Well-posedness for a higher-order, nonlinear, dispersive equation: a new approach

April 5, 2019

Colette GUILLOPÉ

In collaboration with Hongqiu CHEN and Jerry L. BONA

A class of higher-order models for unidirectional water wave of the form

$$\begin{aligned}\eta_t + \eta_x - \gamma_1 \beta \eta_{xxt} + \gamma_2 \beta \eta_{xxx} + \delta_1 \beta^2 \eta_{xxxxt} + \delta_2 \beta^2 \eta_{xxxxx} \\ + \tfrac{3}{4}\alpha(\eta^2)_x + \alpha\beta\Big(\gamma(\eta^2)_{xx} - \tfrac{7}{48}\eta_x^2\Big)_x - \tfrac{1}{8}\alpha^2(\eta^3)_x = 0\end{aligned} \tag{1}$$

was derived by Bona, Carvajal, Panthee and Scialom [1]. With appropriate choices of the parameters $\gamma_1, \gamma_2, \delta_1, \delta_2$ and γ, this equation serves as a model for the propagation of small-amplitude, long-crested surface waves moving to the direction of increasing values of the spatial variable x. Here α is a typical ratio of wave amplitude to depth, β is a representative value of the square of the depth to wavelength and t is proportional to elapsed time. The dependent variable $\eta = \eta(x,t)$ is a real-valued function of $x \in (-\infty, \infty), t \geq 0$ representing the deviation of the free surface from its undisturbed position at the point corresponding to x at time t. This model subsists on the assumptions that α and β are comparably-sized small quantities while η and its first few partial derivatives are of order one. Moreover, γ_1 and γ_2 are restricted by $\gamma_1 + \gamma_2 = \frac{1}{6}$.

The new result is that when $\gamma = \frac{7}{48}$, $\delta_2 > \delta_1 > 0$ and the initial data

$$\eta(x,0) = \eta_0(x,0) \tag{2}$$

lies in H^1 and not too big, then the initial-value problem of (1)-(2) is globally well posed and the H^1-norm of the solution is uniformly bounded for $t \geq 0$.

References

[1] J. L. Bona, X. Carvajar, M. Panthee and M. Scialom, Higher-order Hamiltonian model for unidirectional water waves, *Journal of Nonlinear Science,* **28** *(2018), no. 2, 543-577.*

(J. L. Bona) Department of Mathematics, Statistics, and Computer Science, The University of Illinois at Chicago, 851 S. Morgan St. MC 249, Chicago, IL 60607,

United States
E-mail address: **bona@math.uic.edu**

(Hongqiu Chen) University of Memphis, Department of Mathematical Sciences, Memphis, TN 38152, United States
E-mail address: **hchen1@memphis.edu)**

(C. Guillopé) Laboratoire d'Analyse et de Mathématiques appliquées, CNRS et Université Paris-Est Créteil, 61 avenue du Général de Gaulle, 94010 Créteil Cedex, France
E-mail address: **colette.guillope@u-pec.fr**