Topological two-body states in SSH chains with interactions

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Summary

The Su-Schrieffer-Heeger (SSH) model, for its simplicity, has often been regarded as an archetype of topological insulators. However, electronic interactions introduce complexity in its behavior and, for arbitrary filling, little is known about the topological characterization of its spectrum.

Here we study half-filled SSH open chains with nearest-neighbor (NN) interactions [1] and show that:

- For one-hole excitations, the presence of interactions translates into a passivation potential at the edge sites which induces phase transitions between topologically different regimes.
- For two-hole excitations, a mapping can be constructed, in the large interaction limit, of two-hole SSH eigensubspaces into one-particle states of a non-interacting 1D tight-binding model, with interfaces between different regions.

Particle-hole transformation

The Hamiltonian of an open SSH chain with NN interactions is given by

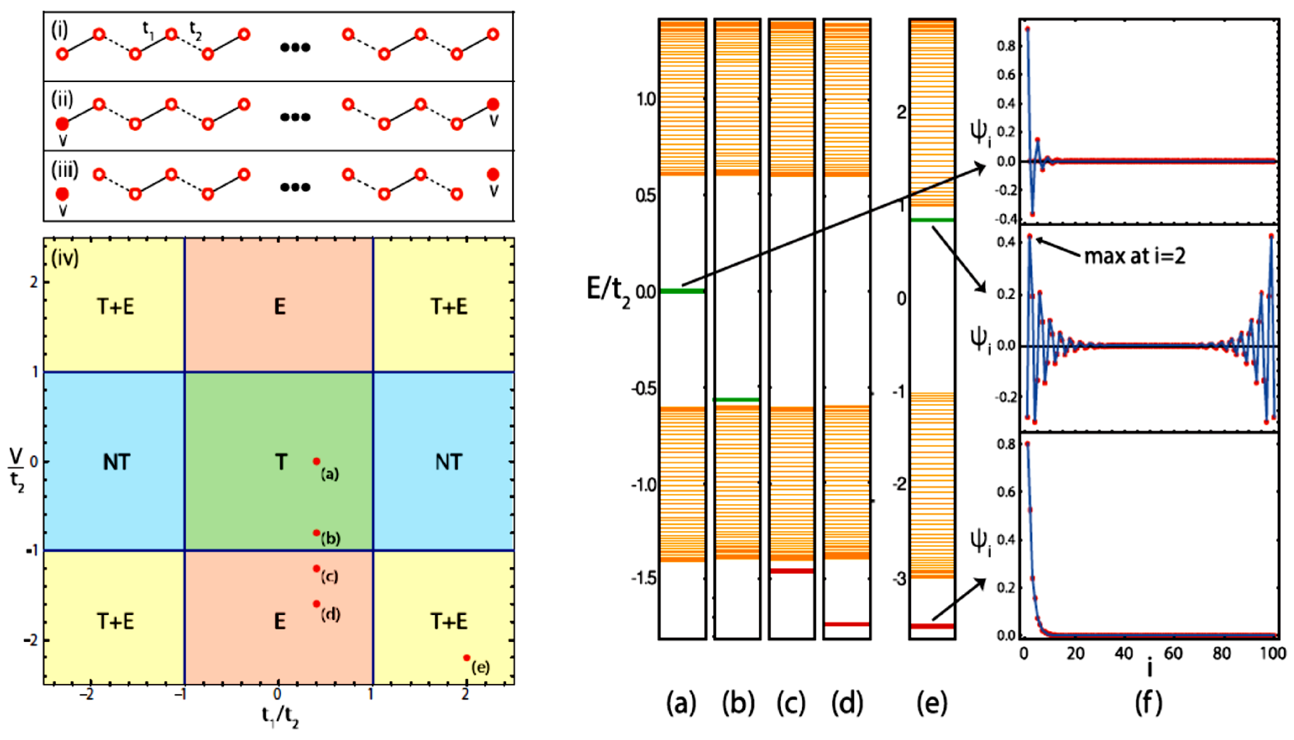
$$H = -t_1 \sum_{j=1}^{N/2} (c_{2j-1}^\dagger c_{2j} + H.c.) - t_2 \sum_{j=1}^{N/2-1} (c_{2j}^\dagger c_{2j+1} + H.c.) + V \sum_{j=1}^{N-1} n_j n_{j+1} \, . \tag{1}$$

For one-hole excitations in a half-filled chain, we perform a particle-hole transformation, $c_j^\dagger (c_j) \to h_j (h_j^\dagger)$, to arrive at

$$H = t_1 \sum_{j=1}^{N/2} (h_{2j-1}^\dagger h_{2j} + H.c.) + t_2 \sum_{j=1}^{N/2-1} (h_{2j}^\dagger h_{2j+1} + H.c.) + V(n_1^h + n_N^h) \, , \tag{2}$$

where the last term shows the conversion of the NN interactions into an impurity (passivation) potential at the edge sites

One-hole states



(i-iii) As V is turned on, impurity potentials are ascribed to the edges and, for $V \to \infty$, the edge sites become independent of the smaller inner chain. (iv) V/t_2 vs. t_1/t_2 topological phase diagram for one-hole states in an infinite open chain. Regions T (NT) and E indicate the presence (absence) of topological or impurity edge states in the energy spectrum, respectively.

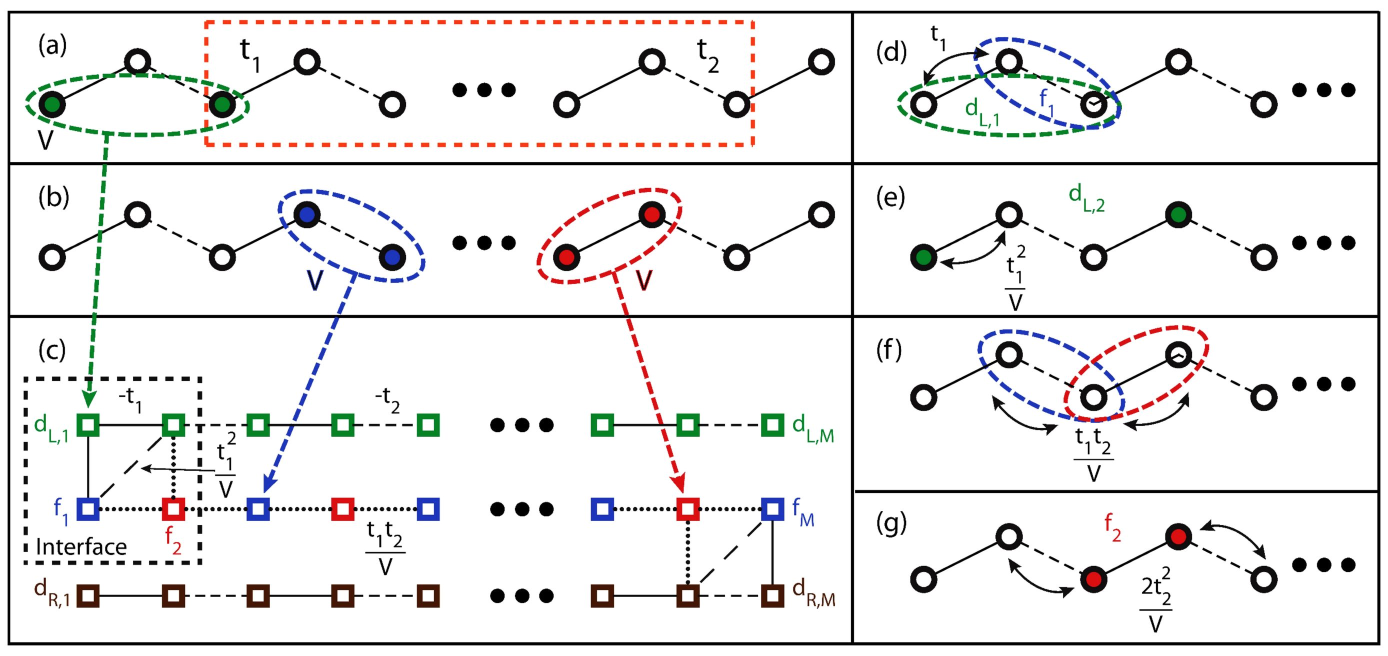
(a-e) Energy spectrum for the corresponding labeled points in (iv). Green (red) levels indicate topological (impurity) states. (f) Normalized spatial distribution of the states indicated by the arrows for a chain with N = 100 sites.

References

[1] A. M. Marques and R. G. Dias, first version available at arXiv:1610.04510 (2016). Submitted.

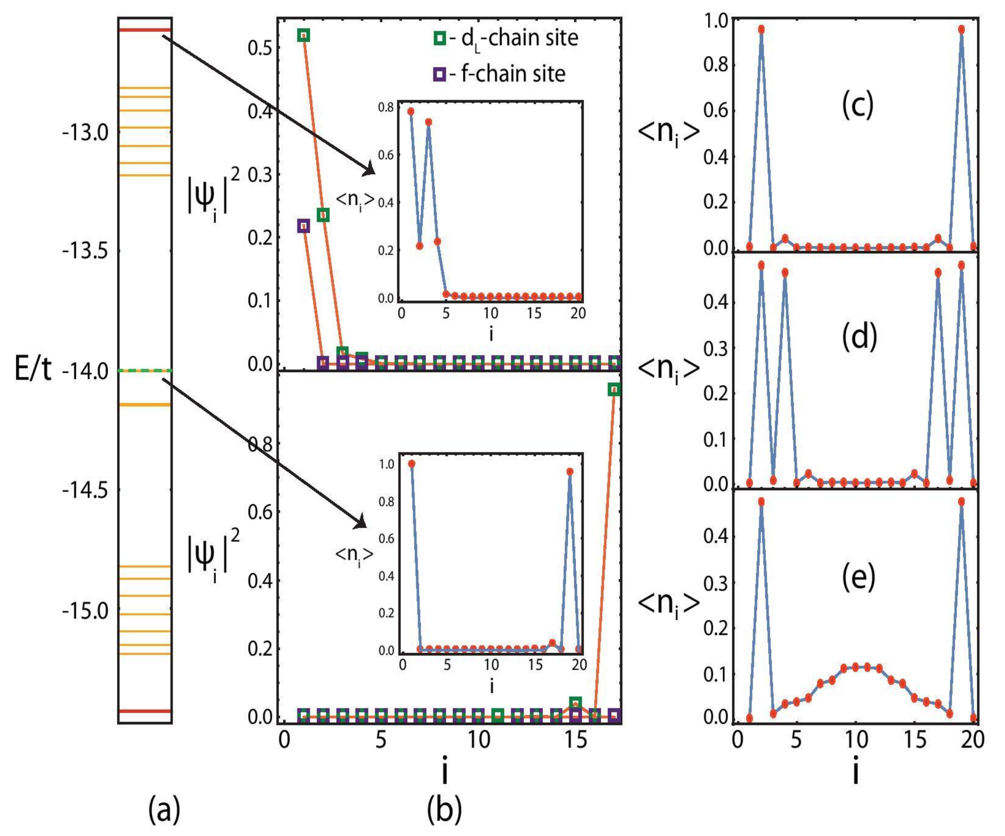
Two hole states for $V \gg t_1, t_2$

We define the states with two holes located at non-consecutive bulk sites as the new zero of potential subspace. Then, in the V potential subspace, we find two types of states that become, using 2nd-order perturbation theory, the sites of an equivalent chain that can harbor states of topological origin.



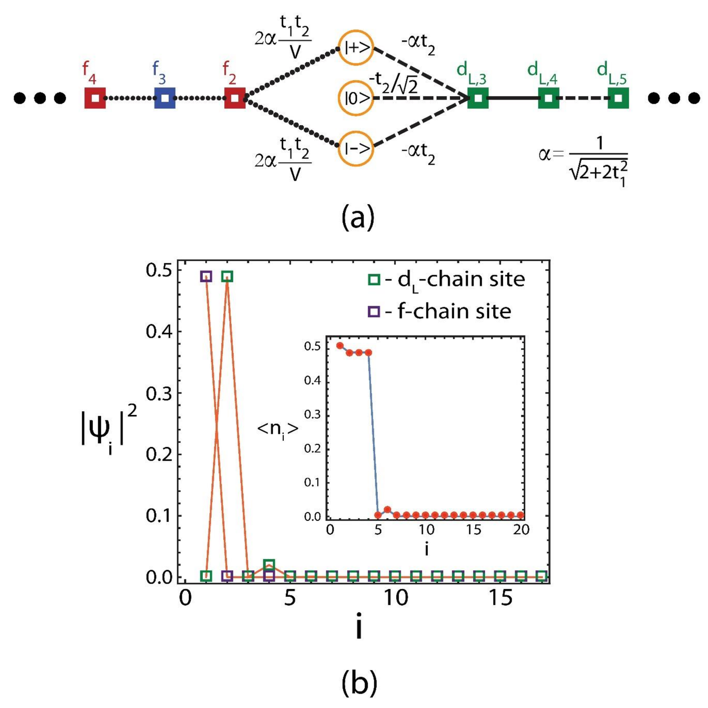
(a) One hole localized at the left edge and the other at any of the sites within the orange dashed box. (b) Two examples, with different inner hoppings, of two-hole states with the holes localized at adjacent bulk sites. (c) The two-hole states in (a) and (b) are translated as sites in an equivalent chain, divided in a d_L-chain, a f-chain and a d_R-chain, where particle creation corresponds, in the original chain, to $d_{L(R),i}^\dagger \to h_{i+2}^\dagger h_{1(N)}^\dagger$ and $f_i^\dagger \to h_{i+2}^\dagger h_{i+1}^\dagger$, with $i = 1,2,\ldots,M$ and $M = N-3$. (d-g) Illustration of some hopping and on-site potential constants at given sites of the equivalent chain.

SSH chain ↔ Equivalent chain correspondence for $N = 20$.



(a) Energy spectrum for two-hole states for $(V, t_1, t_2) \to t(-14,1,0,2)$. Red and dashed green levels represent, respectively, impurity-like and topologically originated states. (b) Probability amplitude squared at the sites of the equivalent chain for the states indicated by the arrows. The insets show how they translate in terms of mean occupation in the original SSH chain, bearing an almost exact agreement. (c)-(e) Examples of other two hole states of topological origin in the zero potential subspace.

Renormalized interface for $V \gg t_1 \gg t_2$



(a) For $t_2 \ll t_1 \ll V$, the diagonalization of the Hamiltonian written in the $\{|f_1>, |d_{L,1}>, |d_{L,2}>\}$ basis yields three different states: $|0> = \frac{1}{\sqrt{2}}(-1,0,1)$ and $|\pm> \approx \frac{1}{\sqrt{2+2t_1^2}}(1, \pm\sqrt{2t_1}, 1)$, with energies $E_0 = 0$ and $E_\pm \approx \frac{t_1^2}{2V} \pm \sqrt{2t_1}$, considering $t_1 \approx 1$. The hoppings form these new "sites" to the f and d_L chains become renormalized. (b) The same as in (b) of the above figure, but for the state of topological origin with a large weight on $|0>$.

Conclusions

We have studied the effects of NN interactions in the topological characterization of hole excitations in half-filled open SSH chains. Our main findings were:

- For one-hole excitations, the edge sites effectively decouple from the rest of the chain for high enough V. In the smaller inner chain the topological nature is reversed (edge bonds switch from t_1 to t_2).
- In the $V \gg t_1, t_2$ limit, the two-hole states of a given eigensubspace can be translated as one-particle states in a non-interacting tight-binding chain which, in the case studied, has non-trivial topology. Under this mapping, we propose a topological characterization of two-hole states in interacting SSH chains based on their corresponding one-particle states in the equivalent chain.

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