Brownian Motion on a n-Dimensional Sphere

Technological Educational Institute of Peloponnese
Antikalamos
24100, Kalamata, Greece.
e-mail: dimkouloubou@teipel.gr

Abstract

We evaluate explicity certain quantities regarding the Brownian motion process on the n-dimensional sphere of radius a. We start with the transition densities of the process. Then we calculate some probabilistic quantities (e.g. moments) of the exit times of specific symmetric domains. We also discuss the reflection principle on S_n. The reflection principle can help to calculate the distribution functions of certain exit times. We can apply this theory to provide a novel method to estimate in a closed-form solution the option prices of various exotic options. (Joint work with Vassilis G. Papanicolaou, Professor of Mathematics, National Technical University of Athens and Stelios Bekiros Professor AUEB / IPAG / EUI).

keywords

Brownian motion, n-dimensional sphere, transition density, Hitting times, Option pricing.

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