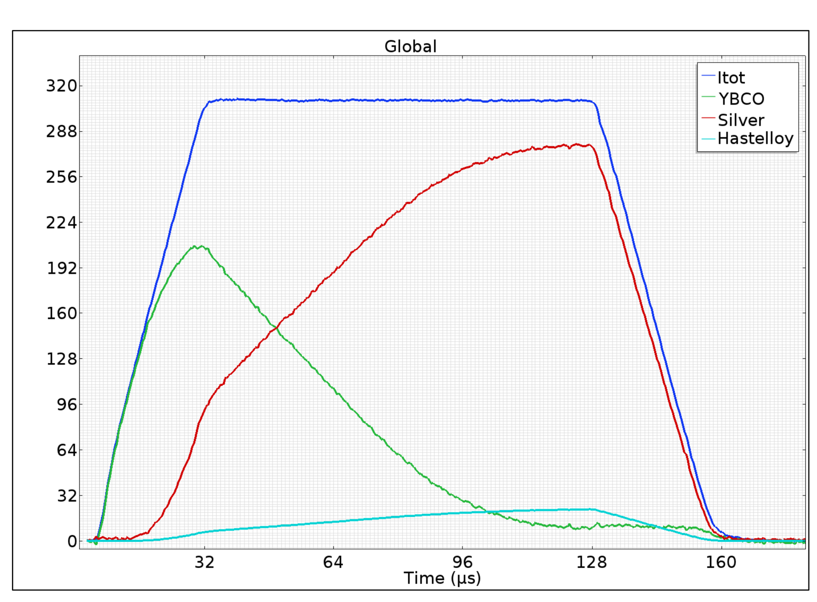
- A 2D (cross section) *purely thermal + current sharing current model* is solved to retrieve the averaged resistivity of the YBCO from the pulses.
- Everything but the resistivity of YBCO is known!

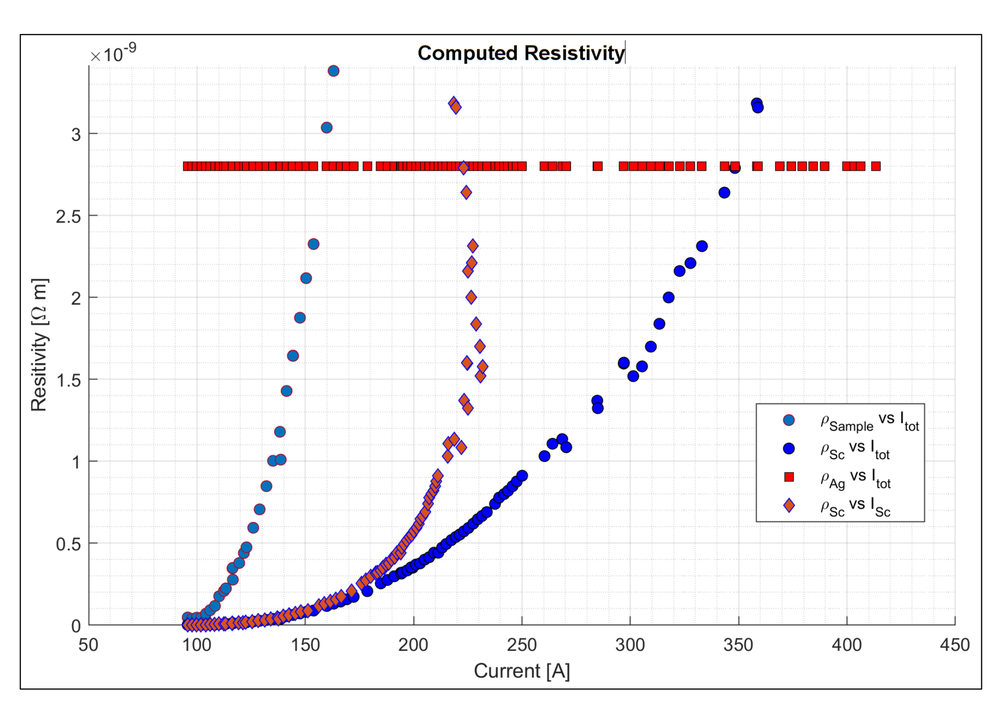
Thermal + Current Sharing

$$\rho_m(T)C_p(T)\frac{\partial T}{\partial t} + \vec{\nabla}\cdot\left(-k(T)\vec{\nabla}T\right) = \vec{J}\cdot\vec{E}$$

$$|E(t)| = \frac{V_{meas}(t)}{L} \qquad |J(t)| = \frac{|E(t)|}{\rho(T(t))}$$







- The analysis of each pulse requires several iterations, therefore reduce the overall time of the simulation can benefit the analysis of large dataset.
- Using eigenvalues obtained by the thermal problem described above, COMSOL with MatLab LiveLink is able to solve *reduced model* and apply time dependent loads as the injected current. (Modal Solver node)
- In the table below, an example of ROM applied in COMSOL on one pulse

$$\rho_{YBCO}(T(t)) = \langle E(t) \rangle \cdot \frac{A_{YBCO}}{I_{YBCO}(T(t))}$$

	HF (single run)	*RB* (single run)	*HF* (98 run)	*HF* (98 run)
Time[s]	~225	~80	~$33 \cdot 10^3$	~$11 \cdot 10^3$

	ROM
Acc. Factor	~2.81

Conclusions and Next Steps

- The results shown above are for a purely thermal model. The tool provided is useful, despite not strictly necessary for such an application.
- The real benefit for the superconducting modelling, shall be the implementation of the RB for Electro-Thermal/Electro-Magneto-Thermal problems.
- Nevertheless, these formulation contain coupled parametrized PDEs and non-affine parameters. In order to apply the RB to such a problems, the mathematical tools are very complex…
  - Next steps will be:
    - Study the feasibility of an implementation of electro-thermal problems in RB [1]
    - Develop a COMSOL-MatLab electro-thermal model with simple linear resistivity
    - Develop a COMSOL-MatLab electro-thermal model with power law resistivity

References

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3. F. Sirois, B. Dutoit et at, *Flux flow characteristics of 2G HTS coated conductors* measured with microsecond-range pulsed currents
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