EPCO 2021

Portuguese Meeting on Optimal Control

NOVA School of Science and Technology

July 28–29, 2021

Almada, Portugal

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Main Talks

Mean-field BSDEs with jumps and global risk measures

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We study mean-field Backward Stochastic Differential Equations with jumps, when the drift contains a generalized mean-field operator that can capture higher order interactions. Indeed, in many applications (and specifically in interacting systems), it may be desirable to incorporate the intensity of system interactions as well, and not only the average state. We interpret the BSDE solution as a dynamic risk measure for a representative bank whose risk attitude is influenced by the system. This influence can come in a wide class of choices, including the average system state or average intensity of system interactions. We provide convergence results of finite interacting systems to the limit mean-field BSDE. We state properties for the mean-field BSDE, in particular a strict comparison theorem which is used to verify the no arbitrage condition of our global risk measure. Furthermore, using Fenchel-Legendre transforms, we prove a dual representation for the expectation of the associated global risk measure.

An interior point approach for risk-averse PDE-constrained optimization with coherent risk measures

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The prevalence of uncertainty in models of engineering and the natural sciences necessitate the inclusion of random parameters in the underlying partial differential equations (PDEs). The resulting decision problems governed by the solution of such random PDEs are infinite dimensional stochastic optimization problems. In order to obtain risk-averse optimal decisions in the face of such uncertainty, it is common to employ risk measures in the objective function. This leads to risk-averse PDE-constrained optimization problems. We propose a method for solving such problems in which the risk measures are convex combinations of the mean and conditional value-at-risk (CVaR). Since these risk measures can be evaluated by solving a related inequality-constrained optimization problem, we propose a log-barrier technique to approximate the risk measure, which leads to a new continuously differentiable convex risk measure: the log-barrier risk measure. We prove consistency of the approximation via a variational convergence technique. Using the differentiability of the log-barrier risk measure, we derive first-order optimality conditions reminiscent of interior point approaches in nonlinear programming. We study the associated Newton systems in full and reduced form and provide a sufficient condition for local superlinear convergence in the continuous setting. For the discretization of the problem, we employ novel low-rank tensor methods. The presentation concludes with numerical examples.

Conference Abstracts

Ensuring stability and safety by combining optimization and nonlinear control tools

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In this talk I will address the control design problem for critical control systems. In particular, the presentation will focus on combining optimization, optimal control and nonlinear Lyapunov based tools that results in powerful frameworks with formal guarantees of robustness, stability, performance, and safety. Illustrative examples in the area of motion control of autonomous robotic vehicles will be presented.

Fractional COVID-19 model with vaccination

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In the end of 2019, the World Health Organization (WHO) reported a novel coronavirus in China, which causes severe damage to the respiratory system. The virus was first found in Wuhan city, and was named as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) [2]. The epidemic began in the city of Wuhan, China in late December 2019 and quickly spread around the world. The COVID-19 pandemic is the world's biggest health threat. At the time of last revision, according to a WHO report released on April 04, 2021, the cumulative number of confirmed infected cases have risen to 131,441,030 with 2,860,578 deaths.

To control infectious diseases, proactive steps are needed, especially for diseases that have a vaccine and treatment. With regard to COVID-19, many vaccines are starting to become available [6], but they represent an economic burden for most governments in the world, so one of the important goals is to know how much you need to spend on vaccination to reduce the number of susceptible and infected, while maximizing the number of recovered population from COVID-19, and in the same time, saving the vaccination program cost.

Fractional calculus can be used, among several possibilities, to deal with the memory effect in mathematical modeling. Memory in mathematical modeling makes important what happened in the past to explain the present [3]. In [4], a system of fractional differential equations is used to study the effect of memory on epidemic evolution. Due to the fact that this disease is little known and underreported, parameters such as speed of propagation and recovery and contact rates are difficult to estimate. Consequently, its modeling using classical differential equations can be inappropriate to represent the dynamics of the populations involved. Thus, it is possible to adjust the order of the differential equation to the real data of the spread of the disease [1, 5]. This is why we proposed in this work a SEIHR model which takes into account the rate of infected individuals hospitalized (H), this model considered as a generalized SEIR epidemic model, to describe the spread of COVID-19 pandemic. Next, we investigate the effect of vaccination on the spread of this pandemic. We discuss the necessary optimality conditions for a general fractional optimal control problem described in the Caputo sense derivative. Our goal is to minimize the number of infected population and the cost of the vaccination strategy, while maximizing the number of recovered population from SARS-CoV-2 virus. Using the fractional version of Pontryagin's maximum principle, we characterize the optimal levels of the proposed control. The resulting optimality system is solved numerically in the Matlab numerical computing environment.

The rest of the paper is arranged as follows. In Section 1, we present our fractional mathematical model. Section 2 recalls the fundamental definitions and the main result of fractional optimal control. In Section 3, we derive an optimal control problem consisting of a cost function to be minimized and the fractional model described by fractional differential equations. While numerical results are discussed in Section 4. We finish with some remarks and conclusion, in Section 5.

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Optimal Market Making Models with Stochastic Volatility for High-Frequency Trading

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In this study, we develop optimal market making strategies for high-frequency trading with the help of stochastic control theory. We first assume that the underlying asset follows the Heston stochastic volatility model including jump parts in price dynamics to see the effect of the arrival of the market orders. We use two types of utility functions in order to arrive at the goal of the market maker which is maximizing the terminal wealth: quadratic and exponential with a risk aversion degree. Then, we set up a model with the stochastic volatility stock price dynamics where the jump parts occur in the volatility. We obtain the optimal prices for both models by writing and solving the related Hamilton-Jacobi-Bellman (HJB) equations. For the numerical solutions, we apply finite differences and linear interpolation as well as extrapolation methods on the HJB equations. In the applications, we show the risk metrics of the models such as profit and loss distribution (PnL), standard deviation of PnL and Sharpe ratio to make decisions on the strategies. Then, we examine the behaviour of the optimal prices for each inventory level of the trader. Moreover, we compare our strategies with the existing ones in the literature. Lastly, we apply our models on a real high-frequency data of an emerging market and explore the results.

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Riemannian cubics close to geodesics at the boundaries

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Riemanian cubics are the critical points c of the bienergy functional J defined by

$$J(c) = \int_0^T < \frac{D^2 c}{dt^2}, \frac{D^2 c}{dt^2} > dt.$$

These curves are applied to concrete problems such as path planning of the rigid body in robotics or spacecraft control in space science. In these applications, the curves are required to satisfy the boundary conditions on position and velocity given by

$$c(0) = p, \quad \frac{dc}{dt}(0) = v, \quad c(T) = q, \quad \frac{dc}{dt}(T) = w.$$

In this talk we present local existence and uniqueness conditions for Riemannian cubics given by the boundary data (p, v) and (q, w). To this end we consider a correspondence between initial and boundary data defined by the biexponential map. This approach has already been presented for boundary data with sufficiently short length in [1]. An extension to more general data is explored in [2] by considering the boundary data in a neighborhood of geodesic boundary data.

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[1]This work was partially supported by the Centre for Mathematics of the University of Coimbra – UIDB/00324/2020, funded by the Portuguese Government through FCT/MCTES.

[2]F. Silva Leite acknowledges Fundação para a Ciência e Tecnologia (FCT-Portugal) and COMPETE 2020 program for the financial support to the project UIDB/00048/2020.

Optimal Control of a District Heating Substation

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The design and control of a district heating substation has an important impact on the energy exchange between the thermal grid and the building heating system. A properly controlled energy exchange will improve the economy of the entire system while minimizing the pollutant emissions and the fossil fuel consumption; the main goal of the European energy plan till 2030 [1]. A substation typically consists of a plate heat exchanger equipped with pumps and control valves that connect the district heating primary side with the secondary side i.e, the building heating system. The temperature of the secondary side is controlled by regulating the flow rate of water at the primary side of the heat exchanger. As a result, an optimal heat transfer from the primary side of the network to the secondary side can be achieved [2, 5]. Existing methodologies are capable to provide steady state form of the secondary supply temperature but they cannot provide the desired low return temperature from the substation to the network. However, achieving the low return temperature from the substation is also an important aspect of the district heating system as it can reduce the operational cost by $10-15\%$ of fuel by saving fuel at the production side [4]. Hence, in this research work, a model-based control methodology is employed to achieve low primary return temperature while satisfying the desired comfort levels in the building. In the designed approach, first, a control-oriented mathematical model of a plate heat exchanger is developed, since the existing mathematical model of a plate heat exchanger consists of nonlinear partial differential equations (PDEs) and such model cannot be used directly in model-based control framework due to its complexity and computational inefficiency [2]. Thus, nonlinear PDEs are approximated by using a finite difference method to obtain control-oriented nonlinear ordinary differential equations (ODEs). The ODE model is validated with the experimental data given in [3] and then implemented with the model predictive control (MPC). The low return temperature from the substation is considered as a constraint in the MPC framework. Furthermore, the constraint on input flow rate of water through the substation is also added to reduce the pumping cost of the network. The designed MPC will provide the desired supply temperature of the secondary side while satisfying all the above mentioned constraints and yields less operating cost. The proposed methodology is also compared with an existing proportional integral (PI) controller. The comparison between the designed MPC and PI controller shows that MPC performs better against the variations of input flow rate and

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temperature.

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Geometric Optimal Trajectory Tracking of Nonholonomic Mechanical Systems

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For trajectory tracking, the usual approach of stabilization of error dynamics cannot be utilized for nonholonomic systems. This is because there does not exist a C^1 (even continuous) state feedback which can stabilize the trajectory of a nonholonomic system about a desired equilibrium point. Existence of such feedback would violate Brockett's necessary condition, which states that for any system of the form $\dot{x} = f(x, u)$ the image of the map $(x, u) \mapsto f(x, u)$ must contain a neighborhood of zero.

In this talk I will present a recent method proposed in [1] for tracking of a trajectory for a nonholonomic system by recasting the problem as a constrained optimal control problem. The cost function is chosen to minimize the error in positions and velocities between the trajectory of a nonholonomic system and the desired reference trajectory, both evolving on the distribution which defines the nonholonomic constraints. The problem is studied from a geometric framework in terms of the Levi-Civita connection associated with a Riemannian metric projected over the nonholonomic distribution. Optimality conditions are determined by the Pontryagin Maximum Principle and also from a variational point of view, which allows the construction of geometric integrators. Examples and numerical simulations will be shown to validate the results.

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Adaptive Optimal Control with accessible disturbances[5]

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This work considers a situation in which the dynamics of the plant to be controlled is unknown, but there is available a state-feedback gain that yields a reasonable performance in closed-loop. at least stabilizing the plant. It is assumed that the state is available for direct measure. In order to adapt the controller gain such as to improve the performance, and making it approach the optimal one, a possibility is to use a combination of approximate dynamic programming (ADP) and reinforcement learning (RL). ADP propagates forward in time an approximation of the optimal value function $V(x)$, with x the state, by considering the state trajectories being followed with the current control policy. This value function approximation is then used as a measure of performance, maximized by a RL strategy [5]. In order to avoid a causality problem associated with the computation of the optimal control estimate u from the approximate value function, a function $Q(x, u)$ is used instead of $V(x)$. This so called Q-function is propagated in time according to a Bellman-like equation and allows to compute the optimal control by maximizing it, resulting in a, possibly nonlinear, state feedback policy $u = \pi(x)$. Furthermore, $V(x) = Q(x, \pi(x))$.

Although it can be applied to nonlinear plants, for linear plants with quadratic costs, this approach yields a linear state feedback that approximates the solution of the corresponding linear quadratic (LQ) problem, without requiring the availability of the plant model. It has been proved [2] that, under a persistency of excitation condition, the adaptive control policy converges to the optimal LQ control when the Q-function is approximated by a linear combination of quadratic basis functions made of second order polynomials in the components of the current state and control variable. In [3] the above Q-learning regulation algorithm has been extended to the tracking problem, by following an approach in which the plant dynamics is extended with the state of a system that generates the reference to track, even when this one is unstable.

This presentation extends the algorithm to adaptive feedback/feedforward controllers for plants with accessible disturbances, in which the disturbance is assumed to be modelled by a, possibly unstable, model.

[5]This work was performed within the framework of the project HARMONY, Distributed Optimal Control for Cyber-Physical Systems Applications, financed by FCT under contract AAC n2/SAICT/2017 - 031411, project IMPROVE - POCI-01-0145-FEDER-031823.

[6]The work of Rita Cunha was also supported by pluriannual LARSyS funding UIDB/50009/2020.

[7]The work of João Miranda Lemos was also supported by pluriannual INESC-ID funding UIDB/50021/2020.

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Model Predictive Control Scheme to Improve Performance of a Guidance Logic Controller for Trajectory Tracking

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This work addresses the problem of trajectory-tracking and path-following control for nonholonomic vehicles. We propose a Model Predictive Control (MPC) scheme that is used to improve the performance and enlarge the domain of attraction of an existing path-following guidance method, while maintaining its stability guarantees.

The trajectory-tracking and path-following control for nonholonomic vehicles is relevant within mobile robotics, using wheeled ground robots, unmanned surface vehicles, autonomous underwater vehicles, fixed-wing and multirotor unmanned aerial vehicles (UAV's), among others. The problem is particularly suited to be addressed by MPC since nonholonomic systems require some degree of planning to be properly controlled, and frequently have limits on the inputs, such as maximum angles of the actuators. However, the high nonlinearity of such systems, allied to fast dynamics and to robustness requirements, poses some challenges to the use of optimizer based techniques. A compromise is to use an established feedback law as the basis controller and add a convenient MPC controller to it.

We use as basis a path-following guidance control law, which was shown to have interesting performance characteristics and local stabilizing properties [1][2].

On top of such control law, we add a term computed within an MPC algorithm. The overall controller can only improve performance with respect to the basis controller. The objective functions for the optimal control problems involved in the MPC scheme are the Lyapunov control pair of functions that was used to verify the stability of the guidance control law. Therefore, it is guaranteed that the stabilizing properties are preserved with the added MPC term.

The use of MPC on top of an already adequate stabilizing controller has been reported in the literature in [3] and [4], on the control of an inverted pendulum and of a nonholonomic mobile robot, respectively. This complementary use of MPC brings about several advantages:

1. The stability of the MPC strategy can be guaranteed, by means of a constructive procedure to select stabilizing MPC design parameters;

2. The performance of the combined controller can only improve upon the basis solution;
3. The optimization problems within MPC strategy are numerically solved in a very efficient way, since the process already starts from a feasible solution;
4. The method is robust even when the optimizer fails to provide an adequate solution in time, since at least the feasible solution known from the beginning can be used.

In this presentation we will describe the main characteristics of the guidance logic control, acting as the basis controller, briefly introduce the framework for the sampled-data MPC and discuss the stability of each of the control terms and of the overall system. Finally, we will discuss its application on an UAV with bounded inputs to properly illustrate the proposed methodology.

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A data-driven method for formation control based on quadratic programming

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Accurate dynamics models are necessary for optimal control of robotic systems. Classical modeling methods are usually based on parametric approaches where the model structure must be known a priori and unknown parameters are computed or estimated. Sometimes it is very challenging to find a suitable parametric model a priori and, as a consequence, classical modeling approaches become often very time-consuming or even unfeasible. On the other hand, data from the systems can easily be collected due to the advancements in information and storage technology, so this data can be used to enhance the modeling and control of complex systems.

Moreover, in the context of robotics many systems can be modelized as Lagrangian systems. Some advances have been made in Lagrangian dynamics modeling using data, as for instance in [5], where the authors studied the problem of finding the dynamics of a system when it is governed by a Lagrangian dynamics and the data is provided by a single trajectory, by solving a nonlinear optimization problem.

In our case, the interest is focused on multi-agent control systems. In particular, decentralized control strategies for multiple robotic systems have gained attention in the last decades. Among the vast literature on decentralized formation control, two main approaches can be distinguished: position-based and distance-based formation control. Distance-based formation control focuses on stabilizing inter-agent distances based on the rigidity theory, which allows controlling specific geometrical shapes described by the group of agents provided that they have access to the relative positions of their neighbors [2], [4]. Such formations are useful in cases where agents have no shared knowledge of global coordinates. In [3], a more general class of systems has been considered by describing the dynamics of agents in the formation through a Lagrangian function together with non-conservative (dissipative) forces.

In this talk we extend the results given in [1] to multi-agent systems. Hence, we propose in [6] a method based on quadratic programming with linear constraints to approximate the Lagrangian function which describes the dynamics for a distance based formation control problem. Specifically, we use available data from positions, velocities and accelerations of the agents to construct an (unknown) Lagrangian function whose forced Euler-Lagrange

[8] Manuela Gamonal acknowledges the financial support from the Spanish Ministry of Science and Innovation, under grants PID2019- 106715GB-C21, MTM2016-76702-P, the "Severo Ochoa Programme for Centres of Excellence"in R&D (CEX2019-000904-S)

[9] The project that gave rise to these results received the support of a fellowship from "la Caixa" Foundation (ID 100010434). The fellowship code is LCF/BQ/PI19/11690016. The authors also acknowledge financial support from the Spanish Ministry of Science and Innovation, under grants PID2019- 106715GB-C21, MTM2016-76702-P, the "Severo Ochoa Programme for Centres of Excellence"in R&D (CEX2019-000904-S) and I-Link Project (Ref: linkA20079) from CSIC (CEX2019-000904-S).

equations approximate the solutions of the unknown Lagrangian system.

The proposed problem can be solved as a quadratic optimization problem, and therefore the solution can be found very quickly. This solution allows to construct a decentralized control law for the formation control problem based on data.

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Distributed Optimal Control Problems for Allocation of a Common Resource: Application to Temperature Control of Buildings

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We consider the problem of determining the optimal aggregate energy consumption for a group of thermostatically controlled loads, such as air conditioners, while satisfying thermal comfort constraints of each building. We formulate this problem in a continuous-time optimal control framework, and apply the maximum principle to analytically characterize the structure of its solution. The characteristics of the problem – linear dynamics and an isoperimetric constraint – imply that the multiplier associated with the isoperimetric constraint is constant and can be interpreted as shadow price of the resource – the total energy consumed – to be distributed among the several buildings. Moreover, given the shadow price, the optimal strategy to be followed at each building can be decided locally. In the special case, when the price forecast is monotone and the loads have equal dynamics, we show that it is possible to determine the solution in an explicit form. The non-monotone price case is handled by considering several subproblems, each corresponding to a time subinterval where the price function is monotone, and then allocating to each subinterval a fraction of the total energy budget. By doing so, at each time subinterval, the problem reduces to a simple convex optimization problem with a scalar decision variable, so the resulting optimization problem can be efficiently solved.

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Variational Collision Avoidance on Riemannian manifolds

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In this talk I will present a path planning strategy with collision avoidance for multi-agent systems evolving on complete Riemannian manifolds from variational principles. That is, paths satisfying some prescribed boundary conditions are constructed by minimizing some energy functional, among a set of admissible curves, which depends on an artificial potential function used to avoid collision between the agents.

Necessary conditions for extrema are derived, and the existence of global minimizers to the variational problem is demonstrated. I will provide conditions on the artificial potential under which it is possible to ensure that agents will avoid collision within some desired tolerance, and define a family of potentials that permit collision avoidance within arbitrary tolerances (constrained only by the geometry of the underlying manifold and the desired boundary conditions). I will further address the problem where trajectories are constrained by uniform bounds on their derivatives, and will derive alternate safety conditions for collision avoidance in terms of these bounds.

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[11] The project that gave rise to these results received the support of a fellowship from "la Caixa" Foundation (ID 100010434). The fellowship code is LCF/BQ/DI19/11730028.

[12] The project that gave rise to these results received the support of a fellowship from "la Caixa" Foundation (ID 100010434). The fellowship code is LCF/BQ/PI19/11690016. The authors also acknowledge financial support from the Spanish Ministry of Science and Innovation, under grants PID2019- 106715GB-C21, MTM2016-76702-P, the "Severo Ochoa Programme for Centres of Excellence"in R&D (CEX2019-000904-S) and I-Link Project (Ref: linkA20079) from CSIC (CEX2019-000904-S).

Sub-Riemannian problems on Lie groups

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We consider sub-Riemannian problems on connected and compact Lie groups, which consist of finding the shortest curve that connects two given points and is tangent to a given distribution. These problems can be tackled using a Lagrangian approach, but instead they will be formulated as optimal control problems and solved using the geometric version of the Pontryagin's Maximum Principle (PMP) adapted to Lie groups. The particular case when the curve is required to be horizontal will be presented in detail. The projection of these special curves to manifolds on which the Lie group acts transitively have interesting properties that will be highlighted.

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[13] The author acknowledges Fundação para a Ciência e Tecnologia (FCT) and COMPETE 2020 program for the financial support to the project UIDB/00048/2020.

Non-smooth optimal control applied to irrigation: analytical analysis and illustrative results

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We consider an optimal control problem to determine the minimum quantity of water used for irrigation of a certain agriculture crop. Our optimal control problem is an adaptation of one introduced in [1]; we do not consider an upper limit with respect to state variable x which represents the quantity of water in the soil. The dynamic of such problem is described via field capacity modes, allowing to better describe the variation of water in the soil, since its behaviour depends on the field capacity. The formulation also guarantees that the field crop is kept in a good state of preservation and growth, by considering that $x(t) \geq x_{\min}$ for all instant of time t under study, where $x_{\min}$ is the hydrological need of the crop. So, the quantity of irrigation water introduced in the soil, translated by control variable u, is positive whenever $x(t) = x_{\min}$.

The dynamic via fields capacity modes yields the non-smoothness of the optimal control problem. Consequently, the study of the analytical solution to this problem is not trivial. We consider several scenarios covering different state profiles; those are shown in Figure 1. Note that the state, x, does not have to be necessarily represented by a composition of line segments. The plots of Figure 1 are only schemes to illustrate the main idea of the eight possible behaviours related with x. Applying the corresponding optimality necessary conditions (see [2, Theorem 9.3.1]), we construct the analytical state, control and multipliers for each scenario of Figure 1. We present here scenarios with only one boundary arc (see Figure 1), because if we obtain a state trajectory, associated with x, with more than one boundary arc, we only have to do a composition of the analysed scenarios and proceed to a

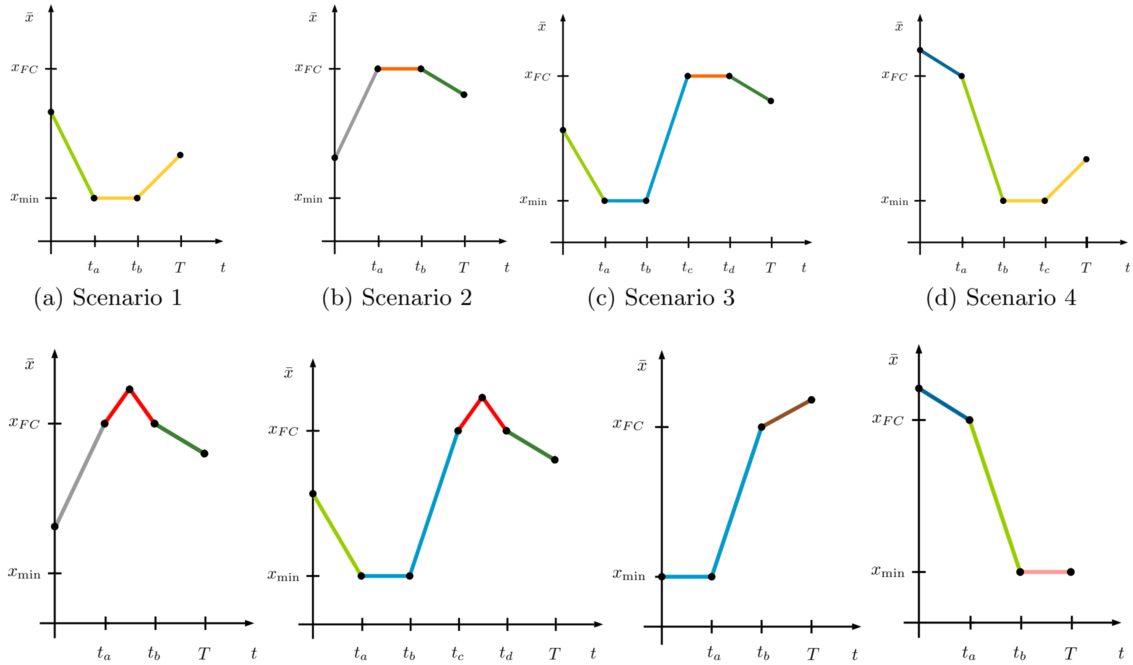
[14] Lemos-Paião is supported by the FEDER/COMPETE/NORTE2020/POCI/FCT funds, under project To CHAIR - POCI-01-0145-FEDER-028247, and the Portuguese Foundation for Science and Technology in the framework of the Strategic Funding through CFIS and CIDMA, under projects UID/FIS/04650/2019 and UIDB/04106/2020, respectively.

[15] Lopes is supported by the FEDER/COMPETE/NORTE2020/POCI/FCT, under project To CHAIR - POCI-01-0145-FEDER-028247, and the Portuguese Foundation for Science and Technology in the framework of the Strategic Funding through CFIS and SYSTEC, under projects UID/FIS/04650/2019 and UID/EEA/00147/2019, respectively.

[16] de Pinho is supported by the the FEDER/COMPETE/NORTE2020/POCI/FCT funds, under project To CHAIR - POCI-01-0145-FEDER-028247, and the Portuguese Foundation for Science and Technology in the framework of the Strategic Funding through SYSTEC, under project UID/EEA/00147/2019.

composed analytical study. Furthermore, in order to avoid repetitive calculations along all our work, the trajectories of Figure 1 with similar analytical solutions, with respect to state variable x and to respective adjoint function, as well as to control variable, are drawn with the same colour.

We end our work by solving some numerical examples with the purpose to illustrate and partially validate the computational results.



(a) Scenario 1 (b) Scenario 2 (c) Scenario 3 (d) Scenario 4

(e) Scenario 5 (f) Scenario 6 (g) Scenario 7 (h) Scenario 8

Figure 1: Schemes of the eight possible solutions with respect to the quantity of water in the soil, where $x_{\min}$ is the hydrological need of the crop and x_{FC} is the water's amount retained in the soil, after it was drained. Note that state trajectory, associated with state variable x, does not have to be represented necessarily by a composition of line segments. The plots above are only schemes to illustrate the main idea of the eight possible behaviours related with x.

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Vaccination dynamics against morbidity risks

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For diseases in which vaccination is not compulsory, individuals take into account different aspects when deciding between to vaccinate or not. Namely, the decision depends on the morbidity risks from both vaccination and infection, and also depends on the probability of being infected, which varies with the course of the disease and the decisions of other individuals.

Using some basic game theoretical concepts, we study the evolution of the individual vaccination strategies depending upon the morbidity risks and upon the parameters of the basic reinfection SIRI model. In [1], it was introduced the evolutionary vaccination dynamics for an homogeneous vaccination strategy of the population, where the individuals change their strategies along time, such that their payoffs increase. Here, we also introduce the dynamical evolution of the morbidity risks, and we analyze the changes provoked on the vaccination strategy of the population.

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Constrained higher-order variational calculus for the optimal control of multi-agent mechanical systems

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We consider multiple Lagrangian systems evolving on a Lie group as in [1] and we describe geometrically a path planning method for multi-agent (mechanical) systems as an optimal control problem. The optimal control problem is understood as a constrained higher-order variational problem and necessary conditions are obtained via the method of Lagrange multipliers. The proposed method, which is inspired by [2] and [3], is illustrated with an application for the shape control with the flocking behavior of Lagrangian models of drones interpolating the dynamics for the centroid of the desired formation.

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Adaptive optimal control of planar biped locomotion

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Bipedal robots [1] currently attract increasing attention, not only because they provide rich test-beds that raise difficult problems and allow the demonstration of advanced control strategies [2], but also as technology components *per se*, seen as sophisticated machines able to perform complex tasks, like the Boston Dynamics Biped robots [3]. Although rigid bodies based dynamics is useful, the assumption that a biped is composed by rigid bodies connected via joints is often a harsh simplification. Flexibilities can be introduced by including additional terms, *e.g.* second order models. Furthermore, mechanical wear can be modelled by additional terms, including nonlinearities (dead-zones, hysteresis, etc). While such terms can be defined using *a priori* information, *e.g.* defined from the mechanical design specifications, in general there will be uncertainties that motivate the use of adaptive control, and an approach in which an initial set of controller gains is updated from on-line observations to learn the dynamics and improve controller performance.

Dealing with the uncertainties in biped locomotion control can be done in different ways. The time propagation of a performance index, in the form of Q-function, using approximate dynamic programming, and its optimization according to a reinforcement learning approach [5], is an alternative to model-based control strategies, resulting directly from Lyapunov analysis and even incorporating robustness terms (see for instance [7]).

In this work, a biped model, developed according to known techniques [4] is used to illustrate adaptive optimal joint control for gait movement of a planar biped robot. The controller relies on a Q-learning algorithm [5], fed with the current state and outputting a correction term that adjusts *a priori* applied controller gains. The Q-functions of the different joint controllers are approximated by linear combinations of quadratic functions. A directional forgetting parameter estimation algorithm has been used to tackle identification problems. Another feature embedded in the control algorithms is the inclusion of a feedforward term from accessible dynamics, that allows to coordinate the action of interacting joints. Furthermore, the velocity algorithm to include integral action, described in [6], and modified in

[18] Pluriannual ISR funding UID/EEA/50009/2013
[19] Pluriannual INESC-ID funding UIDB/50021/2020

order to provide the data for Q-.function regressor identification has been used. The presentation describes the biped model and focus on the above modifications of the basic Q-learning adaptive optimal controller that were found to be crucial for the problem considered.

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Maximum Principle for Bilevel Sweeping Control Problems

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This talk concerns a recent approach to necessary conditions of optimality in the form of a Maximum Principle in Gamkrelidze's form, [1, 2] for bilevel optimal control problem exhibiting sweeping processes at the lower level, [3]. Examples of applications of bilevel dynamic optimization problems, featuring control sweeping processes arise in motion of crowds organized in groups such as teams of robots, motion of groups of pedestrians in a confined space, advanced nanoferro-electric technologies for functional improvement of mobile electronic devices, among many others, [4, 5, 6, 7].

This optimal control paradigm brings together two challenging classes of optimal control problems: sweeping control processes featuring set-valued dynamics that are not Lipschitz continuous in the state variables, [8, 9], and bilevel optimization whose flattening entails constraints that typically are not endowed with reasonable regularity conditions, notably well established constraint qualification conditions, [10]. The approach to these challenges, as well the role played by Gamkrelidze's form will be discussed in detail. For now, we provide just the formulation of the problem to be discussed.

Let $N \in \mathbb{N}$ and Q_i, $i = 1, \ldots N$, and Q be closed spheres in $\mathbb{R}^n$ with radius R_i and R, centered at q_i and q, respectively, with $R \gg R_i$. Denote (y, v), taking values in $\mathbb{R}^{Nn \times Nm}$, and (x^i, u^i), taking values in $\mathbb{R}^{n \times m}$, by the control processes, respectively, for the higher level optimal control problem, and for the lower level control problem i. The articulation among the higher level problem and the lower level problem is made via the relations $y(t) \in Q$, $x_i(t) \in Q_i + y^i(t) \subset Q$, and $Q_i + y^i(t) \cap Q_j + y^j(t) = \emptyset$ for $i \neq j$. We have that $y^i(t)$ takes values in $\mathbb{R}^n$ and $y = col(y^1, \ldots, y^N)$.

The higher level and the lower level optimal control problems are described as follows:

$$\begin{array}{ll} (P_H(x,u) & \text{Minimize } g_H(y,T) \\ \text{subject to} & \dot{y}(t) = f_H(yt), v(t)) \quad \text{a.e. in } [0,T] \\ & y(0) = y_0 \in Q \subset \mathbb{R}^{nN}, \quad y(T) \in E \subset \partial Q \\ & v \in \mathcal{V} := \{v \in L^1([0,T]; \mathbb{R}^{mN}) : v(t) \in V\} \\ & Q_i + y^i(t) \subset Q \quad \Psi^i_L(T, y^i) \neq \emptyset \end{array}$$

Here, y_0 is given, $g_H : \mathbb{R}^{nN} \times \mathbb{R} \to \mathbb{R}$, $f^i_L : \mathbb{R}^{nN} \times \mathbb{R}^{mM} \to \mathbb{R}^{nM}$, $V \subset \mathbb{R}^{mN}$ is compact, ∂Q is the boundary of Q, E is closed and connected, $(x, u) = col((x^1, u^1), \ldots, (x^N, u^N))$, and

$$\Psi^i_L(T, y_i) := \{(x^i, u^i) \text{ feasible process for } (P_L(T, y^i))\}.$$

Unlike $(P_H(x, u))$, when the state variable of the low level problems $(P^i_L(T, v))$ hits the

[20] Thanks the support of SYSTEC, and of MAGIC, and Harmony FCT R&D projects

boundary the sweeping phenomenon may arise.

$$\begin{array}{ll} (P_L^i(T, y^i)) & \text{Minimize } g_L^i(x^i(T), T) \\ \text{subject to} & \dot{x}^i(t) \in f_L^i(x^i(t), u^i(t)) - N^{M_i}_{Q_i + y^i(t)}(x^i(t)) \quad \text{a.e. } t \in [0, T] \\ & x^i(0) \in Q_i + y_0^i, \quad u^i \in \mathcal{U}^i := \{u \in L^\infty([0, T]; \mathbb{R}^m) : u^i(t) \in U^i\} \\ & x^i(t) \in Q_i + y^i(t) \; \forall t \in [0, T], \end{array}$$

where $g_L^i : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, $f_L^i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $U^i \subset \mathbb{R}^m$ is compact, the truncated cone $N_A^M(z) := N_A(z) \cap MB(0)$, being $N_A(z)$ the Mordhukovich normal cone to the set A at point z, [11], $B(0)$ the unit ball at the origin, and the $M_i > 0$ are given constants.

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Interpolation on the Essential Manifold using Rolling

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Interpolation problems involving data on non-Euclidean spaces is a hot topic in a growing number of applications in different areas of our society, ranging from computer vision and robotics, to industrial and medical applications, and research in these areas has been booming over the past few years.

For instance, the problem of recovering structure and motion from a sequence of images, also known as stereo matching, is a crucial problem in computer vision that continues to be one of the most active research areas with remarkable progress in imaging and computing hardware. We refer, to Bressan [1], Ma et al. [6], Szeliski [8] and references therein for details concerning multiple applications.

Such examples served as a motivation for our study and the main goal of this talk is to present an approach for solving interpolation problems on the Generalized Essential manifold, which is entirely based on rolling motions. That manifold is the product of the Grassmann manifold $G_{k,n}$, consisting of all k-dimensional linear subspaces of the real n-dimensional Euclidean space, and the Lie group of rotations SO_n. One particular case of that is the Normalized Essential manifold, corresponding to $k = 2$ and $n = 3$, which plays an important role in image processing. The classical problem of reconstructing a scene, or a video, from several images of the scene can be formulated as an interpolation problem on that manifold, since it encodes the epipolar constraint. Typically, it is given an ordered set of time-labelled essential matrices, $E_1, \ldots, E_j$, relating j different consecutive camera views (*snapshots*), and the aim is to calculate a continuum of additional virtual views by computing a smooth interpolating curve through the E_i's, $i = 1, \ldots, j$.

The basic idea of the present approach, which was inspired by the work of Hüper [5], is to project the data from the original non-Euclidean manifold to a flat manifold, using rolling and unwrapping techniques, solve the problem in that simpler manifold, and then reverse the process to obtain the interpolation curve on the original manifold. This approach has obvious advantages over others methods, in particular those that are variational in nature, such as, Crouch and Silva Leite [3], Camarinha [2] and Grohs [4], since it produces an interpolating curve that is given in closed form, as opposed to highly nonlinear differential equations that are extremely hard to solve. More details can be found in [7].

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The replicator dynamics approach: democracy and corruption

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In this paper we propose a game theoretic model with three populations, namely a government, officials who serve the state, and citizens, to analyse the evolution of corruption in a society. The influence of democracy in corruption is modelled through the action of the citizens who exercise influence in the government because of their elective power since corruption causes a great displeasure in the citizens which can result in a vote against a ruler elite that promotes or is an accomplice to corruption. When immersed in a society in which corruption is a common occurrence, citizens may behave in a complacent manner with corruption because of a lack of valid alternatives to this behaviour even if they oppose corruption. Indeed, this complacent behaviour may also be observed in democratic societies and can lead to periods of growing and diminishing corruption. We are thus able to get a better understanding of some causes for the evolution of corruption and how the evolution may be halted and the effects of democracy and influence in this.

[22]The authors thank the financial support of LIAAD-INESC TEC and FCT-Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) within the project PTDC/MAT-APL/31753/2017.

An optimal control problem for a non-autonomous predator-prey model with disease in the prey

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In this paper we consider an optimal control problem for a non autonomous predator-prey model with disease in the prey. Existence and uniqueness of the solution and the optimal control is proved by using Pontryagin's Optimality Principle. The aim is to minimize the number of infected elements in the population at the final time period. This is done by separating the infected and non infected preys. In order to achieve this objective, two controls are then applied to the system: the first one, a separating control, is intended to separate the non infected from the infected prey population, and the second control serves as a treatment control that is used to decrease the rate of death caused by the disease. Numerical simulations are provided. The numerical simulation shows that there must not be any contact between the healthy and infected elements of the prey population. The second control that serves as a treatment must be applied during part of the time, that is, it is not applied during the entire time interval. This work is a joint work with César M. Silva (email csilva@ubi.pt).

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[23] P. Rebelo was partially supported by FCT through CMUBI project UIDP/00212/2020
[24] S. Rosa was partially supported by FCT through Instituto de Telecomunicações (project UID/EEA/50008/2019)

State estimation for semilinear parabolic equations

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Part of this talk is based on recent work [1], where the estimation of the full state of a nonautonomous semilinear parabolic equation is achieved by a Luenberger type dynamical observer. The estimation is derived from an output given by a finite number of average measurements of the state on small regions. The state estimate given by the observer converges exponentially to the real state, as time increases. The result is semiglobal in the sense that the error dynamics can be made stable for an arbitrary given initial condition, provided a large enough number of measurements, depending on the norm of the initial condition, is taken. The output injection operator is explicit and involves a suitable oblique projection. The results of numerical simulations are presented showing the exponential stability of the error dynamics. At the end of the talk we shall also discuss "the" optimal placement of the sensors, in other words, "the" best locations for the measurements.

As an illustration, let us be given a scalar parabolic equation as

$$\tfrac{\partial}{\partial t}y + (-\Delta + \mathbf{1})y + ay + b\cdot\nabla y - y^3 = f, \qquad \mathcal{G}y|_{\partial\Omega} = g, \tag{1}$$

evolving in a bounded, convex polygon (polyhedron) $\Omega \subset \mathbb{R}^3$. The functions $a = a(t,x) \in \mathbb{R}$, $b = b(t,x) \in \mathbb{R}^3$, $f = f(t,x) \in \mathbb{R}$, and $g = g(t,x) \in \mathbb{R}$ are given and known, for $(t,x) \in [0,+\infty) \times \Omega$. By $\mathbf{1}$ we denote the identity operator, and $\mathcal{G}$ stands for either Dirichlet or Neumann boundary conditions, $\mathcal{G} \in \{\mathbf{1}, \frac{\partial}{\partial \mathbf{n}}\}$.

The state $y = y(t,x)$ is unknown. In particular, the initial state $y(0,x)$ is unknown.

The main goal is to obtain an estimate $\widehat{y} = \widehat{y}(t,x)$ for the state $y = y(t,x)$.

Obtaining an estimate $\widehat{y}$ for the state y is of interest for applications, for example, in the implementation of feedback controls, where the control u is a function of the state as $u = K(y)$. Since y is usually unknown, the approximated feedback $\widehat{u} = K(\widehat{y})$ is used instead.

To construct such a state estimate $\widehat{y}$ we will use a Luenberger observer based on a finite number of sensor measurements. As sensors we take the indicator functions in the set

$$W_S := \{\mathbf{1}_{\omega_{S,i}} \mid 1 \le i \le S\} \subset L^2(\Omega), \qquad \mathcal{W}_S = \operatorname{span} W_S, \qquad \dim \mathcal{W}_S = S, \tag{2}$$

where $\omega_{S,i} \subseteq \Omega$, and we take average-like measurements as

$$w_{S,i}(t) := (\mathbf{1}_{\omega_{S,i}}, y(t,\cdot))_{L^2} = \int_{\omega_{S,i}} y(t,x)\,\mathrm{d}x.$$

The vector $w(t) \in \mathbb{R}^{S\times 1}$, $w(t) = \begin{bmatrix} w_{S,1}(t) \\ w_{S,2}(t) \\ \vdots \\ w_{S,S}(t) \end{bmatrix} =: \mathcal{Z}_S y$ is called the output.

[25]The author is supported by ERC advanced grant 668998 (OCLOC) under the EU's H2020 research program. The author also acknowledges partial support from the Austrian Science Fund (FWF): P 33432-NBL.

The result. For an arbitrary pair (μ, R) of positive constants. the estimate $\widehat{y}(t)$ given by the Luenberger observer/estimator

$$\tfrac{\partial}{\partial t}\widehat{y} + (-\nu\Delta + \mathbf{1})\widehat{y} + a\widehat{y} + b\cdot\nabla\widehat{y} - \widehat{y}^3 = f + \mathcal{I}(\mathcal{Z}_S\widehat{y} - \mathcal{Z}_S y), \quad \mathcal{G}\widehat{y}|_{\partial\Omega} = g, \tag{3a}$$

with an output injection operator $\mathcal{I}$ in the explicit form

$$\mathcal{I} := -\lambda A^{-1} P_{\mathcal{W}_S}^{\widetilde{\mathcal{W}}_S^\perp} A^2 P_{\widetilde{\mathcal{W}}_S}^{\mathcal{W}_S^\perp} \mathbf{Z}_S, \tag{3b}$$

converges exponentially to the state $y(t)$ as time increases, as

$$|\widehat{y}(t,\cdot) - y(t,\cdot)|_{H^1(\Omega)} \le \mathrm{e}^{-\mu t} |\widehat{y}(0,\cdot) - y(0,\cdot)|_{H^1(\Omega)}, \quad t \ge 0, \tag{4}$$

provided that $|\widehat{y}(0,\cdot) - y(0,\cdot)|_{H^1(\Omega)} \le R$ and that $\lambda > 0$ and $S \in \mathbb{N}_+$ are both large enough. Here $\widehat{y}(0,\cdot)$ can be chosen/set as an initial guess we might have for $y(0,\cdot)$, S is the number of appropriately placed sensors (e.g., as in [2]), and $\widetilde{\mathcal{W}}_S$ is an "appropriate" auxiliary space (e.g., the span of "regularized" actuators).

The operator $\mathbf{Z}_S$ is defined by $P_{\mathcal{W}_S}^{\mathcal{W}_S^\perp} z = \mathbf{Z}_S \mathcal{Z}_S z$, and P_X^Y denotes the oblique projection in $L^2(\Omega)$ onto X along Y.

Finally, note that the fact that the output injection operator $\mathcal{I}$ is explicitly given, makes it attractive for applications.

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Modelling optimal control of fractional order Cholera model

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A Caputo type fractional-order mathematical model for the metapopulation cholera transmission was proposed in [1]. In this work, a sensitivity analysis of model is done to show the accuracy relevance of parameter value estimation. A novel fractional optimal control problem (FOCP) is formulated and numerically solved. A cost-effectiveness analysis is performed to assess the relevance of studied control measures. Such analysis assess the cost and the effectiveness of those control measures during the intervention. Effective measures are what we are looking for in order to control the disease. Hence, ineffective measures are discarded. Finally, a system that combines the classical system with the fractional order system, a variable-order fractional system, is proposed. Such variable-order fractional system showed to be effective in the control of the disease, when compared with integer and fractional order systems.

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[26]Supported by FCT through IT, reference UID/50008/2020.
[27]Supported by FCT through CIDMA, reference UIDB/50008/2020.

Reduction for symmetry breaking cost functions in the optimal control of multi-agent systems.

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In this meeting, we present our work on symmetry reduction of optimal control problems for affine multi-agent control systems on Lie groups, with partial symmetry breaking cost functions. More precisely, cost functions that break some of the symmetries but not all. We develop the reduction for optimality (necessary) conditions in the optimal control problem.

In particular, we advance on the results of [1] and [2] by extending the reduction for optimality conditions in the optimal control problem to multi-agent systems with cost functions that are not left invariant under the action of the entire Lie group, but it is invariant under the action of a subgroup of the Lie group. This occurs for instance when we consider obstacles in the workplace that agents should avoid while they also avoid mutual collision with barrier functions. Our approach emphasizes the role of variational principles. Specifically, we recast the optimal control problem as a constrained variational problem with a partial symmetry breaking Lagrangian. We restore the symmetry of the system so that we can take advantage of the symmetry provided by Lie groups and we obtain the Euler-Poincare equations from a variational principle.

Furthermore, by using the Legendre transformation, we obtain a reduced Hamiltonian function and we derive the Lie-Poisson equations by the Lie-Poisson bracket in the same context and the reduction of the Pontryagin maximum principle. This also helps to show the connection between the Euler-Poincare and Lie-Poisson equations. We illustrate the employment of the results with an example of three unicycles avoiding collisions and obstacles.

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[28]The project that gave rise to these results received the support of a fellowship from "la Caixa" Foundation (ID 100010434). The fellowship code is LCF/BQ/DI19/11730028. The authors also acknowledge financial support from the Spanish Ministry of Science and Innovation, under grants PID2019-106715GB-C21, MTM2016-76702-P, the "Severo Ochoa Programme for Centres of Excellence"in R&D (CEX2019-000904-S) and I-Link Project (Ref: linkA20079) from CSIC (CEX2019-000904-S).

Model Predictive Control and Adaptive Kalman Filter design for p53 recovery

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Cancer remains one of the most prominent causes of death around the globe. p53, a tumor suppressor protein, plays an essential role in protecting the genomic integrity of cells during DNA damage. Hence, rightfully called the "guardian of the genome" [1]. In its wild-type form, p53 transcribes various genes involved in DNA damage repair, senescence, apoptosis, and cell cycle arrest. This cell fate information is encoded in the dynamics of p53. The response patterns of p53 are broadly classified into sustained and oscillatory responses. p53 oscillatory response leads to either DNA repair or the cell cycle arrest, while its sustained response leads to permanent cell death. Considering the vital role of p53 in regulating cellular mechanisms, precise control of its level is crucial. In about 50% of cancer cases, it has been observed that p53 is either inactivated or mutated. The inactivation of p53 is attributed to the hyperactivity of its negative regulator protein Mdm2. Hence, the most promising strategy to elevate levels of p53 is by preventing the interaction of Mdm2 with p53 [2]. In the literature, various small molecules-based drugs have been investigated that block the p53-Mdm2 interactions. Nutlin is one such drug that has the potential to restore p53 to the desired levels and inhibit the cancerous cell growth in a dose dependent manner [3].

In this research work, a nonlinear model of p53 pathway (cf. [3]) is employed to design a model predictive control (MPC) which administers the dosage of Nutlin to revive the concentration of p53 protein. The nonlinear programming (NLP) problem is solved such that the p53 concentration tracks a desired trajectory in both of its dynamic behaviors, i.e., sustained and oscillatory responses. Furthermore, the physical constraints on the states and input are also satisfied. In order to make the model-based control design possible an adaptive Kalman filter (AKF) is designed to obtain the estimates of the unmeasured states. Moreover, AKF is also employed to estimate an input disturbance to the p53 pathway. The design of MPC and AKF is based on the nominal model, however, to assess the robustness of the control scheme, parametric variations are introduced in the p53 plant. The performance of the designed control scheme is also compared with already developed techniques in the literature. The simulation results and the quantitative analysis show that the proposed control scheme yields better performance in terms of tracking error and control energy utilization.

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Gradient feedback stabilization of fractional diffusion systems: Decomposition approach

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We consider the state gradient stabilization concept of fractional time diffusion systems. We present sufficient conditions to achieve the gradient Mittag–Leffler and strong stability. Then, we characterize some feedback controls that ensure the state gradient stabilization of fractional time diffusion systems using the decomposition method. Our analytical results are performed through various examples and simulations.

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