Optimal Reinsurance of Dependent Risks

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The management of risk has always been the business of insurers. Reinsurance consists in transferring part of the liabilities of the insurance company to another insurer, the reinsurer. By doing so, the ceding insurer company protects itself from the risk of large variations in its undertaken liabilities, thus reducing the amount of capital needed to cover them. Hence, reinsurance is a risk mitigating tool, usually reducing minimum regulatory capital requirements, and an important instrument in the management of risk of an insurance company. Of course, reinsurance has a price, the reinsurance premium, and reinsuring all the risk is not an option as in such case the insurer would be just an intermediary. In addition, there are several forms of reinsurance. So, when an insurance company decides to reinsure part of its risk, at least two questions arise "What form of reinsurance is optimal?" and "How much reinsurance is optimal?". Answers to these questions constitute solutions to the optimal reinsurance problem, and clearly depend on the underlying risk and on the reinsurance premium. Moreover, from the vast range of literature intended for the financial and insurance community, it is widely accepted that dependencies play a determinant role in risk assessment and management, namely when analysing the optimal reinsurance problem. The introduction of dependencies strongly increases the complexity of the optimal reinsurance problem from the mathematical view point.

We study the optimal reinsurance problem for two dependent risks, from the point of view of the ceding insurance company. The problem consists in finding the optimal reinsurance treaty, for each risk, that maximizes the expected utility or the adjustment coefficient of the total wealth of the insurer. It is known that these two criteria are connected and moreover the adjustment coefficient is related to the ultimate probability of ruin of the insurer through the Lundberg inequality.

First we consider a combination of two types of reinsurance, quota-share and stop loss treaties, and assume depencies between the two risks are modeled by means of copulas. Copulas constitute a convenient and elegant mathematical framework to describe dependencies between two or more random variables where the joint distribution function is expressed as a parametric function of the marginal distribution functions. Due to the complexity introduced by dependencies, results are obtained numerically. Sensitivity of the optimal reinsurance strategy to

several values of the dependence parameter, to different distributions of the underlying risks and to a variety of reinsurance premium calculation principles are performed in three families of copulas describing different tail behaviours of the joint distribution function.

Afterwards we assume a generic dependecy structure, so that the optimal solution depends on the joint distribution. Due to dependencies, the optimal level of reinsurance for each risk involves a trade-off between the reinsurance premia of both risks. We study the shape of this trade-off and characterize the optimal treaties.