Heteroclinic Dynamics in Game Theory

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ABSTRACT

Game theory studies interactions among agents. The dynamics of these interactions may be modelled through differential equations. In the case of the Rock-Scissors-Paper (RSP) game played by two agents, the dynamics exhibit a heteroclinic network made of three heteroclinic cycles nearby which robust long-term complex behaviour takes place. In particular, the study of stability properties of such objects adds information to the dynamics. The stability of heteroclinic cycles may be obtained from the value of the local stability index along each connection of the cycle. We establish a method that, under one mild assumption, provides an expression for the values of the local stability index for *quasi-simple heteroclinic cycles*: cycles whose connections are one-dimensional and contained in flow-invariant spaces of equal dimension. We ilustrate our results with the cycles present in the heteroclinic network of the RSP game. These cycles are shown to be as stable as possible for a wide range of parameter values, depending on the players' payoff received for a tie. Using some applications to price setting models, we propose what could be a starting point for the contribution of the RSP game to the understanding of cyclic dominance in two-player games.